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# REVIEW AND COMPARISON OF ROCK MASS CLASSIFICATION SYSTEM

**BY**
**Saffet Yagiz**
**Saad Cheema**

# REVIEW AND COMPARISON OF ROCK MASS CLASSIFICATION SYSTEMS

Saffet Yagiz [1] , Saad Cheema[2]

## ABSTRACT

In this paper, the history of the rock mass classification is discussed generally. Since the development of Tezaghi's Rock-Load Classification System, several scholars have introduced quite different classification systems, which are briefly described. Most recent modifications of the Q systems was done by Palmstrom, (1995). Unal, (1990) modified the RMR for weak rock. The detailed modifications are explained in this article, which include the development of RMI and modified RMR. Importance of site investigation and rock type has been evaluated for applying any of the classifications for design purposes. In the conclusion, advantages and disadvantages of the rock mass classification systems are discussed.

## INTRODUCTION

The best-known and oldest classification system is Karl V. Terzaghi's Rock-Load Classification the was introduced in 1946. In 1958 H.Lauffer, an Austrian engineer, suggested a new method of classification called Stand-up Time, in which he divided rock mass in seven classes and related them to biggest span of tunnel and stand-up time. Many engineers have since modified Lauffer's classification. Then, the Rock Structure rating (RSR), was offered by G. E. Wickham et. al., in 1972. In 1973, Bieniawski developed the Rock Mass Rating System. After RMR, N., Barton et. al., introduced the Q classification system in 1974. They used as many parameters as possible to have closer definition of ground conditions, and to suggest more accurate loads for tunnel supports. After the Q and RMR system, both classifications were modified many times by scholars and practical engineers. E., Unal modified RMR in 1990. He proposed Modified Rock Mass Rating Systems, which is applicable for weak Rock, startified rock, anisotropic, and clay bearing rock. A., Palmstrom (1995) proposed Rock Mass Index (RMI) which was based on the Q system. RMI has been developed for strength characterization of rock masses for use in rock engineering and design.

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[1] Research Assistant, Mining & Earth Systems Engineering, Colorado School of Mines, Golden, Colorado, USA

[2] Senior Research Associate, Earth Mechanics Institute, Colorado School of Mines, Golden, Colorado, USA

## EARLY ROCK MASS CLASSIFICATION SYSTEM

### TERZAGHI'S ROCK LOAD CLASSIFICATIONS

Terzagi proposed a simple rock classification system for use in estimating the load to be supported by steel arches in tunnels in 1946. He described various types of rock loads for various ground conditions.

His concepts , which estimates, the rock load to be carried by steel arches used to support a tunnel, is in the figure 1. The loosening rock which the darkened area of the graphic,(abcd), will tend to move into the tunnel. This movement will be resisted by frictional forces along the lateral boundaries a, c, and b, d , and the friction force transfers of major portion of the overburden weight (W), onto the material on either side of the tunnel. The roof and sides of the tunnel are required only to support the balance, which is equivalent to the height of Hp. The width, B, of the zone of rock in which movement occurs depends on the characteristics of the rock mass and on the tunnel dimension.

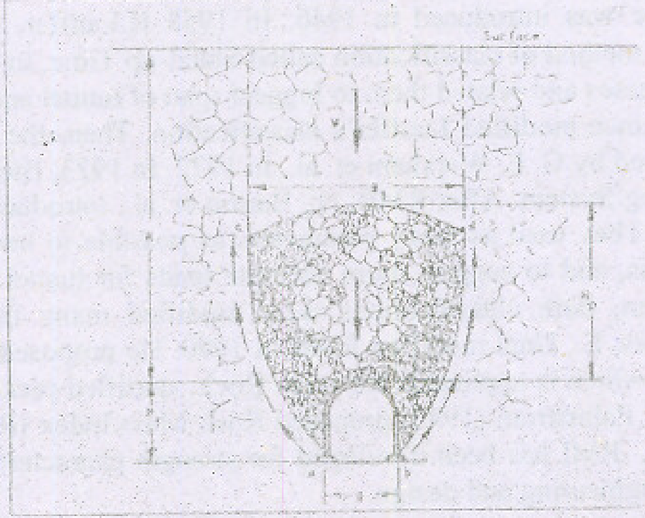

Figure 1. Simplified diagram representing the movement of loosened rock into a tunnel and the transfer of the load onto the surrounding rock (After Terzaghi)

On basis of his experience and test results, Terzaghi introduced the range of the rock-load values and rock condition classification for the steel support tunnels.

### STAND UP TIME CLASSIFICAITON

Lauffer emphasized the importance of stand up time of the active span in tunnels in 1958. Stand up time for an unsupported excavation is related to the quality of the rock mass in which the span is excavated. Stand up time is the length of the time that an underground opening will stand unsupported after excavation. The active span is the largest unsupported span in the tunnel section between the face and the support.

In the figure 2, the letters refers to the rock classes, A is the best rock, which corresponding to Terzaghi's intact rock, while G is the worst rock , which corresponds to Tezaghi's squeezing or swelling rock. Number of the factors that affect stand up time, such as orientations of tunnel's axis and shape of cross-section, excavation methods, and support methods.

