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# DESIGN OF WEIRS ON SAND FOUNDATIONS.

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### Historical.

The theory of the design of weirs on sand foundations is at present in a very unsettled condition. Prior to 1930 one theory, that of Bligh, was almost universally accepted. It was simple and easy to use and comprehensively specified the features of a work supposed to be necessary to ensure safety against the action of sub-soil flow. Under these conditions engineers gave little attention to the theory of sub-soil flow, no experimental work was done, and no advance was made in our knowledge of the subject.

At the end of the last decade, however, trouble occurring on certain works designed in accordance with Bligh's principles revived interest in the subject in the Punjab and recently a certain amount of research has been done there. The first cases to attract attention were those of certain syphons on the Upper Chenab Canal then in charge of Mr. A.N. Khosla and the trouble occurring here and the failure of the usual remedial measures to give satisfactory results led to the experimental work the results of which were published in Congress Papers Nos. 138 and 142. These contained, amongst other matter, records of the actual pressures under many existing works both syphon outlets and weirs and a comparison of these with the pressures estimated from Bligh's theory showed that the latter was far from reliable.

Apart from Khosla's work many examples might be quoted of works which have failed, apparently through the action of sub-soil flow, while Bligh's requirements have been met, and of works which are standing without sign of weakness where they are lacking. A notable example of the former is the case of the Deg Diversion regulator on the Upper Chenab Canal. The cause of the failure of this work, which occurred when it was first brought into use was undoubtedly sub-soil flow, but the hydraulic gradient across it at the time was of the order of 50 to 1.

Subsequent to Khosla's work, Dr. Bose published a paper, No. 140 of this Congress, on the kindred subject of sub-soil flow in relation to tube wells, and in this, treating the subject mathematically, developed general equations for sub-soil flow. The solution of these equations when applied to the complicated cases occurring in practise, is not, however, practicable.

In order to investigate more closely the nature of flow beneath weirs, the Director of Irrigation Research instituted a series of experiments on models, an interim report on which was published before this Congress last year. These experiments produced a great deal of valuable information, and considerable use will be made of them in the course of this paper.

**Present Position of the Subject.**

While, as recorded above, a considerable amount of research has been carried out in recent years, very little has been done in the way of interpreting and applying the results. The chief practical deduction to be made from Khosla's work was that vertical cut offs were more effective, and horizontal floors less effective, in practice than Bligh's theory anticipated. In a composite work the hydraulic gradient between pile lines is much flatter, and the drop at the pile lines greater, than Bligh estimates.

The only constructive proposal for applying this to design that the writer has seen came from Mr. Ivan Houk, of the U. S. Reclamation Service, who suggested the modification of Bligh, to the extent of assuming the resistance to vertical flow to be greater than that to horizontal flow. The coefficients he proposed, however, were so selected as to give the same average gradient as Bligh, and, of course, he followed the same principle, measuring the path of flow along the surface of the work and limiting the gradient along it.

Dr. McKenzie Taylor's experiments have shown, however, that this basis is fundamentally wrong ; at the same time they provide additional material on which, in conjunction with other available experimental work, a new theory may be based; and the purpose of the present paper is to endeavour to state such a theory, and to consider its application to design. In doing this, however, it is realised that available data is by no means complete and portions of the theory advanced require experimental confirmation.

In order to design a weir which is safe against sub-soil flow, it is necessary to know

(*a*) the distribution of streamlines and pressures in the material on which the work is founded, and

(*b*) the conditions of pressure and streamline distribution which are dangerous when applied to the material and hence to be avoided.

The research referred to above deals almost exclusively with the former group and nothing has been done on the latter subject. This is perhaps unfortunate, as while the old Bligh theory was unreliable in estimating the length of flow and intensity of pressure under a work it was rarely in error by more than 50% ; in estimating dangerous conditions, however, it was, in the writer's opinion, completely astray. With a properly designed work gradients three or four hundred per cent steeper than those permitted by Bligh would be safe and conversely it is possible to devise a work in which gradients considerably flatter than Bligh's would not be safe.

Before considering this question, however, we will state the theory of streamline and pressure distribution which emerges from recent experimental work. This, for brevity will be referred to as the "Streamline Theory." The writer claims no originality for it: it is merely the general application of Darcy's Law the possibility of which has been recognized by others. As far as he is aware, however, a descriptive theory in relation to weir design has not so far been published.

As will be seen the theory is still very incomplete.

**The Streamline Theory.**

The fundamental basis of this theory may be stated as follows:

"At any point in a submerged permeable medium in which the hydraulic pressure varies from static, flow will take place in the direction of the maximum rate of change of excess (over static) pressure, and will vary directly with this rate and with the permeability of the medium."

This statement probably requires a little explanation. The static pressure at any point in a body of water at rest is of course that at the base of a column of water of height equal to the vertical depth of the point below the free surface. If the water is in motion the free surface will be at different levels at different points. If we wish to define the static pressure under these conditions therefore, it is necessary to specify the location of the free surface from which the depth is measured. This would usually be taken as the source, *i.e.*, in the case of a weir as the upstream water level. The 'excess pressure' referred to in the statement is the difference between the actual pressure at the point under consideration and static. Normally, of course, this difference would be negative. If the rate of change of excess pressure between two points is uniform it is the difference between the excess pressures divided by the distance between the points. In calculating this it is unnecessary to specify the static pressure datum: all that is necessary to know is the difference of level of the points. The difference in excess pressure is the difference in actual pressure less the difference in level.

The permeability of a material will vary with the size, shape and grading of the particles of which it is composed and also with the degree of compaction. Generally speaking permeability decreases with size; given the same average size, irregularly shaped particles will pack better than uniform ones and hence have a lower permeability. Particles of uniform size do not pack as well as those of different sizes, hence the importance of grading. Compaction can make a very great difference to the permeability in the case of soils containing clay or loam and the difference is appreciable in the case of pure sands. The permeability of a medium is measured by its 'transmission constant,' which is the discharge which will pass through a specimen of unit area and length under unit difference of pressure. For average Punjab silt the transmission constant is of the order of 0·003 in English units. A further factor which appreciably affects the permeability of a medium is the temperature of the water, but this is not of great importance from the point of view of weir design.

The transmission constant is related to grain size by the formula

$$k = 0{\cdot}038\ d^2 . \frac{t+10}{60} \qquad \text{(Hazen)}$$

where '$d$' is the diameter in millimetres of a sphere of the same volume as a particle of 'effective size.' The latter term is used to denote the size such that ten per cent. of the specimen consists of smaller grains; '$t$' is the temperature in degrees, Fahrenheit. The range over which this formula is accurate is between effective sizes of 0·1 and 3·0 mm.

As previously mentioned, the basis stated above is a generalization of Darcy's Law. This law, which states that the discharge through a permeable medium is directly proportional to the head, was originally developed from experiments on horizontal parallel flow only. That it is generally applicable is, however, obvious theoretically and amply confirmed experimentally. The streamline theory comprises the application of this basis to the determination of the flow confirmation under various conditions, and, more particularly to those found in practice.

Accepting the basis, it follows that given the boundaries, open and closed, and the transmission constants of a body of permeable material, and the pressure head or heads to which it is subject, the discharge, streamline conformation and pressure distribution will be fixed. Although for any given case the solution is unique, its determination, save in certain simple cases, is not susceptible to mathematical treatment. The theory can therefore only be developed to a limited extent mathematically and outside these limits resource has to be had to model experiments which are the only method of obtaining a knowledge of the flow conformation of the complicated cases which occur in practice. Apart from this, however, experiments on the simple cases for which mathematical solutions are practicable afford useful confirmation of the theory.

We will next consider the

**Mathematical Development of the theory.**

Expressed mathematically, the fundamental basis gives the equation

$$q = k.a.\frac{dp}{ds} \qquad \ldots \qquad \ldots \quad (1)$$

where '$a$' is the cross sectional area and '$s$' is the length of a stream tube, '$p$' is the excess pressure, and '$k$' is the transmission constant. A stream tube is, of course, the space covered by any appreciable volume of water in flow while the locus of a particle of water is a streamline.

