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SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

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ABSTRACT

The objective of the study was to observe the influence of shear span to effective depth ratio (a/d) on shear strength of reinforced concrete beams. Ten beams, of six inches wide and 12 inches deep with varying span of 6 ½' to 7 ½' without shear reinforcement were tested. The shear span to effective depth ratios (a/d) beams were ranging from 3.2 to 3.9. It was observed that all of the beams failed in shear with a crack pattern similar to the beams failing in diagonal tension. A statistical analysis was done using data from these experiments and an equation was developed to estimate the shear strength of RC beams without web reinforcement. Some data of other researchers was also included to raise the confidence level of statistical results.

INTRODUCTION

The prediction of stresses in concrete is difficult due to its heterogeneous nature. The inclusion of reinforcement further complicates the stress distribution. Regarding flexural strength of reinforced concrete elements a rational formula had been developed by different research workers for its estimation. However, the shear strength of reinforced concrete elements is still controversial [(1)]. A lot of research work has been done on the shear strength of reinforced concrete beams. The existing design codes assume that shear resistance of a beam to be provided by the sum of the resistances due to;

i) the concrete contribution, ii) the shear reinforcement.

The concrete contribution [(i) above] is taken as being equal to the shear force at the commencement of diagonal cracking. This concrete shear strength has been formulated by different researchers using equations fitting the experimental data and involving the major variables (parameters) affecting the shear strength such as reinforcement ratio, concrete strength, effective depth and shear span to effective depth ratio. While the format and importance given to the parameters differs somewhat from code to code. These variables are taken into account in different ways. These equations are as follows.

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i) **BS8110** [2]

$$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{1/3}(\rho)^{1/3}\left(\frac{400}{d}\right)^{1/4} \quad for\ a/d > 2.0 \quad \text{and} \qquad (1)$$

$$v_c = \left(2.0\frac{d}{a}\right)0.79\left(\frac{f_{cu}}{25}\right)^{1/3}(\rho)^{1/3}\left(\frac{400}{d}\right)^{1/4} \quad for\ a/d < 2.0 \qquad (2)$$

ii) **ACI CODE** [3] $v_c = 1.9\sqrt{f'_c} + 2500\rho\frac{Vd}{M} \le 3.5\sqrt{f'_c} \quad for\ l/d > 5.0 \qquad (3)$

$$v_c = \left(3.5 - 2.5\frac{M}{Vd}\right)\left[1.9\sqrt{f'_c} + 2500\rho\frac{Vd}{M}\right] \le 3.5\sqrt{f'_c} \quad for\ l/d > 5.0 \qquad (4)$$

iii) **EUROPEAN CODE 2 (EC2)** [4] $v_c = \tau_{Rd} K(1.2 + 40\rho) \qquad (5)$

where τ_{Rd} is the characteristic concrete strength and $K = 1.6d \le l$ and 'd' in meters.

iv) **CEB–FIP MODEL (90) CODE** [5]: $v_c = 0.15\left(\frac{3d}{d}\right)^{1/3}\left[1 + \sqrt{\frac{200}{d}}\right](100\rho f_{ck})^{1/3} \qquad (6)$

v) **Theodore Zsutty's Equation** [6] $v = 59bd(f'_c \rho d/a)^{1/3}$

Where a = shear span (portion of the span length subjected to maximum shear), v_c = shear stress causing diagonal cracking, b = width of the member, d = effective depth of tensile reinforcement, f_c' = cylinder strength of concrete, f_{cu} = cube strength of concrete, ρ = tensile steel ratio and (Vd/M) or (d/a) = the ratio of shear to moment multiplied by effective depth at location where shear is calculated.

It is clear from the above equations that differences exist between different design codes or practice for the shear strength. The effect of concrete strength, tensile reinforcement ratio, a/d ratio and the effective depth on the shear strength and the general format of the prediction equations differ from code to code. Each equation has its own merits and demerits. The purpose of this study was to collect some more experimental data and to propose an equation to predict shear strength of RCC beam having a/d ratio between 3.2 to 3.0.