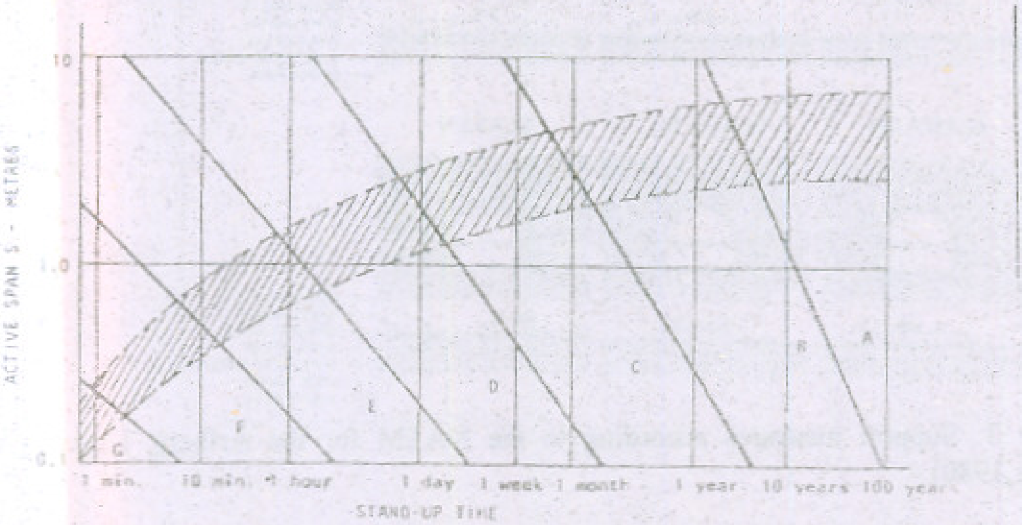

Figure 2. Relationship between active span & stand-up time for different rock mass classes(After Lauffer).

## NEW AUSTRIAN TUNNELING METHODS ( NATM)

Lauffer's original classification has been modified by number of Austrian engineers, including Rabcdwicz, Pacher, and Muller in 1964. The new Austrian Tunneling Method includes number of techniques for safe tunneling in rock conditions in which the stand-up time is limited before failure occurs. These techniques are applicable in soft rocks like mudstone, shale, in which the squeezing and swelling problems are likely occur.

In practice, NATM classification is related to ground conditions, excavation procedures, and tunnel support requirement.

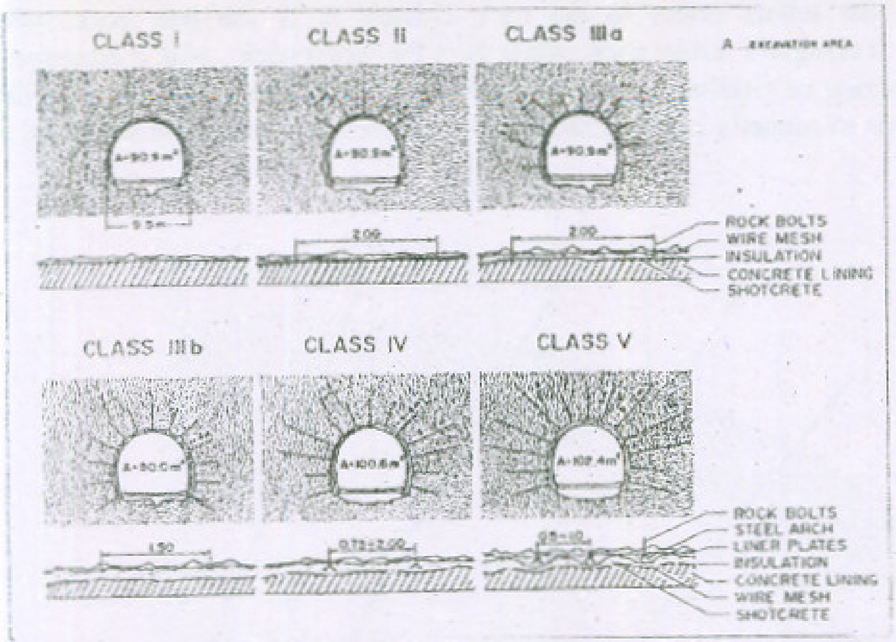

Figure 3: Support measures according to the NATM for the Arlberg Tunnel (After Pacher,1980).

## ROCK STRUCTURE RATING ( RSR)

The RSR concept was developed in United States in 1972 by Wickham, et al,. The rock structure rating system presents a quantitative method for describing the quality of rock mass and selecting appropriate ground support. The method was a step forward in the number of respects. First, it was a quantitative classification, unlike Terzaghi's qualitative one. This is unlike Lauffer's classification that relies on practical experience to decide on a rock mass and which gives an output in terms of the stand up time and span. The main contribution of the RSR concept was that it introduced a rating system for rock mass. The rating system was developed on the base of case histories as well as review of various books and technical papers dealing with different aspect of ground support in tunneling.
The RSR concept considered two general categories of factors influencing rock mass behavior in tunneling.

1- Geologic Parameters: rock type, joint patterns, joints orientation(strike and dip), and types of discontinuity, major fault/fold, rock material properties, weathering.

2- Construction Parameters: size of tunnel, direction of drive, methods of excavation.

All these factors were grouped in the three parameters, A,B,C. these three parameters are as follows:

A-) Parameter: General evaluation of rock structure based on rock type, origin, hardness and geological structure.

B-) Parameter: Effect of discontinuity pattern with respect to the direction of the tunnel drive based on joint spacing, orientation, direction of tunnel drive.