The mean velocity of the water through space may be obtained from the above, *i.e.*,

$$v=\frac{q}{a}=k.\frac{dp}{ds} \quad .. \qquad .. \quad (2)$$

The velocity through space must be distinguished from the actual velocity through the interstices of the material. If the interstices occupy $1/n$ of the volume, the component of the latter velocity in the direction of flow will be $nv$. The actual velocity will be somewhat greater owing to the greater length of the winding path actually followed through the interstices.

In endeavouring to develop the theory from this basis the writer has been able to find complete solutions for only two forms which might be used in practice, the simple floor and the single pile line. These will be given later, but we will first consider certain cases which might be termed 'symptomatic,' *i.e.*, conditions of flow which may occur, or be approximate to, locally in the general conformation of a complicated practical case and which consequently it is useful to examine. In Appendix A the mathematics of six cases are worked out and five of these will now be commented on. What we are particularly interested in is the effect of the form of flow on the discharge and the pressure gradient.

Two cases of converging flow are dealt with. In Case I the area varies directly as distance. This case is approximated to in practice when flow approaches any obstruction such as the end of a floor or a line of piles. The converse case of diverging flow, the mathematics of which are the same, occurs when flow leaves a similar obstruction. The case applies to flow converging in two dimensions only, *i.e.*, convergence in space on to a line. In Case II the area varies as the square of the distance representing convergence in space on to a point. This is approximated to in practice when we have converging flow both in section and in plan, a common example being the case of a syphon inlet or outlet where spring level is high.

For Case I we see

$$q=\frac{h}{L}.k.a_0.\frac{m-1}{\log_e m}$$

where $a_0$ is the area at exit and $ma_0$ that at entry.

At exit

$$\frac{dp}{ds}=\frac{m-1}{\log_e m}.\frac{h}{L} \text{ for convergence}$$

$$=\frac{1}{m}.\frac{m-1}{\log_e m}.\frac{h}{L} \text{ for divergence.}$$

The following table gives values of $\frac{m-1}{\log_e m}$ and $\frac{1}{m} \cdot \frac{m-1}{\log_e m}$ for different values of $m$:—

| $m$ | $\frac{m-1}{\log_e m}$ | $\frac{1}{m} \cdot \frac{1-m}{\log_e m}$ |
|---|---|---|
| 1·5 | 1·23 | 0·82 |
| 2·0 | 1·45 | 0·73 |
| 5·0 | 2·48 | 0·50 |
| 10·0 | 3·90 | 0·39 |

The important point to observe here is that the pressure gradient at exit may be considerably greater than average in the case of converging flow and less than average when flow is divergent.

For Case II

$$q = \frac{h}{L} \cdot k.a._o \sqrt{m}$$

and at exit $\frac{dp}{ds} = \sqrt{m} \cdot \frac{h}{L}$ for convergency

and $= \frac{1}{\sqrt{m}} \cdot \frac{h}{L}$ for divergency.

It will be noticed that the discharge and rate of change of pressure for convergency in this case, while still above average are less than in the previous case.

In Case III we have two lengths of parallel flow of different sections. This was considered in order to investigate the conditions where flow at exit is obstructed say, by concrete blocks or wells. It assumes that the joints are full of the same material as exists below the work. It will be seen that where the length of the path through the obstruction is short in comparison with the total length of percolation flow, the discharge is practically unaffected and the gradient is increased in proportion to the reduction in area.

In Cases IV and V we have the combination of parallel flow with the two kinds of converging flow. The results are the same as in the last case, *i.e.*, the discharge is unaffected and the pressure gradient is increased inversely in proportion to the area.

In practise these cases are met where flow at exit is obstructed but the passages between the obstructions are free. The former case would apply to flow in the vicinity of the gaps between concrete blocks of wells; the latter to that in the vicinity of relief pipes or weepholes.

In Case III of course the conditions of Case IV or V would always be present also, *i.e.*, there would always be a short length of converging flow between the two lengths of parallel flow.

## Solution of Certain Cases with aid of Confocal Conics.

The above examples are useful only in indicating in a general way the pressure distribution which is associated with certain types of flow. As we will see later, certain forms of pressure distribution are dangerous and should be avoided in practice; a knowledge of the type of flow which is associated with these conditions is, therefore, useful.

These cases cannot be applied, however, to give a complete solution of the flow distribution under a work, such as might be met in practice. As has been stated above, as far as the writer is aware, mathematical solutions of the more complicated practical forms is not possible. In the following, however, a method is given by means of which the common cases of the simple floor and the single line of piles can be solved. The theoretical case of a foundation the section of which is an ellipse, and of which Mr. Beresford's cylindrical foundation is a special case, is also covered.

In this method use is made of the properties of confocal conics. For the suggested use of them in this connection the writer is indebted in the first instance to Morley Parker's "Control of Water", where they are used—wrongly, in the writer's opinion, since conditions (*c*) and (*d*) *infra*, are not satisfied—in conjunction with seepage flow under earthen dams.

The streamlines which represent seepage flow constitute a system of curves. Pressure distribution can be similarly represented by a system of surfaces (or curves, if we are considering a section) over each of which the excess pressure is constant. The curves of each system are not necessarily parallel, but have the following characteristics:—

(*a*) Every curve of one system intersects every curve of the other orthogonally.

This follows from the fact that flow takes place in the direction of the greatest rate of change of pressure.

(*b*) At any point, the normal distance between curves of one system representing constant increments of discharge or pressure is proportional to the normal distance between curves of the other system.

This follows from Darcy's Law.

$$q = k.\ a.\ \frac{dp}{ds}$$

If $q$ and $dp$ are constant, $ds$ is proportional to $a$.

It follows therefore that if two systems of curves satisfy the abov conditions, and if

(*c*) The fixed boundaries of sub-soil flow (including any fr water surfaces) are curves of one system, and (*d*) The surfaces of entry and exit give curves of the other syste the curves of the one system will be streamlines and those of t other contours of equal excess pressure.

Examples of such systems are : (*i*) Two sets of parallel lines at rig angles, and a set of concentric circles (or spheres) and their radii, u of which has been made in the above examples. These systems hav however, a very limited application in practice.

A more useful example is provided by systems of confocal conics the mathematics of which are dealt with in Appendix B. Here are giver

(1) a proof of the well-known fact that the hyperbolas and ellipses of such a system intersect each other orthogonally,

(2) expressions for the normal distances between curves, and

(3) proof of the proportionality of the normal distances between curves of the two systems representing equal increments of discharge and pressure.

Thus conditions (*a*) and (*b*) above are satisfied, and the curves may be used to represent seepage flow provided conditions (*c*) and (*d*) are met. The general equation of the curves is :

$$\frac{x^2}{a^2}+\frac{y^2}{a^2-b^2}=1$$

which gives an ellipse or a hyperbola according as $a$ is greater or less than $b$. In either case $2b$ is the distance between the foci and $2a$ is the length of the major axis, that of the minor axis of the ellipse being $\sqrt{a^2-b^2}$. If $a=b$, we get the special case where the curves reduce to the straight line, $y=0$. Between the foci this is the limiting case of the ellipses, while outside the foci it is the limiting case of the hyperbolas. Again, if $a=0$, we get the special case of the hyperbolas representing the straight line through the centre at right angles to the major axes.

Consider now Fig. V, which represents a system of confocal conics. The points A A are the foci and the length of the line AA is $2b$. Full lines are the confocal ellipses and the dotted lines the hyperbolas of the system. It is clear that this can be used to represent sub-soil flow under a simple floor if the ellipses be taken as streamlines and the hyperbolas as contours of equal pressure. In this case the surfaces of entry and exit will be the lines A C, which as we have seen, is a limiting case of the hyperbolas. The boundary on the upper side will be the floor, AA, which as we have also seen, is a limiting form of the ellipses. For the boundary on the lower side we have to imagine an ellipse infinitely removed, which represents with sufficient accuracy the conditions obtaining in practice when the sub-foundation is uniform to a considerable depth. Conditions (*c*) and (*d*) above are thus satisfied.