SHEAR SPAN TO EFFECTIVE DEPTH RATIO (a/d)

The distance from the plane of nearest concentrated point to the plane of support is known as shear span (a). Shear strength is very sensitive to the range of values of shear span to effective depth ratio (a/d).

The effect of the shear span to effective depth ratio (a/d) on ultimate shear strength of the beam is well pronounced and is briefly described as below:

The shear valley (the plot of the ratio of ultimate shear strength to pure flexure strength versus a/d ratio) exists between a/d =1 to a/d = 7 as reported by different researchers.

There exists a minimum shear strength corresponding to a certain critical value of the a/d ratio [normally $(a/d)_{critical}$ = 2 to 3]. The other parameters such as reinforcement ratio affect the variations of the curve significantly. For example, there is a shift in the position of the valley at different levels of reinforcement ratio.

The shear strength increases very sharply due to arch action for lower values of a/d, whereas for the values of a/d greater than $(a/d)_{critical}$, the increase in strength is relatively slower. The curve is flatter for the values of a/d greater than 5 to 7.

ACI Code underestimates the effect of a/d, and CEB-FIP Model Code. Bazant's and Zsutty's equations are considered to predict relatively reasonable results. BS8110 ignores the effect of a/d ratio on shear strength for values of a/d ratio greater than 2. In flexure-shear interaction models, beam is considered to behave in two different ways
(i) Arch Action $(a/d)<(a/d)_{critical}$ (ii) Beam Action $(a/d) > (a/d)_{critical}$

Focus of this study is to see the effect of a/d ratio on shear strength of beams without web reinforcement. The ratio a/d for all the beams was greater than 3 to eliminate any chance of arch action to exist.

EXPERIMENTAL INVESTIGATION

The test specimens consisted of 10 beams rectangular in section without shear reinforcement. The web width was taken as 6" and height 12". The percentage of steel ratio varied from 0.52% to 1.47%. Two series of beams (A&B) were cast with lengths 7' for series 'A' and 8' for 'B'. The spans were 5'-6" and 6'-6" for series 'A' and 'B' respectively. The a/d ratio was in the range of 3.2 to 3.9. In order to improve the bond between steel and concrete and to prevent the slippage of reinforcement steel during test procedure, beam was extended 9" behind the supports. Deformed steel bars of Grade 60 (series A) and Grade 40 (Series B) were used as the tensile reinforcement.

Concrete mix ratio of 1:2:4 by weight was used with three different water cement ratios for achieving the specified concrete strength of 3000 psi. The coarse aggregate passing through a ¾" sieve and retained on ½" with a ¾" maximum was used. The beams were tested in a straining frame. Dial gauge was installed at mid span for measuring mid span deflection. Load was applied incrementally at mid span and mid span deflection was recorded corresponding to each increment of load. Also cracks were observed and marked on the beams. The testing time of single beam was approximately 45 minutes.

OBSERVATIONS, RESULTS AND DISCUSSION

When application of load started on the beam, there were no cracks on the face of the beam. Deflection gauge showed very small deflections as whole of the beam section participated in resisting the moment. Beams behaved quite in an elastic manner at this stage. It was observed that deflection increased suddenly near the flexural cracking load. The first crack was too narrow to be visible and it was observed and marked at a load higher than actual flexural cracking load.

When the load was increased at an increment of 1 kip flexural cracks were developed in the section of beam. These cracks (flexural cracks) propagated towards load point at an inclination less than 90°. These crack patterns were marked on all the beams and it was observed that mostly the pattern of cracking was almost the same for all the beams and number of cracks seemed to depend on the amount of longitudinal reinforcement. The beams having low reinforcement exhibited a few but wider cracks. It was noted that most of these cracks were developed within the middle half of shear spans and after cracking beam supported significant magnitude of load before its final failure. Critical crack was observed as an extension of the last flexural crack. By increasing the load, this crack propagated upwards towards the point load and downward towards longitudinal steel. When the crack reached the longitudinal reinforcement, it propagated along longitudinal reinforcement for short distance.