C-) Parameter: Effect of groundwater inflow based on overall rock mass quality due to parameters A and B, joint condition, amount of water inflow

Finally, total rock mass rating value is:

$$RSR = A + B + C$$

Although the RSR system is not widely used practically in mining, Wickham et al.'s works played a significant role in the development of the classification system for future

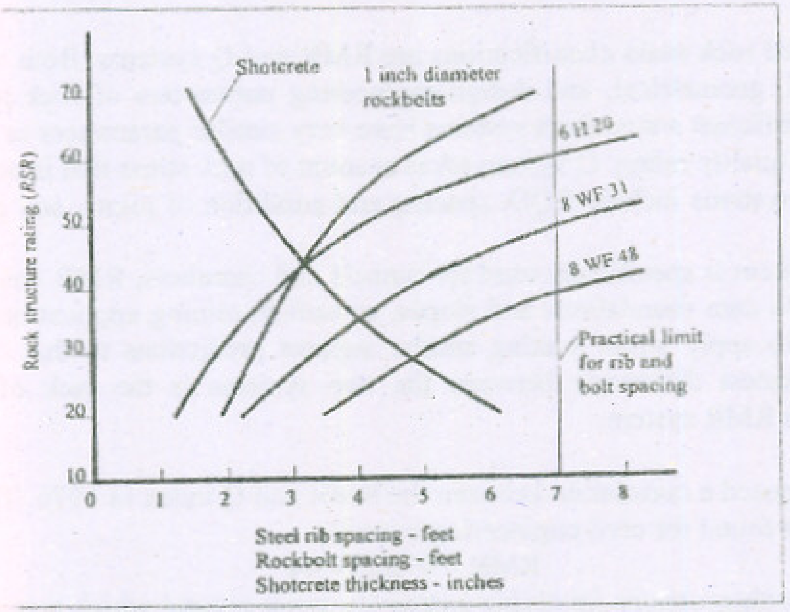

Figure 4. RSR support estimates for a 24-foot diameter circular tunnel (Wickham et al. 1972).

## RMR AND Q CLASSIFICATIONS

### ROCK TUNNELING QUALITY INDEX (Q)

On the basis of an evaluation of a large number of case histories of underground excavation stability, Barton, et al. Proposed an index for the determination of the tunneling quality of rock mass in Norway in 1974. The system consists of six parameters, which are RQD, number of joint sets, (Jn), joint roughness number, (Jr), joint alteration number,(Ja), Joint water reduction,(Jw), and stress reduction factor,(SRF).

The numerical values of this index (Q) are defined :

$$Q=(RQD / Jn) * (Jw / SRF)$$

### ROCK MASS RATING SYSTEM (RMR)

Bieniawski developed a Rock Mass Rating Classification System in 1973. This classification consists of six parameters: uniaxial compressive strength, ROD, spacing of discontinuities, condition of discontinuities, groundwater conditions, and orientation of discontinuities. Detailed information has been given for the RMR and Q classifications in the geotechnical course lecture. Furthermore, this classification is also facilitates estimation of the ground pressure on the support using following formula:

$$P=((100-RMR) / 100)*\gamma*B$$

Where the P is support pressure (kN), B is tunnel width (m), and gamma (kg/cub.m)is the rock density (Unal, 1983). The RMR provides some guidelines for support design and selection for long-term stability of tunnel excavations.

## COMPARISON BETWEEN RMR AND Q CLASSIFICATION SYSTEMS

Two widely used rock mass classifications are RMR and Q systems. Both methods deal with geological, geometrical, and design/engineering parameters of rock mass quality, but in slightly different ways. Both systems have very similar parameters to calculate the final rock mass quality rating. Q system takes account of rock stress that is not considered in RMR. Both systems include RQD, spacing and condition of joints, and ground water conditions.

While the Q system is specifically used for tunnels and chambers, RMR has successfully been extended to dam foundations and slopes, as well as mining applications. RMR was found simpler to apply while making similar support predictions to that of Q system. Finally, the greatest difference between the two systems is the lack of rock stress parameter in the RMR system.

Bieniawski proposed a correlation between the RMR and Q index in 1976. The following relationship was found for civil engineering tunnels:

$$RMR = 9*\mathrm{In}Q + 44$$

When dealing with problems involving extremely weak ground which result in swelling and squeezing rock, it has been found that the Q system gives better and more reliable results in comparison with the RMR.

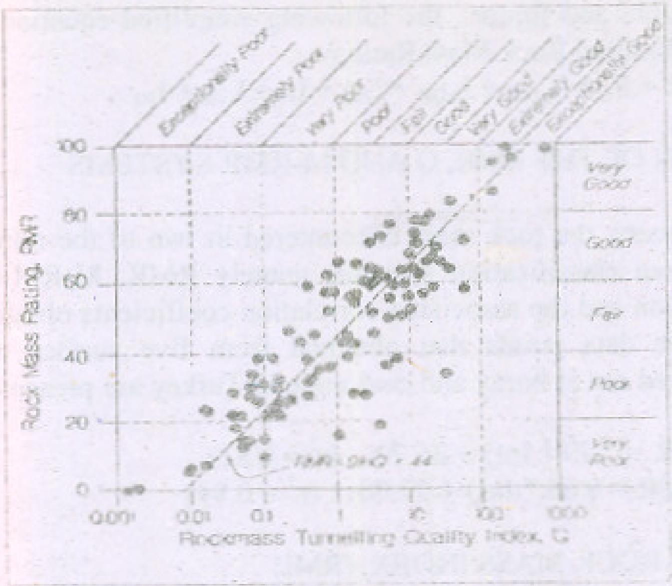

Figure 5: Relationship between the RMR and Q Systems

## AFTER RMR AND Q CLASSIFICATION SYSTEMS

## MODIFIED ROCK MASS RATING SYSTEM, (M-RMR)

Unal proposed the MRMR system for characterization of rock mass in 1990. This system has been developed based on found years of geotechnical investigations carried out in a borax mine and two coal mines regions in Turkey.