It may be noted here that if it is desired to reproduce these conditions in a model experiment, the external boundary of the sand must be made in the form of an ellipse whose foci are ends of the floor. Any other form will distort the flow conformation to some extent in the vicinity of the model.

Reverting to Fig. V, it is also seen that the conditions of flow when erosion has occurred downstream could be accurately represented by the conics if the limits of erosion be one of the hyperbolas. The hyperbola will correspond approximately with natural conditions at some distance from the work: close to it, however, a discrepancy occurs, as the slope of the curve is very steep, being vertical in the limit, a condition which cannot occur in practice. The flatter slope of nature is likely to give a steeper pressure gradient at exit near the floor, than would result from the conic conformation.

It is also clear that if the under surface of the work be one of the confocal ellipses (of which the circle is the special case, $b=0$), the requisite conditions hold. The pressure contours will then be radii. The case is of theoretical interest only, however.

Though outside the scope of this paper, it may be mentioned that the converse case, where the ellipses represent pressure contours, and the hyperbolas streamlines has a possible application to seepage flow from a canal.

Consider now Fig. VI, which represents the application of the conics to the case of the simple pile line. In this case the length of the pile is $b$, one focus being situated at its bottom end. As before, the ellipses represent streamlines, and the hyperbolas contours of equal pressure. The surfaces of exit and entry are the limiting case of the hyperbolas, the line $x=0$, and the inside boundary is the limiting ellipse, the line $y=0$, resulting from $a=b$. The outer boundary must again be supposed to be an ellipse infinitely removed. All the requisite conditions are thus satisfied and the conics will represent flow with mathematical accuracy.

The method can be applied to eroded conditions if the limits of scour correspond with one of the hyperbolas, and in this case there is nothing unreasonable in the assumption since no impossible slopes are involved.

In Appendix B, (*iv*) and (*v*), expressions are developed for the discharge between two ellipses and the change of pressure between two hyperbolas. Also for the rate of change of pressure and velocity at any point.

These are:

(*a*) The change of pressure between two hyperbolas is given by

$p=\text{H}.\ \frac{u}{\text{T}}$, where $u=\sin^{-1}\frac{e}{b}-\sin^{-1}\frac{e'}{b}$, and $e$ and $e'$ are major axes of the hyperbolas.

(*b*) The discharge between two ellipses is given by

$q=\frac{H}{TT}k.\ v$, where $v=\cosh^{-1}\frac{c}{b}-\cosh^{-1}\frac{c'}{b}$, and $c$ an $c'$ are the major axes of the ellipses.

(*c*) The rate of change of pressure at any point is give by $\frac{dp}{ds}=\frac{H}{TT/(c^2-e^2)}$, where $c$ and $e$ are the majo axes of the ellipse and hyperbola passing through the point

(*d*) The velocity at any point is given by

$$V=k.\ \frac{dp}{ds}=\frac{K.H.}{TT/(c^2-e^2)}$$

With the help of the formula, (*A*) and (*B*), a system of confoca conics can be drawn which will give the streamline and pressure distribution in the vicinity of any floor or pile line. This has been done in Fig. VII: to apply this diagram in practice, all that is necessary is to express the co-ordinates of the point in terms of $b$. The ellipses and hyperbolas shown represent equal increments of discharge and pressure, and are graduated in terms of $\frac{v}{TT}$ and $\frac{u}{TT}$.

For example, in the case of a pile line 50′ deep, in sand, the transmission constant of which is 0·01.

(*i*) To find the pressure at a depth of 60′, 20′ *d.s.* of the line, under a head of 20′.

The point corresponds with $x=1{\cdot}2b$, $y=0{\cdot}4b$, and is marked A in Fig. VII. It lies on the hyperbola, $\frac{u}{TT}=0{\cdot}14$, which means that, in the case of a pile line the excess pressure at the point will be 0·36H or 7·2′. The total pressure at the point with water at the floor level on the *d.s.* side will be 7·2′.

(*ii*) To find the rate of change of pressure at a point 5′ below the surface and 10′ *d.s.* of the line, under the same head.

The point corresponds with $x=0{\cdot}1b$, $y=0{\cdot}2b$ and is shown in Fig. VII as B. It lies on the hyperbola, $\frac{u}{TT}=0{\cdot}47$ or $e=0{\cdot}09b=4{\cdot}5'$ and the ellipse, $\frac{v}{TT}=0.06$, or $c=1{\cdot}03b=51{\cdot}5'$. Hence

$$\frac{dp}{ds}=\frac{H}{TT/(c^2-e^2)}=\frac{20}{TT/(51{\cdot}5^2-4{\cdot}5^2)}=\frac{1}{80{\cdot}5}$$

Again, for a floor, 160′ long, under the same head and on the same material,

(*iii*) To find the seepage discharge per foot of weir in a 90′ length of river bed.

The *d.s.* end of the river bed in question is 170′, *i.e.*, 2·13*b*′ from the centre of the system. It is shown on Fig. VII at C and lies on the ellipse

$$\frac{v}{\pi} = 0{\cdot}44.$$

Hence $q = H.k.\frac{v}{\pi} = 20 \times 0{\cdot}1 \times 0{\cdot}44 = 0{\cdot}088$ cs. per foot.

(*iv*) To find the velocity at a point 5′ deep and 10′ *d.s.* of the end of the floor.

The point corresponds with $x = 1{\cdot}125b$ and $y = 0{\cdot}0625b$, and is shown as D in Fig. VII. It lies on the ellipse, $c = 1{\cdot}13b = 90{\cdot}5'$, and the hyperbola, $e = 0{\cdot}98b = 78{\cdot}5'$.

$$\text{Hence } V = \frac{H.k}{\pi/(c^2 - e^2)} = \frac{20 \times 0{\cdot}01}{\pi/(90{\cdot}5^2 - 78{\cdot}5^2)} = \frac{1}{710} \text{ ft./sec.}$$

It is interesting to compare the rate of change of pressure at this point, *viz.*, $\frac{1}{7{\cdot}1}$, with that in Example (*ii*) above.

From (C) above it may be generally noted that the gradient increases as *c* and *e* approach each other, *i.e.*, as the point approaches the end of the floor or pile line. At these points the gradient is theoretically infinite. In the case of the upstream end of the floor or the bottom of the piles stability would be obtained from the containing material on the downstream side, but in the case of the downstream end of the floor local failure must occur on the application of the slightest head.

So much for the mathematical development of the theory ; we will now turn to the experimental side.

## Flow Conformation as Determined by Experiment.

It is not the intention of the writer to give any account of the experimental work itself as the reader desiring information on this subject can refer to the original publications. For our purposes it will suffice to give the results of the experimental determination of flow conformation in four cases. These are contained in the diagrams of Plates I and II. These have been prepared from the photographs of Dr. McKenzie Taylor's experiments published with his Congress paper of last year. Figs. I, II, III and IV are prepared from Figs. III, V, VIII, and XI of the paper respectively.

The photographs of course indicated the streamlines only and the diagrams were prepared by the following method. The streamlines of the photographs were traced and the lines of equal pressure computed from them graphically. This was done by dividing each stream tube, represented by the space between the streamlines, into units of length proportional to the width : each unit then represents an equal drop of pressure, and it is a simple matter to divide the total length into zones containing an equal number of units. The stream tubes were thus divided into ten zones, each of which represents a drop of pressure equal to ten per cent. of the total head.

After completing this, the zones themselves were similarly treated, thus dividing them into stream tubes of equal discharge, though, in this case, for clearness, the number of tubes has been limited to five, *i.e.*, each gives twenty per cent. of the total discharge.

Great accuracy is not claimed for these diagrams, as the small scale of the photographs, and, in some cases, the obscurity of the original markings, made it difficult to work with precision. This inaccuracy, coupled with certain defects in the original experiments which the writer has commented on elsewhere, accounts, in his opinion, for the lack of symmetry in the simple forms of Figs. I and II, and the non-orthogonal crossings. The diagrams are, however, considered sufficiently accurate to justify the general conclusions drawn from them.