Beams collapsed finally when the crack extended well into the compression zone. In some cases it was observed that spalling of cover below the longitudinal reinforcement took place. The critical section was marked and its distance from the support was measured for all the beams. The results obtained from tests are shown in table 1. The shear failure of beams can be classified into two categories:

(i) Diagonal Tension (ii) Shear Compression

The "Diagonal Tension" group consisted of the beams failing just after the formation of first diagonal crack. Most of the test beams failed in category (i) above i.e. in Diagonal Tension Failure. Following Table 1 shows the results of tested beams.

Table 1: Results of Tested Beams

Beam #	% Steel ratio	fc' (psi)	Initial Cracking Vc (psi)	Diagonal f^*s (ksi)	At Failure Vu psi	M/Vd	fs ksi	Vu/Vc	Vc/Vaci	Vc / V_{zsutty}
3-A-1	0.52	3166	79.86	48.87	115.80	1.60	70.86	1.45	0.69	0.78
3-A-2	0.53	3332	76.62	46.42	110.89	1.60	67.19	1.45	0.65	0.74
3-A-3	0.63	3297	82.69	41.98	116.59	1.61	59.19	1.41	0.69	0.75
3-A-4	1.29	3118	97.15	23.38	134.56	1.65	34.30	1.39	0.77	0.71
3-A-5	1.35	3254	99.43	24.19	138.21	1.65	33.63	1.39	0.77	0.71
3-B-6	0.63	3214	82.10	49.26	82.10	1.90	49.26	1.00	0.71	0.79
3-B-7	0.89	3405	89.32	37.99	125.94	1.89	53.58	1.41	0.73	0.76
3-B-8	0.99	3326	98.82	38.67	142.05	1.95	55.58	1.44	0.80	0.82
3-B-9	1.18	3062	110.89	36.40	154.13	1.95	50.59	1.39	0.92	0.89
3-B-10	1.47	3125	119.25	31.58	169.33	1.95	44.84	1.42	0.95	0.89

REGRESSION ANALYSIS

As mentioned earlier many researchers have proposed the equations for calculating shear strength of reinforced concrete beams. Zsutty's equation(6) is considered as the most appropriate that is based on best fit of the experimental results. So the format of this equation was adopted for regression analysis in this paper. Results of 312 beams tested by other researchers (7,8,9,10) were also included.

All the beams considered for statistical analysis are simply supported. The following equation was developed.

$$Vc = 45.5\,(f`c\;\rho\;d/a)^{1/3}$$

With $\in$ = Coefficient of variation = 9.85% and r = coefficient of correlation = 0.87. The results from the proposed equation have also been compared and given in table 1.

MULTIPLE REGRESSION NEGLECTING M/Vd

Multiple Regression was performed by taking ρ and $f`c$ as independent variables. The equation obtained as a result of regression analysis is as under.

$Vc = e^{3.34}\,(\rho)^{0.328}\,(f`c)^{0.329}$ or $Vc = 37.864\,(\rho f`c)^{1/3}$ With $\in = 13.54\%$ and $r = 0.88$

Comparing the results with the equation obtained when M/Vd (or a/d) was also considered; it was found that by taking the effect of M/Vd, both Vc and 'r' are improved.

LOWER LIMIT OF M/Vd

During the statistical analysis the minimum adopted value of M/Vd was 1.23. There it is recommended that for design purposes the value of M/Vd should not be taken less than 1.23. i.e., $M/Vd \geq 1.23$.