The M-RMR system enables determination of a quality-rating index (M-RMR) for characterization of rock mass. In general, M-RMR system is based on RMR; however, new features are added to the system for better characterization of wide ranges of rock conditions, including weak rock, stratified, anisotropic and clay-bearing rock masses(Unal,1990).

In the M-RMR system, the total rating suggested by the original RMR system for rock individual parameters has not been changed; however, after correction, due to the weathering effect, the M-RMR quality rating index may go up to 110.

In order to determine the M-RMR parameter index manually, the rating of the six basic input parameters, the weathering coefficient and the two adjustment factor $A_b$ and $A_w$ should be considered. The six parameters considered in the M-RMR system are uniaxial compressive strength, RQR, spacing and condition of discontinuities, ground water condition of discontinuities. In order to determine the M-RMR value initially, the geotechnical data must be converted to numerical values that reflect the rating assigned to the input parameters. This could be accomplished by following chart in figure 7. Also figure 6 and tables 2, 3, 4 would be helped to get M-RMR values (Unal, 1990). After the value is read from related table and figures, the following simplified equation may be used to calculate the Basic Modified Rock Mass Rating:

$$BM\text{-}RMR = F_C * (I_{USC} + I_{RQD} + I_{JS}) + I_{JS} + I_{GW} + I_{JO}$$

## COMPARISON OF THE RMR, Q AND M-RMR SYSTEMS

During the classification process, the rock mass encountered in two of the mine region was evaluated based on three classification systems, namely RMR, M-RMR and Q values. The regression equation and the associated correlation coefficients obtained from statistical evaluations of the data points that obtained from five surface and nine underground bore-holes carried out at borax and coal mine in Turkey are presented in the following equations.

$$RMR = 7.79 * \mathrm{InQ} + 36.70 \quad (r^2 = 0.81)$$

$$M\text{-}RMR = 9.66 * \mathrm{InQ} + 37.90 \quad (r^2 = 0.84)$$

## ROCK MASS INDEX, (RMI)

The rock mass index, (RMI) introduced by Palmstrom in 1995, has been developed to characterize strength of the rock mass for construction purposes, such as tunneling. RMI uses the following input parameters: compressive strength of the rock, block volume, and joint characteristics as roughness, size and alteration ( Table 5). RMI is based on the reduced rock strength caused by jointing, expressed as:

$$RMI = \sigma_C * JP$$

$\sigma_C$ = the uniaxial compressive strength of the intact rock measured on 50 mm samples, JP = the jointing parameter, which is the reduction factor representing the block size and the condition of its face as representing by their friction properties and the size of the joints. The influence of the JP has been found by using calibrations from tests results (Palmstrom, 1995). So, after the calibrations, the following expression is achieved:

$$JP = 0.2 * \sqrt{Jc} * (V_b)^D \qquad D = 0.37 * jC^{-0.2}$$

Where $V_b$ is block volume given in the and D is the block diameter.

The value of the JP is from near 0 for crashed rocks to 1 intact rock. In above equation, JC is the joint condition factor that is given by:

$$JC = jL / (jR / jA)$$

Where is jL (joint size and condition factor), jR (joint roughness factor), jA (Joint alteration factor). For rating of these parameters please see table 5 in the appendix.

The factor jR and jA are similar to the joint roughness number (jr) and the joint alteration number (Ja) in the Q system respectively. Joint size and continuity factor, (jL). Introduced in RMI to represent to scale effect of the joints. At below, the table shows the values of RMI:

## CLASSIFICATION OF RMI

| RMI | Related to rock mass strength | RMI value |
|---|---|---|
| Extremely Low | Extremely Weak | < 0.001 |
| Very Low | Very Weak | 0.001 – 0.01 |
| Low | Weak | 0.01 – 0.1 |
| Moderate | Medium | 0.1 – 1 |
| High | Strong | 1.0 – 10 |
| Very High | Very Strong | 10 – 100 |
| Extremely High | Extremely Strong | > 100 |

The RMI system could be used for blasting and support design in rock mass, and also it is useful to evaluate tunnel boring machine (TBM) performance.

## CONCLUSIONS AND RECOMMENDATIONS

The main objectives of rock classification systems are to identify the most significant parameters influencing the behavior of a rock mass and divide a particular rock mass formation into group of similar behavior. It is provide a basis for understanding the characterization of each rock mass class and relate the experience of rock conditions at one side to the others where the conditions and experience encountered. The classification system derive quantitative data and guideline for engineering design and provide a common basis for communication between engineers and geologists; The classification scheme approaches do not always totally evaluate significant aspect of the problems. So, those systems and not rigid guidelines, but are useful tools to use for underground excavations. Finally, before the classification system is used for design purposes, a site area should be evaluated very carefully. After that, at least two of the classification systems should be applied to achieve reliable results for any design purpose.