The sub-soil flow in the models is, of course, affected by the external boundaries which do not occur in the prototype, but the effect of these on the conformation in the vicinity of the internal boundaries will be slight. The external boundaries are shown in Figs. I, III and IV : in the case of Fig. II, however, the dotted line represents the limits of the photograph which did not include the external boundaries of this experiment.

The first use we have for these diagrams is to test the results arrived at in the mathematical development of the theory, and for this purpose we have to compare Figs. I and II with Figs. V and VI, respectively. On doing so, the general similarity of the flow conformation as determined experimentally and theoretically is at once obvious. Exact agreement could hardly be expected, as, in addition to the sources of inaccuracy in the preparation of the diagrams noted above we have the following major discrepancies.

(*a*) The floor and pile lines of the models have an appreciable thickness, whereas the theory applies to a floor, the bottom of which is level with the ground surface, and to a pile line the thickness of which is negligible.

(*b*) The outer boundaries of the models do not correspond with those postulated by the theory.

In view of these discrepancies we may conclude that the agreement between theory and practice is as good as could be expected.

Figs. III and IV give the flow conformation for two other types of work. Fig. III is that of a floor with a pile line at one end and Fig. IV that of a floor with pile lines at both ends. Use will be made of these diagrams later when considering the relative merits of different types of defense. Before doing this, however, we must investigate the second part of the subject, *i.e.*, the conditions of flow which are dangerous and to be avoided.

**The Criterion of safety.**

Bligh's theory, after defining the most dangerous path of the water as that following the surface of the work, limits the average pressure gradient along it to figures supposed to be safe for specific materials. The American theory referred to above follows Bligh closely in this respect, the limiting gradients proposed being chosen to correspond with Bligh's when applied to the new theory. The writer knows of no theory which advances beyond Bligh in the matter of the safety criterion.

Bligh's theory in this respect appears to have been based on the popular misconception of the way in which 'piping' *i.e.*, the removal of material from under a work under the action of sub-soil flow, occurs; that is, that it is effected by excessive velocity. It is assumed that for a specified material there is a limiting velocity, and that if this is exceeded, the particles of which the material is composed will be carried away. The hydraulic gradients laid down by Bligh are supposed to be such that this limit will not be reached.

As far as the writer is aware there is no experimental evidence in support of this theory, and, as a theory, it is most unconvincing. Consider, for instance, the actual magnitude of the velocities involved. As the writer pointed out in the discussion on Khosla's paper in 1930, the actual velocity through the interstices of a fairly coarse material under a gradient as high as 1 to 1, is of the order of 1/50th of a foot per second and it is obvious that such small velocities could not transport the finer particles of which the material is composed, even were they free to move. That any particles held in position by the surrounding material should be moved in this manner is out of the question. Incidentally particles not so held can be contributing nothing to the support of the overlying work, and their removal is immaterial. Again, the velocity theory takes no account of the direction of flow relative to gravity: disintegration is supposed to occur under the same gradient whether flow is horizontal or vertical. The idea that the capacity of a material lacking cohesion to withstand the action of sub-soil flow is independent of the direction of the flow can, however, easily be disproved. The finest silt arranged in a vertical tube and subject to water pressure from below will stand a gradient of 2 to 1 or higher without failure, while a coarse material arranged in a slope corresponding with its angle of repose will fail on the application of the slightest gradient from within.

The writer submits the following as a more rational explanation of the cause of failure.

'In a body of uniform material subject to a hydraulic head, failure can occur at the surface of exit only, and will take place when the excess pressure set up by the sub-soil flow on any section exceeds the containing forces due to the weight and frictional resistance of the material down-stream thereof.'

This statement negatives the idea of any transfer of the material through its own interstices, but it will be noted that it applies only to uniform bodies. Any non-uniform body can be divided up into a number of smaller uniform bodies, and in this case it is conceivable that failure may occur at the surface of exit of one of the constitutent bodies and the material be transported through the interstices of other bodies lying downstream. This affords an explanation of springs and tunnels carrying material in suspension such as are occasionally seen in practice, possibly as a prelude to the failure of a work. A necessary condition for failure to occur in this way is that the downstream material should be so coarse that the material failing should be able to pass through its interstices. This subject will be reverted to later.

## Standard flow and bursting Gradient.

With the above conception of the way in which failure occurs it is possible to calculate the gradient which will result in failure under certain standardized conditions, in terms of the physical properties of the material.

The standard conditions are:

(*a*) The streamlines must be straight and parallel. From this it follows that the cross section of any stream tube will be uniform.

(*b*) The direction of flow must be vertically upwards. From this it follows that the surface of exit will be horizontal.

(*c*) The material must be uniform. This involves the same transmission constant, void percentage, and density throughout.

In a stream tube, in which these conditions are met, consider a horizontal section at depth '$y$.' The total uplift acting on the section due to the seepage flow will be $a.y.\frac{H}{L}$. The forces resisting failure are the weight of the material in the stream tube above the section plus the friction between the sides of the tube and the surrounding material. If the internal pressure exceeds these forces failure will occur. If the depth $y$ is small in comparison with the cross sectional area, $a$, or if the tube be surrounded by other material similarly stressed to near the point of failure, the frictional term may be neglected and we have simply the internal pressure balanced by the weight of the material. Expressed mathematically—

$$\frac{H.y.a}{L}=\frac{y.a.r}{w} \text{ or } \frac{H}{L}=\frac{r}{W}$$

where $w$ is the weight of unit volume of water, and $r$ is the weight in water of unit volume of the material.

It will be noticed that the above expression is independent of $y$. If this gradient is exceeded, therefore, failure will take place simultaneously throughout the tube. The breakdown will be sudden and not progressive.

Bligh's limiting gradients are flatter for finer materials : it is evident, however, that the quantity, $r/w$, is largely independent of the size of the particles of which the material is composed, but depends more on their specific gravity, and the extent of the voids. If $d$ is the former and $n$ the percentage of the latter, we have

$$\frac{r}{w}=(d-1)\ (1-n)$$

Outside limits for $d$, as found in nature, would be about 1·8 and 2·8, and the corresponding values of $n$ would be 0·4 and 0·2. These values would give $r/w=1{\cdot}44$ for the heavy compact material, and 0·48 for the light diffused one, corresponding with gradients of 0·7 and 2·0 to 1 respectively. The weight of the material, dry, in air, is given by

$$\frac{W}{w}=\text{d.}\ (1-n)$$

The weights of the above materials would thus be 140 and 68 lbs. per cubic foot respectively.

Denoting the ratio $r/w$, the bursting gradient as B, we have

$$\text{B}=\frac{W}{u}-1+n.$$

W and $n$ can easily be determined practically for any material.

Since fine silts have generally a low density, it is obvious that the specific gravity of the particles of which they are composed must be low. To this extent, therefore, the permissible gradient may be said to depend on the particle size.

The above figures illustrate what high gradients may prevail before failure occurs. Support for them may be found in the experimental work of Terzaghi in the United States, who found gradients of this order to be necessary for failure in models of weirs on sand foundations. It is understood, of course that gradients of this order apply to the condition of failure, and that a safety factor would be necessary to give a safe working gradient for design. There is no reason, however, why the safety factor should be excessive, and gradients of say 1 in 3 for gravel and 1 in 6 for silt should be practicable for the condition of standard flow.

Having established the circumstances when failure will occur under the conditions of standard flow, we will now consider the effect of varying these conditions. The first of these is that flow shall be parallel and we will now investigate the

**Effect of Variation of the Cross Section.**

The possible variations from parallel flow are, of course, infinite and we can, therefore, only form some general conclusions and investigate a few simple cases in detail.

In the first place, it is obvious that any variation in the cross sectional area will produce a corresponding variation in the pressure gradient, a reduction of section being accompanied by an increase of gradient and *vice versa*. If the reduction takes place at the downstream end of a stream tube there is danger of local failure under the higher gradient. Failure will not, however, necessarily be complete, since, while the total length of flow is reduced and hence the average gradient is increased, the result of the local failure may be to increase the area at exit and hence to reduce the local gradient more rapidly than the average is increased. Failure will continue, however, as long as the local gradient exceeds the bursting gradient for the material.