CONCLUSIONS

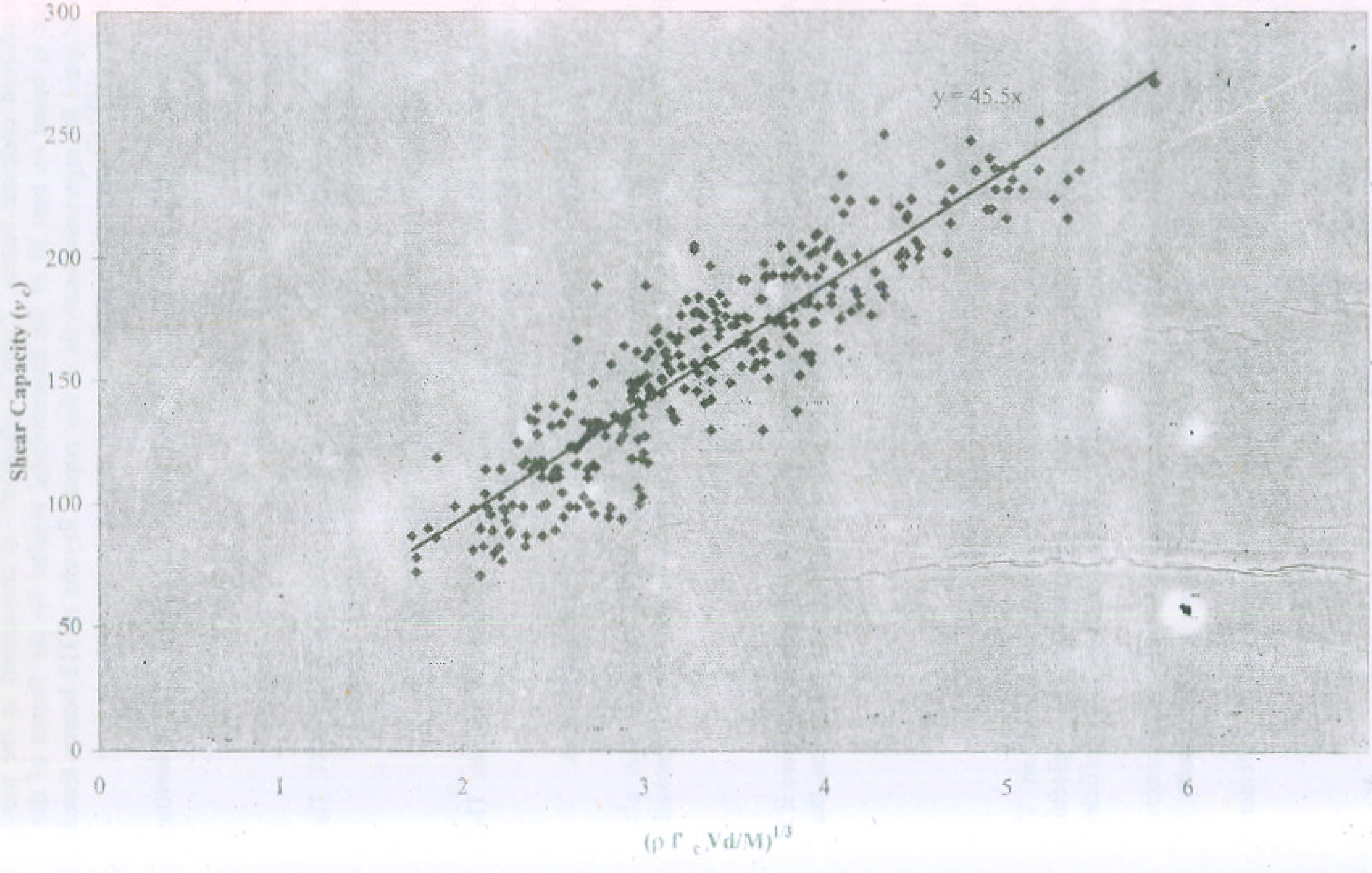
The following conclusions have derived from this research work.

1) The investigations confirmed a pronounced influence of M/Vd on Vc. Multiple regression analysis is performed by considering and neglecting M/Vd; it was concluded that by taking effect of M/Vd both Vc and coefficient of correlation are improved.
2) The results of ACI code equation are less conservative for reinforced concrete beams with $\rho < 0.01$ whereas equation provided some reliable results for beams having $\rho > 0.01$ (see table 1).
3) The shear capacity of beams without web reinforcement can be calculated from the following proposed equation (see Graph 1).

$Vc = 45.5\,(\rho\,f`c\;d/a)^{1/3}$ when M/Vd or $a/d \geq 1.23$

GRAPH 1:- BEST FITTING CURVE FOR THE FORMAT OF EQ. 8

(FOR ALL 312 No. BEAMS)



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