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# NONLINEAR SYSTEM ANALYSIS AND DESIGN USING MATLAB AND SIMULINK- TUTORIAL

Dr.Farrukh Nagi[1]
Fawad Ahmad

## ABSTRACT

SIMULINK is a package used in MATLAB environment for modeling, analyzing and simulation of dynamic systems. It supports linear and nonlinear systems modeled in continuos or discrete time or a hybrid of the two. This paper describes the use of MATLAB and SIMULINK for stabilizing a nonlinear plant. The plant consist of an inverted pendulum mounted on a moveable cart. Phase plane method is used for analyzing the nonlinear and linear models of the plant. The linearized model is controlled around an equilibrium point using full-state feedback technique. The state feedback gains are determined using pole placement and LQR methods. The effect of initial condition on the stability of the closed loop dynamics is demonstrated along with other pertinent details.

## INTRODUCTION

Simulink is a graphical simulation package for use with MATLAB, the popular numerical analysis program. Due to the inherent nature of nonlinear systems, obtaining their analytlical solutions is either very difficult or not possible MATLAB and SIMULINK provides different tools for the anaysis of nonlinear systems. In addition MATLAB has a Nonlinear Control Design (NCD) blockset, a recent add-on to SIMULINK. NCD allows constrained output design for nonlinear systems.

In this paper we describe the use MATLAB and SIMULINK to analyze nonlinear control systems in context to standard techniques presented in nonlinear control theory [1,3]. The bench mark example of the popular inverted pendulum [2] is taken as the nonlinear plant. The plant is first graphically developed in SIMULINK environment and saved as a Complementary S-function with inport and outport blocks. The plant is than linearized in MATLAB environment around an operating set point $x_n$. Using MATLAB control Toolbox, feedback gains are evaluated for the specified dynamic response. The closed loop response of the plant is then determined using SIMULINK.

## PROBLEM STATEMENT

The inverted pendulum[1,2] mounted on a cart is shown in Fig.1. The plant is to be Balanced upright such that the angular position state, $x_1 = \theta = 0$, velocity state $x_2 = d\theta/dt = 0$ and input $u = f_x = 0$.

The nonlinear state equations,[1,2] for the plant is given as

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[1] National University of Science and Technology Pakistan Navy Engineering College

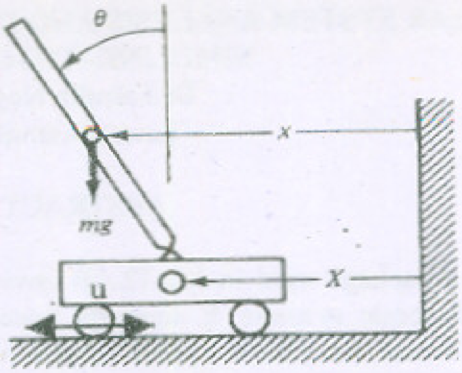


**Figure 1. Non linear inverted pendulum plant**

$$x = \begin{bmatrix} x_2 \\ \dfrac{19.6\sin x_1 - 0.1x_2^2\sin(2x_1) - 0.2\cos x_1 u}{1.33 - 0.2\cos^2 x_1} \end{bmatrix}$$

where $x_1$ and $x_2$ are the state variables representing the plant angular position and velocity respectively.

The objective is to design a control law to keep the plant balanced vertically upwards. And to bring the pendulum back to its initial position if perturbed within a certain bound. The closed loop system is required to have stable poles at $\lambda_1 = -4$ and $\lambda_2 = -5$. The plant in Eq .(1) cannot be solved for a unique solution or an equilibrium point as in case of linear sys. The use of SIMULINK HELPS in obtaining the dynamic behavior and Stability of the plant for different initial conditions, inputs and disturbances.

## SIMULINK REPRESENTATION OF THE NL PLANT

The SIMULINK graphical block diagram representation of the nonlinear plant in Eq(1) is Shown in Fig 2. An input block and two outport blocks are added to the block diagram For the external input 'u' and outputs of the state variables $x_1$ and $x_2$. The diagram is then saved as an m-file in SIMULINK. The stability of the plant is determined by simulation and graphical phase plane method before and after linearization. By changing the value in the integrators, the response of the plant can be simulated for different sets of initial conditions.

## ANALYSIS AND SIMULATION OF THE PLANT

Some of the common techniques to analyze nonlinear systems are linearization of system About an operating point, state feedback linearization, describing functions, Lyaponov Theory and phase plane representation [1,3,4]. Because of two dimension apprehension and ease of use, the phase plane method is very popular for second order systems. Simulation is accomplished using numerical integration techniques. A suitable integration Method along with the integration step size should be selected depending on structure of the plant dynamics. Simulation with a larger step size can lead to unstable trajectories of an other wise stable system.