If a constriction occurs at the upstream end, although the local gradient may be excessive, this is generally immaterial as stability is assured by the weight and frictional resistance of the downstream material. Failure can only occur when the total pressure on any section exceeds these, and if this happens it will be sudden.

In the mathematical development of the theory expressions for the gradient at exit in certain simple cases of variation of the cross sectional area have been obtained. These need not be repeated here, but it is interesting to investigate the conditions under which failure will be progressive and total in these cases.

We have seen that in the case of flow converging on a line the pressure gradient at exit is

$$\frac{dp}{ds}=\frac{m-1}{\log_e m}\cdot\frac{h}{L}$$

and it can be readily shown that

$$\frac{dp}{ds}=\frac{m}{\log_e m}\cdot\frac{h}{s_1}$$

$h$ and $s_1$ remaining constant, this expression is a minimum

$$\text{when}\quad \frac{d}{dm}\left(\frac{m}{\log_e m}\right)=0$$

*i.e.*, when $\log_e m=1$, or $m=2{\cdot}718$.

For values of $m$ greater than 2·718, therefore, the gradient at exit will decrease as failure progresses and total failure will not occur unless $\frac{2{\cdot}718h}{s_1}$ exceeds the bursting gradient. If $m$ is less than 2·718, however, if failure occurs it will be progressive to total destruction.

For flow converging on a point it may be shown that a similar critical point occurs when $m=4{\cdot}0$.

In the case of two lengths of parallel flow, obviously if the gradient in the second part is excessive, failure will be sudden in that part. Failure in the second part will increase the gradient in the remainder but not necessarily to the bursting point.

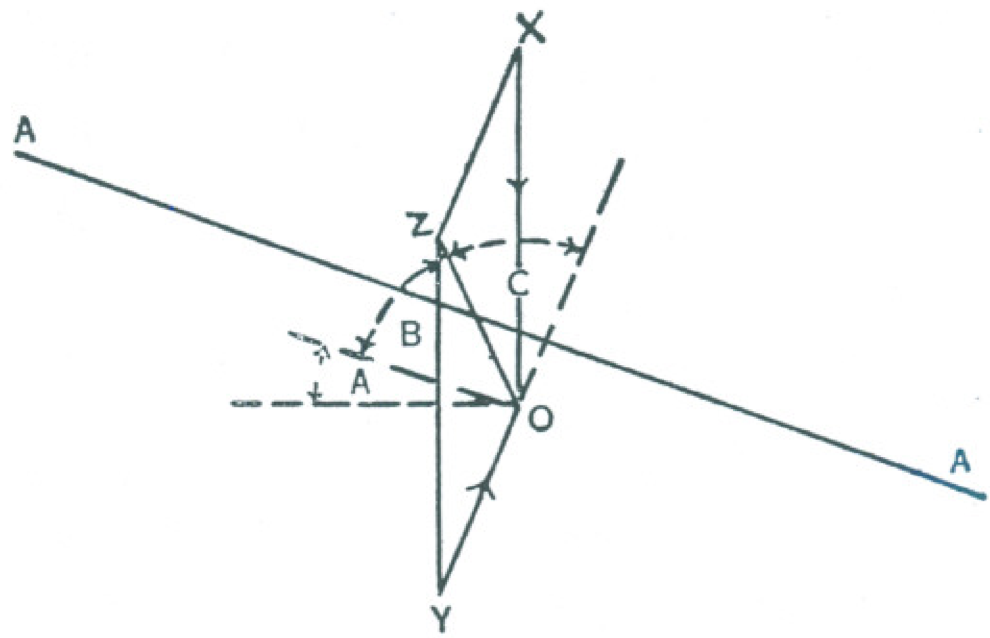
As regards the cases of converging flow appended to parallel flow, assuming the length of converging flow to be small and the average gradient to be reasonably safe, failure will be purely local, as the sectional area will increase much more rapidly than the average gradient. The practical application of this case, however, is to flow emerging through obstructions, such as stones or blocks, in which case the local failure will permit the obstruction to settle and thus reinstate the cause of the trouble. Hence failure would be progressive until controlled by some outside agency.

The second condition of standard flow is that its direction shall be vertical. We will now consider what happens in the case of

**Non-Vertical Flow.**

This case is fortunately capable of a mathematical solution. As the result of the seepage flow, a force, proportional to its volume, acts on the material in the direction of the flow. If flow is not vertical, this force can be combined with gravity to give a single resultant force inclined to the vertical. Stability will persist so long as the inclination of this resultant from the normal to the surface of exit is less than the angle of repose and failure will occur when this angle is exceeded. Obviously, if the surface of the material is already inclined at the angle of repose, no seepage flow can be introduced without disturbing stability; if, however, the surface is inclined at an angle less than that of repose, seepage flow can be supported to an extent depending upon this angle.

Consider the forces acting on unit volume of a permeable material in the vicinity of a surface of exit making an angle 'A' with the horizontal. These are




(*a*) Gravity.

This, represented by OX in the sketch, acts vertically downwards and is equal to

$$r = w(d-1)(1-n)$$

where, as before, *d* is the specific gravity, and *n* is the void percentage of the material, the whole being submerged.

(*b*) Excess pressure, due to seepage flow.

This, represented by OY, must act normally, to the surface of exit, AA, and is equal to

$$w.a.\ dp = w.a.\frac{dp}{ds}.\ ds = w.\ \frac{dp}{ds} \qquad \text{since } a.ds = 1.$$

If 'B' is the angle made by the resultant of these two forces represented by OZ with the surface AA, the horizontal component of the resultant will be $w.\frac{dp}{ds}$. sin A. The vertical component will be

$$r - w.\frac{dp}{ds}.\ \cos\ A.$$

$$\text{Hence } \tan\ (A+B) = \frac{r - w.\frac{dp}{ds}.\ \cos\ A}{w.\frac{dp}{ds}.\ \sin\ A}$$

Hence $\frac{r}{w} \cdot \frac{ds}{dp} = \tan(A+B) . \sin A + \cos A = \frac{\cos B}{\cos(A+B)}$

or $\frac{w}{r} \cdot \frac{dp}{ds} = \frac{\cos(A+B)}{\cos B} = \cos A - \sin A . \tan B$

Failure will take place when B is the complement of the angle of repose. If this be 'C', then tan B=cot C.

To obtain some idea of the effect of inclined flow in practice we may take a practical example. For an average material we may take $d=2{\cdot}3$, $n=0{\cdot}3$ and $C=38°$. Hence $r/w=0{\cdot}91$ and tan $B=1{\cdot}28$.

With these values the following table has been compiled. This gives for different values of A the corresponding bursting gradient, expressed in the form commonly used of the length of flow to unit head, and, in the last line, the ratio of this gradient to that for standard flow.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A° | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| B | 1·10 | 1·24 | 1·34 | 1·74 | 2·15 | 3·01 | 4·88 | 12·95 |
| Ratio | 1·00 | 1·13 | 1·31 | 1·55 | 1·96 | 2·74 | 4·44 | 11·8 |

It is seen that a 20° slope practically halves the bursting gradient, and that the latter decreases rapidly with steeper slopes.

The third condition of standard flow is that the material shall be uniform. Variation in the physical properties of the material at any site are not likely to have a great effect on the seepage flow, except in so far as they are evident in the variation of the permeability. This we will now consider.

**Effect of Variation of Permeability.**

As in the case of variation of the cross section, the possibilities here are infinite and we can only deal with a few simple cases and general inferences.

Consider first the case of a stream tube consisting of two parts, the transmission constants of which are uniform but different. If these parts are arranged in parallel, *i.e.*, if the areas of the parts on any surface of equal pressure are proportional, the pressure gradients through them will be the same. There will be no transfer of water from one part to the other, and the variation in the transmission constant will affect nothing except the discharge, which will, of course, be greater through the part with the greater permeability.

Suppose now the parts are arranged in series. In this case the discharge through each will be the same, but the pressure gradient in the case of the portion having the lower permeability will be higher. If the cross sectional area be constant and the transmission constant in the upstream part, (length '$n$L') be '$m$' times that in the remainder, it may be shown, *vide* Case VI of Appendix A, that

$$h = \frac{q.L}{k.a.} \cdot \left(\frac{n}{m} + 1 - n\right)$$

and that the pressure gradients in the upstream and downstream portions are respectively

$$\frac{p_1}{n.L} = \frac{h}{L} \cdot \frac{1}{n + (1-n).m}$$

and

$$\frac{p_2}{(1-n).\ L} = \frac{h}{L} \cdot \frac{m}{n + (1-n).m}$$

The similarity between these expressions, and those derived in Case III for different areas may be noted.

In the general case, if a stream tube of length, L, be divided into a number of lengths, $L_1$, $L_2$ etc., and the transmission constants, drop of pressure, and cross sectional areas be $k_1, k_2, .. p_1, p_2, .... a_1, a_2, ....$, respectively, then the pressure gradient in any section is given by

$$\frac{p_n}{L_n} = \frac{q}{k_n a_n}$$

where

$$q = \frac{1.}{h}\left( \frac{L_1}{k_1 a_1} + \frac{L_2}{k_2 a_2} + \ldots \right)$$

It will be observed that if a comparison be made between any two lengths of the tube, the pressure gradient is inversely proportional to the permeability. If, however, the permeability be uniformly reduced over a certain length of a stream tube, the increased pressure gradient, compared with the original one, will not be exactly inversely proportional to the transmission constants, but will be somewhat less, the reason being that the introduction of the length of low permeability reduces the discharge.

However, if we know the streamline conformation of a variable medium, it is a simple matter to determine the pressure gradient. This may be done graphically as follows. First mark out the pressure gradient, (in the form of divisions representing equal pressure increments) by the method already given, assuming the permeability to be uniform. Next, mark out the zones of different permeability, and, taking any zone as standard, work out the ratios of the transmission constants of the remaining zones to that of the selected one. Let these be $m_1$, $m_2$, etc. ; then, in any zone, $m$ of the pressure divisions will be equivalent to one in the standard zone. In the non-standard zones introduce new divisions equivalent to those in the standard zone ; total all the divisions and redistribute to obtain any desired pressure increments.

In the earlier part of this paper we have determined the streamline conformation for various cases on the assumption that the permeability is uniform. We must now consider what would be the effect of a variation in the permeability on this investigation. Unfortunately this generally completely alters the streamline distribution and consequently makes our special solutions inapplicable.

That this must be so may be shown by considering what happens when a streamline strikes at an angle the surface between two bodies of different permeability.

In the sketch, let AA be such a surface, and POQ a normal to it, cutting the surface at O. Also let the permeability of the medium Q be *m* times that of P.

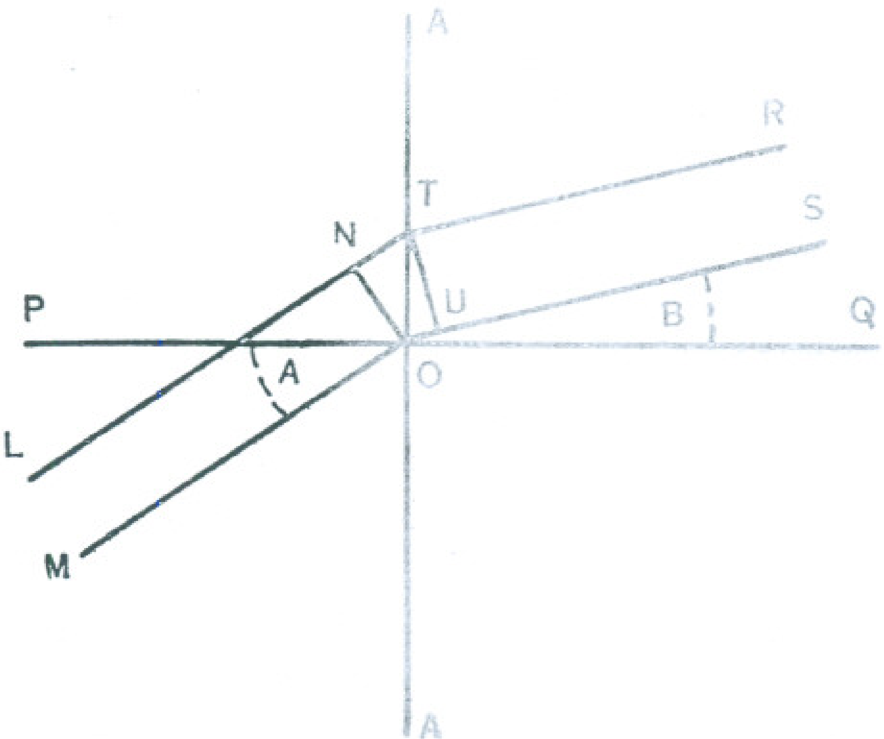
If LMRS be a stream tube making angles A and B with the normal in the mediums P and Q respectively, and ON and TU be perpendicular to LT and OS respectively, then these lines will be contours of equal pressure.

Since $q = k.a. \frac{dp}{ds}$,

we have $q = k.\text{ON}. \frac{dp}{\text{NT}} = m.k.\ \text{TU}. \frac{dp}{\text{OU}}$

Whence $\frac{\text{OU}}{\text{UT}} = m. \frac{\text{NT}}{\text{ON}}$ or $\tan B = m. \tan A.$

Thus it is evident that whenever a streamline passes from a body of high permeability to one of low, if its direction is inclined to the normal to the surface of separation of the two bodies, it will be deflected towards it and *vice versa*. Hence, if, in any body subject to seepage flow, the uniformity of the permeability be disturbed, the streamline conform-




ation will be altered, unless the variation of the permeability be such that uniformity is retained along contours of equal pressure. If the latter condition is satisfied the streamline conformation will be unaltered and the pressure conformation may be calculated with exactitude by the method given above.

In practice, of course, the possibility of this condition being satisfied is very remote and the added complexity makes a theoretical solution of a practical case where variation of permeability has to be taken into account very difficult. Generally speaking, a model experiment is the only way of determining the flow conformation in these cases.

In one important case, however, it appears to the writer that great error would not be involved in neglecting the alteration of streamline conformation due to variation of permeability. He refers to the case of low permeability occurring in the surface layers in the vicinity of a pile line. Alluvial deposits, such as usually occur at weir sites, generally consist of beds of clay, sand, or gravel, and, while the permeability will vary from bed to bed, it will generally be fairly uniform throughout any one bed. As these beds extend for considerable distances horizontally, a pile line driven through them will have similarly distributed material on each side of it. This means that the flow will be symmetrical about the pile, and the excess pressure at the bottom of it will be half the total head. If the material is uniform we know that the pressure gradient decreases towards exit and consequently all that we need do to ensure safety, is to see that the average gradient on the downstream side is safe. Obviously, also, the same criterion is sufficient if the permeability is generally greater in the surface layers. If, however, the permeability is low here, we may get locally dangerous gradients and these require to be calculated. A variation of permeability of this nature, apart from directly increasing the pressure gradient, will have the following secondary effect. Since the surfaces of contact of the beds are sensibly horizontal and the normal thereto vertical, as the streamlines pass from layers of high permeability to those of low, they will be deflected towards the vertical. The result of this will be to narrow the stream tubes and hence to steepen the gradient. As, however, in this region the slope of the streamlines is already very steep, it is obvious that this secondary effect cannot be great and consequently the writer considers it would suffice to redistribute the pressure contours on the basis of the streamline conformation for a uniform material, trusting to the safey factor to cover the small error involved.