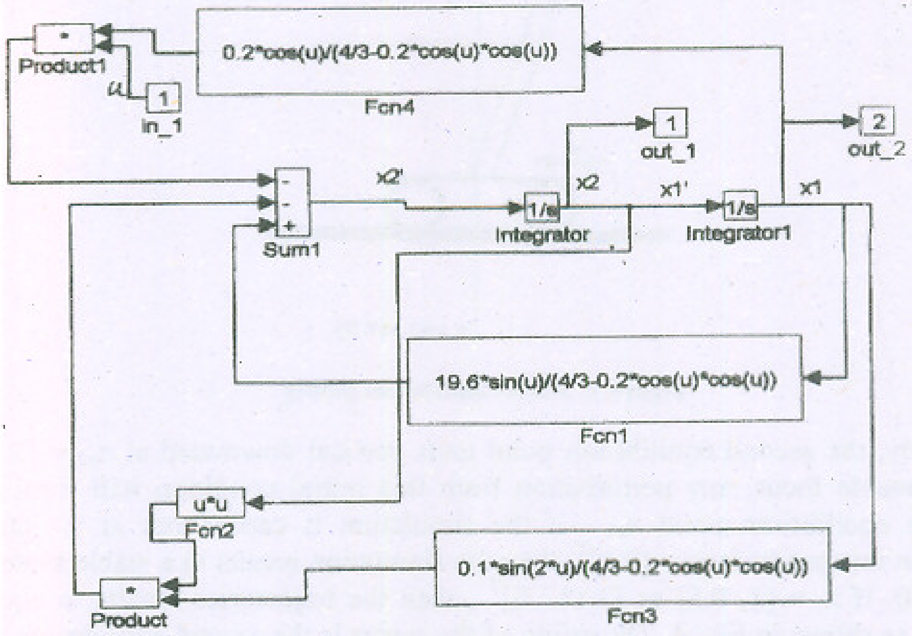


**Figure 2 . Simulink representation of non linear inverted pendulum plant**

## EQUILIBRIUM POINTS [1,3] AND INITIAL CONDITIONS

The state $x_{e1} = [0\ 0]^T$ forms the first equilibrium point and implies the vertical balanced position of the pendulum as shown in fig.3.

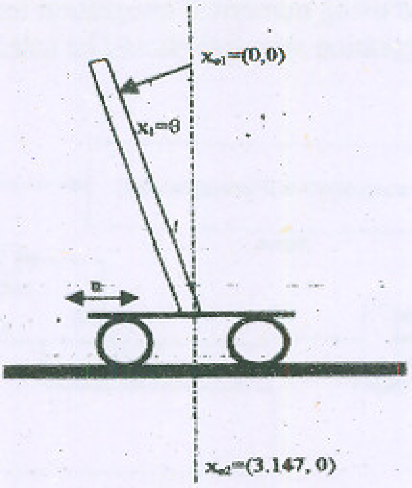


Figure 3. Plant equilibrium points

Heuristically, the second equilibrium point exits vertical downward at $x_{e2} = [3.147\ 0]^T$. $x_{e1}$ is an unable focus, any perturbation from this initial condition will form a center around the equilibrium point $x_{e2}$ . if the simulation is carried out at $x_{e1}$ and initial condition in integrators is $x_i = [0\ 0]^T$, then the simulation results in a stable system at the origin (0 0). If $x_i = [1, 0.5]$ or $[3.147\ 0]^T$ , then the trajectories results in equilibrium center [1], as shown in Fig. 4. The origin of the center is the second equilibrium point $x_{e2} = [3.147\ 0]^T$ and $x_i$ also becomes a point on the trajectory. Three trajectories of the plant at different initial conditions $x_{i1}$, $x_{i2}$ and $x_{i3}$ are plotted in Fig.4. The first two forms an ellipse because of swinging oscillations. State $x_i$ increase with CCW motion and decrease with CW motion. For $x_{i3}$ initial condition, $x_1$ is always increasing because of high initial angular velocity and the pendulum starts rotating in CCW motion.

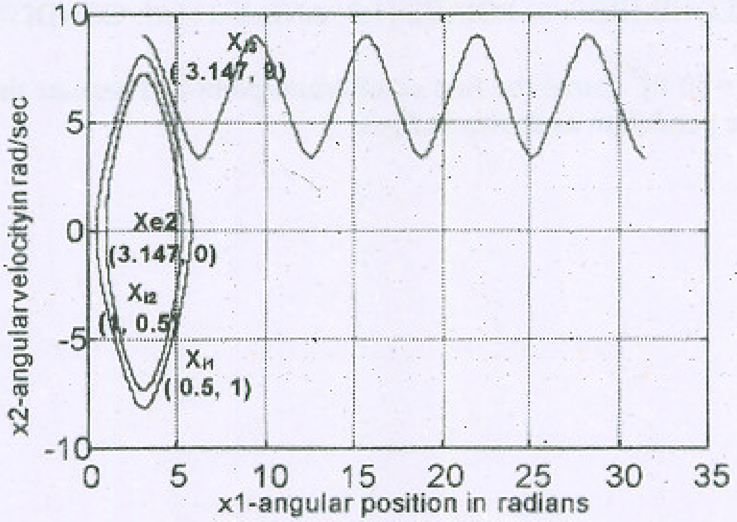


Figure 4. Effects of initial conditions on non linear plant

## EQUILIBRIUM POINTS FROM MATLAB [4,5]

The equilibrium points $x_{e1}$ and $x_{e2}$ can be determined from MATLAB using SIMULINK model. Equilibrium points are determined for zero rate of change of states and zero input [1,3]. From Eq. (1)

$$x = f(x, u = 0, t) \qquad \text{where} \qquad f(x, u = 0, t) = 0 \qquad (2)$$

The MATLAB commend based on this criterion to determine the equilibrium points is:

`>>[xe,u,y,dx] = trim ('NL plant',xi.yi,ui)` (3)

where 'NL plant' is the name of m-file in which the simulink model of Fig.4 is stored. The variable $x_e$ returns the equilibrium point in the vicinity of chosen guess points $x_I$ , y is the output and equals $x_1$ and dx should be equal to zero, as shown in Eq(2). The initial input and outputs are represented by $u_i$ and $y_i$ respectively. To check if origin is an equilibrium point, enter $x_I = [0\ 0]^T$ when $dx = [0\ 0]^T$. To find $x_{e2}$ enter an initial guess of $x_I = [2\ 4]^T$ and observe dx, which should be close to zero. Change $x_I$ to $x_I = [4\ 2]^T$ and $x_I = [3\ 3]^T$ and observe dx. Select that return value of $x_e$ for which dx=0; the seleced value should comes out to be $[3.147\ 0]^T$.