Another case worth considering is that of a pocket of coarse material in a body subject to seepage flow. The main points to be noted here are that flow will be concentrated through the pocket with the results that streamlines will converge prior to entry and will be deflected away from the normal at the pocket surface. At exit the reverse processes will take place. Also the velocity in the pocket will be considerably increased. The degree of convergence will depend on the curvature of the surface : anything after the nature of sharp corners will involve rapid convergence and *vice versa*. Also, the increase of velocity will depend on the shape of the pocket, being greater in the case of a long narrow pocket whose length coincides with the direction of flow.

Now in what way is a variation of permeability likely to give rise to dangerous conditions of flow ?

In the first place, as we have seen above in the case of the simple pile, if there is a zone of low permeability at the downstream end of the seepage path, the pressures in the vicinity of the surface of exit may be increased sufficiently to result in local failure. Similarly a barrier of low permeability, situated anywhere in the path of the streamlines will result in a steepening of the gradient locally, maybe to bursting point. An essential condition for failure in this case, however, is that the material downstream shall be so coarse that the finer material lying upstream can be transported through its interstices.

Let us consider this condition. In an ideal material, in which all the particles are perfect spheres of diameter, '$d$', the largest sphere which can pass through the voids is of diameter, $d/6{\cdot}4$. In actual practice, however, owing to the lack of uniformity of the particles size and the arching effects which would occur, no serious movement could take place unless the relative particle size were considerably greater.

If this condition is satisfied, however, local failure will take place at the surface of exit from the fine material if a local gradient exceeding that the slope of the surface can stand, occurs. In the case of a closed pocket, however, the failure can only progress until the interstices of the pocket are filled. The result of this would be a slight general settlement, the effects of which would not necessarily be serious.

A further possible method of failure is that the velocity in a pocket might be so high as to scour and transport the fine material forming its boundaries. To the writer's mind, however, this is only conceivable in the case of very coarse materials, such as ballast or stone. In connection with flow in open channels the view is now gaining acceptance amongst irrigation engineers that there is a low velocity limit, below which no transportation of silt, however fine, can take place.

This limit is probably about 0·8 foot per second. In any case flow through sand, or finer materials must be streamline, as opposed to turbulent flow. This follows from the fact that the velocities involved are always far lower than the lower critical velocity for pipes of diameters of the order of the interstices, and is confirmed by the direct proportionality of discharge to head, which is also a property of streamline flow. Now turbulent flow is essential for silt transportation, since silt in suspension relies on the upward movement in eddies for support. Hence material can only be moved by velocity in sub-soil flow when the medium is very coarse.

With regard to the settlement which would follow from the filling of a pocket, it is interesting to note that, in the case of a seepage defence based on a pile line, no great harm could result. If dependence is placed on a floor, however, the settlement would be likely to result in the formation of hollows under the floor, greatly reducing its effective resistance.

Pockets in which the condition of penetrability is satisfied are not likely to be found in the natural river bed at weir sites, since it is very improbable that two materials one of which is fine enough to pass through the interstices of the other, should be in contact in alluvial deposits without intermingling. The writer knows of no case where such conditions have occurred. Such pockets may, however, be brought into existence artificially, as for example where deposits of stone or ballast are made as a temporary protection for a damaged weir, and are not moved or filled before permanent repairs are carried out. A more common example is the placing of a layer of ballast under concrete foundations. Any such deposits likely to endanger the resistance of a work to sub-soil flow should of course be avoided, if possible. If unavoidable, they should be removed, or rendered harmless by sand grouting. On the assumption that this will be done it is not considered necessary to provide for their possible occurrence by using an excessive safety factor in design.

We have seen that variation in the permeability of the sub-foundation will affect the conformation of the sub-soil flow and may result in dangerous conditions. We have also seen, however, that if the nature of the variation is known, it may be provided for, at any rate approximately, in design. In order to do this, a permeability survey of the site of all important works seems to be called for. This could be done by making core borings at points suitably distributed over the site, the permeability being estimated from the grain size of the specimens. For this purpose Hazen's formula may be used.

Alternatively it appears to the writer that it should be possible to measure permeability *in situ*. He suggests the use of a hollow probing rod fitted with a standard nozzle, the discharge of which would be measured under a given head. Flow from the nozzle would be approximately spherically diverging, and would vary directly as the transmission constant of the surrounding material. The nozzle would be calibrated in a material the transmission constant of which would be measured by other means. This method would have the advantage that the compaction of the material at site would not be disturbed.

Whatever the method of survey adopted, care should be taken to fill and tamp the holes left after the removal of the tools with material of low permeability. Such holes, if neglected, might easily be the cause of the subsequent failure of the work.

### Sub-soil Flow and Scour in Conjunction.

In searching for factors which might affect the stability of a material under the influence of sub-soil flow, it occurs to the writer that a possible source of danger is the movement of the water into which the sub-soil flow discharges. We can estimate with some accuracy the velocity which silt of a certain grade will sustain without erosion. Accepting the above theory we can also say what pressure gradient at exit a material will stand, assuming the water into which the sub-soil flow discharges to be subject only to such movement as is caused by the sub-soil discharge.

What, however, will be the effect of combining these two factors? As far as the writer is aware there is no experimental evidence on the subject, but it seems certain that, as the sub-soil flow tends to move the material at the surface of exit, scour would take place with velocities below normal.

In the absence of any investigation of the subject, however, one can only note the probability and provide a suitable safety factor, preferably to the depth of scour, where such conditions occur.

## Effect of Curvature of Flow.

In the standard conditions of flow we have assumed that flow will be vertical, from which it follows that it will also be straight. If, instead of this, the streamlines be curved, though terminating vertically, what will be the result? The effect on the discharge and pressure gradient will be nil (assuming the width of the stream tube to be negligible compared with its length). The containing forces will be reduced, however. Instead of having the whole weight of a stream-tube downstream of any section to resist movement as in the case of standard flow, we will only have the weight of the material vertically above the section. Fortunately in practice, the worst conditions on which a design would be based occur where flow is vertical.

## Effect of Resistance of Surrounding Material.

A study of any streamline diagrams, such as those given in Plates I and II, shows that the worst conditions which have to be provided for in design are generally very local. In some cases, as for instance that of flow in the vicinity of a pile line, it is obvious that failure cannot occur at such a point without disturbing the surrounding material, and since this is not so highly stressed, it must be able to exert a restraining influence through soil friction on the critical locality.

The extent of this action cannot be quantitively determined theoretically, though it might be measured practically by model experiment for any particular case. It is probably considerable in the case of deep pile lines, but, for the present, the writer proposes to neglect it, merely drawing attention to it as an additional factor of safety.

**Best form of work for Resistance to Seepage Flow.**

We have seen above that a body of material, uniform in cross section and permeability, can withstand a comparatively steep hydraulic gradient applied vertically. We have also seen that

(*a*) Inclining or curving the flow weakens the resistance of the material.

(*b*) Varying the section affects the pressure gradient. Converging flow has a steeper gradient at exit, while diverging flow has a flatter one.

(*c*) Varying the permeability affects the conformation. A high transmission constant at the downstream end reduces the pressure gradient there and *vice versa.*

We will now consider the application of these principles to practice, first examining the various forms of defence, of the flow conformation of which we have knowledge, with a view to determining their relative merits. In this investigation we shall be concerned with (*a*) and (*b*) only, it being assumed that the permeability is uniform or variable to a similar extent in each case.

For our purpose it is useful to consider the sub-foundation of a weir, through which seepage flow takes place as divided into three parts, that lying directly under the impervious length of the structure and those upstream and downstream of it. These may conveniently be named the upstream, mid, and downstream portions. Obviously we are chiefly concerned with the last, since, in the absence of pockets (which is assumed), so long as this remains in position, no movement can take place in the other two. Our chief interest in the mid-sub-foundation lies in the uplift pressures developed there and acting on the floor. The only function of the upstream portion is to reduce the steepness of the pressure gradients in the downstream portion by adding to the length of flow.