## NON LINEAR (NL) PLANT LINEARIZATION

The plant under consideration is nonlinear with unstable response as shown in Fig.4. It is desired to stabilize the plant near the operating point $x_n = x_{e1}$, which implies vertical upright position. In this paper, linear control theory is applied to stabilize the plant. The nonlinear plant dynamics is first linearized [1,3] about the operating point. The following MATLAB [4,5] command is used:

`>>[a,b,c,d]=linmod ('NLplant',xe,u)` (4)

linmod is used to linearized the plant 'NLplant' about $x_{e1}$ without xe and u input, resulting in

$$a = \begin{bmatrix} 0 & 1 \\ 17.34 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ -0.177 \end{bmatrix} \qquad c = [1\ \ 0] \qquad d = 0$$

which gives the inearized state space model of the nonlinear plant in Eq(1)

## STATE FEEDBACK CONTROLLER

State feedback [1,3] controller is used to stabilize the linearize the linearized plant in Eq(5). The stability of the plant can be determined for state matrix in Eq(5) from MATLAB command:

```
>>eig(a)
```

which results in $\lambda_1 = 4.17$ and $\lambda_2 = -4.17$. The linearized system is unable with one pole in the right half of the s-plane. The desired closed loop poles are $\lambda 1 = -5$ and $\lambda 2 = -4$. Configuration of control system designed [3] by linearization about the set point xn = xe1 is shown in Fig.5. the top left portion of the figure represents the set point or feedforward control law, and the rest of the structures forms the feedback controller. The feedback gains $K_1$ and $K_2$ can determined by pole placement or linear quadratic regulator method using MATLAB.

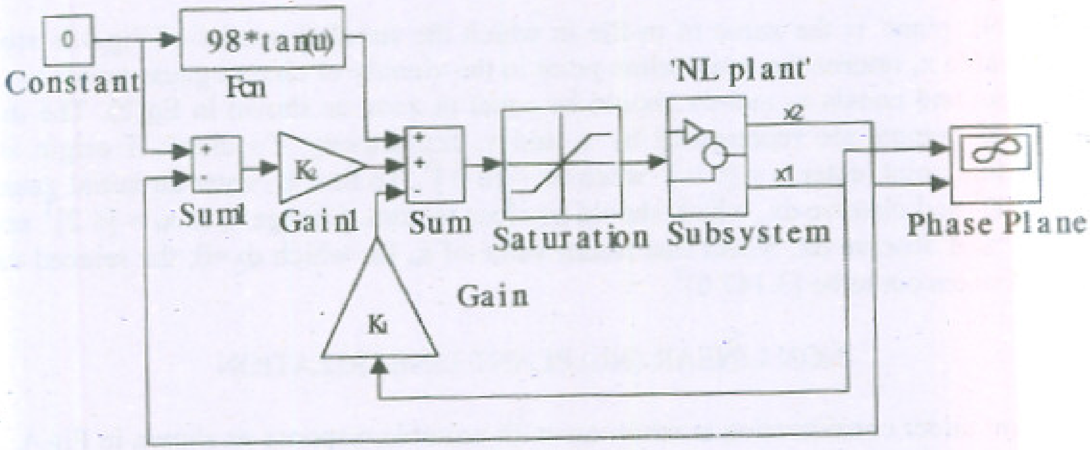


**Figure 5. Linearized state feedback control cystem**

POLE PLACEMENT CONTROLLER DESIGN

The controller state gains are easily found from the following MATLAB [1,4,6] command

```
>>k = place(a,b,c,d,cp)
```

where cp = [-4 -5] is a vector of the desired closed loop poles. This gives k =[-211.33 -51] as the state feedback gains. Subsystem is the grouped 'NLplant' of Fig. 2. Fen is the formed from

$$x(2,1) \Rightarrow 0 = 19.6 \sin(x_1) - 0.1x_2^2 \sin(2x_1) - 0.2 \cos(x_1)u$$

$$x_{e1} = (x_1 , 0) \qquad (6)$$

$u = 98 \tan(x_1)$ where $x_1$ = u of Fcn

Control input u limited by the saturation block, set to 1000 N maximum allowable.

Other technique of infinite time Linear Quardratic [6] – Optimal control can also be used to evaluate the gains k. the MATLAB command is:

```
>>[k,p] = iqr(a,b,Q,R)
```

Here, k and p are gains and closed loop poles based on minimum quadratic performance index. Q is an identity matrix and R = 1. During minimization process, weights can be assigned to states matrix Q or control input R. More weigh on R, lower the control effort. + Similarly by increasing the weigh on the Q matrix, states $x_1$ and $x_2$ are penalized resulting in a damped system.

The linear quadratic gain matrix is k = [-195, -47.235] and with closed loop poles p = [-4.1, -4.3]. The LQ gains can also be used in Fig.5. the system in Fig.5 is perturbed by plant initial conditions of $x_i$ =(0.5, 1) , (1,0.5) and (1.25, 0). The resulting phase plane of state feedback system is shown in Fig.6.

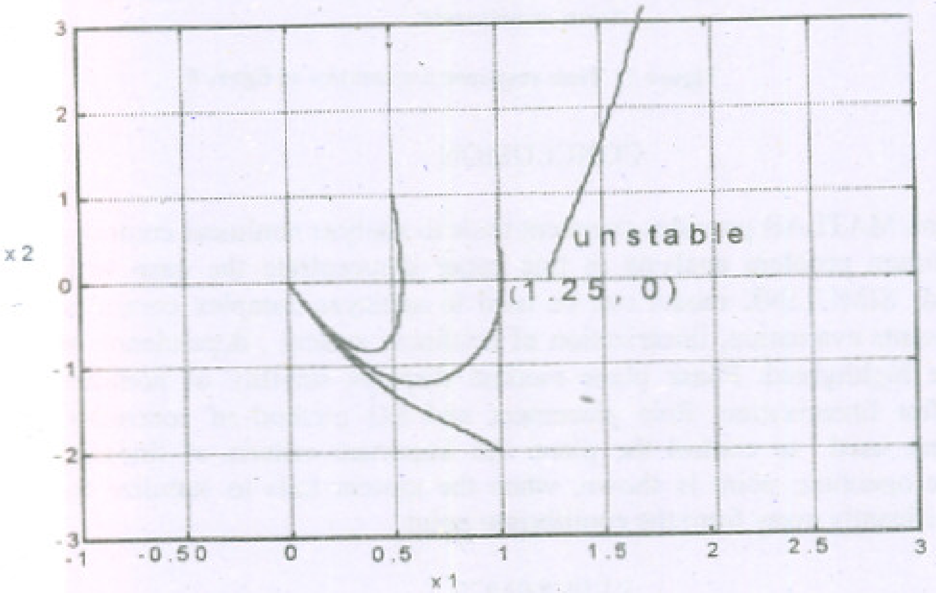
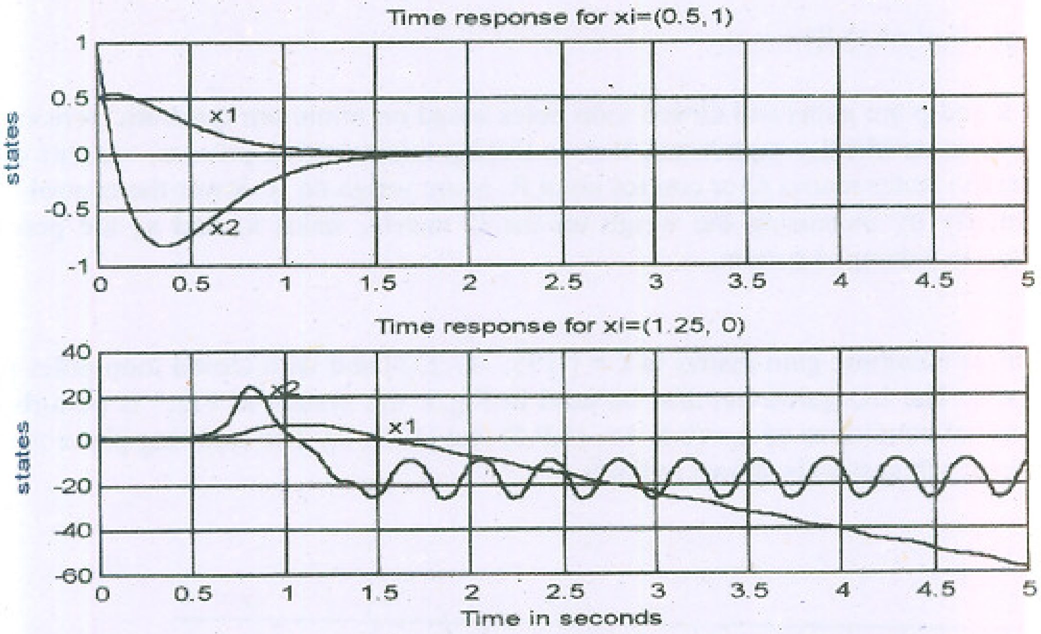


**Figure 6 Phase Plane Linearized System**

In comparison to Fig.4 the unstability limits starts from initial condition (1.25 , 0), the limits of linearization of the plant is only possible in a neighborhood of the operating point $x_n = x_{e1}$.

Time response of Fig. 6 is shown in Fig.7 for two initial conditions. the first condition results in a asymptotically stable equilibrium, since both states come to zero or the pendulum is upright within 2 seconds after perturbation from the initial condition (0.5, 1). In second unstable condition the states are never zero and the system fails to balance the pendulum in the vertical position.


Time response for xi=(0.5,1)

Time response for xi=(1.25, 0)

**Figure 7. Time response trajectories of figure 6.**

## CONCLUSION

SIMULINK and MATLAB provides excellent tools to analyze nonlinear control systems. inverted pendulum problem analysis in this paper demonstrate the case with which MATLAB and SIMULINK model can be used to analayze complex control systems. Equilibrium points evaluation, linearization of nonlinear system , dependency on initial conditions are highlighted. Phase plane method displays stability of nonlinear plant before and after linearization. Pole placement and LQ method of controller design techniques were used to control the plant. An important criteria of linearization in vicinity of the operating point is shown, when the system fails to stabilize for initial condition significantly away from the equilibrium point.

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