The flow conformation resulting from different types of works may be seen from the diagrams of Plates I, II and III, where the case of the plain floor, the simple pile line and two combined types are illustrated. Figs. 1 and 5 give the conformation under a plain floor. Fig. 1 is based on experiment, Fig. 5 on theory. In Fig. 1 it may be noticed that the thickness of the floor used in the model, *i.e.*, its depth below the surface of the sand, is large compared with that likely to be found in practice: although not very obvious from the figure, the result of this must be that the greatest concentration of streamlines is at the corners, and not at the points of entrance and exit adjacent to the work as in Fig. 5. In any case it is obvious from either figure that conditions at the downstream end (the figure should be symmetrical and flow is reversible without disturbing the conformation: so it does not matter which end we consider) are the opposite of what we desire, Flow curves rapidly away from the vertical upstream of the surface of exit, and we have the undesirable converging flow. Apart from general considerations, however, we have proved in the mathematical development of the theory, that, in the theoretical case where the bottom of the floor is level with the surface of the bed, the gradient at the ends of the floor is infinite, hence local failure must occur under any head. Moreover, it seems probable, though the writer is unable to prove it mathematically, that failure will be progressive to total destruction.

For consider what happens. After local failure a new surface of exit will be formed which must be a pressure contour. This cannot coincide with one of the original pressure contours since they were vertical at the upper end and the soil, under the influence of a seepage gradient can only stand a flat slope. It is obvious, however, that a flat slope will entail an increased concentration of flow in the corner and a higher local gradient, requiring a still flatter slope for equilibrium. In the limit, of course, with a level surface of exit, we would again have the infinite gradient, a conformation similar to the original one being produced, on a shorter basis. It is possible, however, that there may be an intermediate stage of equilibrium where the gradient associated with the concentration resulting from the flat slope is such that the material can stand. In the absence of a mathematically exact investigation the possibility cannot be denied, but in any case the equilibrium could not be very stable and the head required to produce a breakdown would be small.

We will now turn to Figs. 2 and 6 which represent flow under a simple pile line. Fig. 2 is of course obtained from a model experiment and Fig. 6 from theory. Examining these diagrams in the light of the general principles stated above, we see that they are very favourable. Concentration of flow and steep pressure gradients occur at the bottom of the piles remote from the surface of exit. Incidentally here again, it may be noted that the degree of concentration at the bottom of the pile is affected by its thickness. In the vicinity of the surface we have vertical flow for a considerable distance and also flow is diverging. In fact it is obvious from the formula developed in Appendix B for the determination of the pressure gradient,

$$\frac{dp}{ds} = \frac{H}{\pi\sqrt{(c^2 - e^2)}}$$

that the gradient in any stream-tube is actually a minimum at exit.

Comparing the two types, floor and pile, it is obvious that the latter has all the advantages, and the former all the disadvantages from the point of view of resistance to seepage flow and is therefore infinitely to be preferred. It may be argued, however, that the theory applies to an infinitely thin floor, while in practice a floor has a definite thickness, which would be placed below ground level: also drop walls would probably be used at the end of the floor. The answer to this argument is that the theory does not apply only to the infinitely thin floor, but to any floor whose under surface coincides with the bed of the stream on the downstream side. While this condition may not apply in the case of a floor as originally constructed it can very easily be brought about by erosion. If a drop wall is provided at the downstream end, the case becomes one of a floor and pile in combination which will next be considered. Unless the drop wall is very deep, however, erosion again will obviously give rise to dangerous conditions very rapidly.

As far as the writer is aware, no weir of importance has yet been constructed depending on a single line of piles for its resistance to subsoil flow: in all existing works where piles are used they are in combination with an impervious floor. The possibilities of the simple pile line, which in the writer's opinion are considerable, will be discussed later, other established types being considered first.

The combination of the pile and the floor is illustrated in Fig. 3, the outcome of a model experiment. In the actual model the pile was upstream, though a reversal of flow would not affect the conformation. Comparing this diagram with Figs. I and II, it is at once obvious that with flow from the pile end, the conformation in the upstream sub-foundation is very similar to that of Fig. II, while at the downstream end we have conditions which resemble Fig. I.

It appears that about 45 per cent. of the total head is absorbed upstream of the pile. The downstream side of the pile takes about 20 per cent, while the floor accounts for only 35 per cent. These figures will, of course, depend on the ratio of the lengths of the pile and floor, which, in this particular case was 0·63. It would be interesting to have the results of experiments on other models in which this ratio was varied: it is high in comparison with anything to be found in practice. The general caution previously given regarding the accuracy of the diagrams applies particularly to these figures which will also be affected by the thickness of the components relative to their lengths. After comparing this diagram with Figs. I and II, and concerning ourselves chiefly with conditions in the downstream sub-foundation, we would, I think, be justified in drawing the following conclusions.

The addition of a floor on the upstream side of a pile line reduces the pressure gradient, but only to a slight extent. For this particular pile to floor length ratio, the improvement appears to be only of the order of 10 per cent.

The addition of a pile line at the upstream end of a floor effects an improvement in the gradient as compared to that obtaining in the case of a simple floor. In Fig. III, the downstream half of the floor carries about 26 per cent of the head, while in the case of the simple floor it would take 50 per cent. The improvement is, therefore, of the order of 100 per cent. Comparing the combination with the simple pile however, the advantage is all with the latter. The conformation in the downstream sub-foundation of the combined type has all the disadvantages of the floor type, though in a lesser degree. If erosion reduces the downstream bedlevel to the bottom of the floor, local failure is inevitable, and will probably be progressive until held by the pile. Meanwhile, however, the floor has been undermined and probably only maintained at considerable cost. In any case, the conformation is not so good as in the case of the simple pile. In other words, the addition of a floor downstream of a pile line detracts from the safety of the work. This is a very important point. When considering the relative merits of floor and pile line in the design of a weir one usually meets the argument that the floor is necessary in any case, so why not make use of it for resisting sub-soil flow? It is not realized that the existence of an impervious floor may actually be a disadvantage from this point of view.

Fig. IV, prepared from Dr. McKenzie Taylor's Fig. XI, shows the case of a floor with a pile line at each end. The piles are of different lengths being about 30 and 50% of the floor length respectively. In this case the head taken by the outer faces of the pile is about 70% of the total; the inner faces take about 20% while the floor only accounts for 10%. The outer face of the larger pile takes about 40%: hence comparing with Fig. II, it is seen that with this pile downstream and of the same length as that in Fig. II, the improvement is of the order of 20% only, to gain which we have to add a second pile, two-thirds the length of the first and a floor of double the length. If the longer pile be placed upstream, obviously the downstream conformation is not so favourable as in Fig. II.

Dr. McKenzie Taylor's experiments covered one other type of weir, that consisting of two pile lines with a floor extending upstream and downstream. A diagram has not been prepared for this type as it is obvious that it cannot give so favourable a conformation as that of Fig. IV.

The above investigation has shown that by far the most suitable form of structure for resisting sub-soil flow is the single pile line, and while admitting that a careful experimental determination of the pressures may reveal errors in the diagrams, the writer considers that their magnitude would not be sufficient to affect this general conclusion.

### Uplift Pressures.

Inseparable from the use of floors to resist the action of sub-soil flow is the disadvantage of having to provide sufficient weight to resist the uplift pressure due to the seepage. These pressures have in the past been estimated in accordance with Bligh's theory, and, as stated above, it was the discrepancy between the theoretical figures and those obtaining in practice which led to the discrediting of the theory.

According to Bligh, the pressure gradient on the under side of a floor should be linear : both theory and experiment, however, show this to be incorrect. In the case of the simple floor, in accordance with the theory of confocal conics, the pressure at any point is given by

$$p = \frac{H}{2} + \left(\frac{H.TT}{2} \sin^{-1}\frac{e}{b}\right)$$

where $e$ is the distance from the centre of the floor and is negative when measured from the downstream side. The curve resulting from this formula, which is, of course, a simple sine curve, is drawn out to scale in Fig. VIII. By means of this the pressure, expressed as a percentage of the total head, at any point under a floor of any length, can be read off directly, if the distance from the centre is reduced to a percentage of the half length of the floor. At the upstream end the pressure is, of course, equal to the total head : the gradient is steep at first, flattens out in the centre, and again falls rapidly towards the toe.

The conditions assumed for the theoretical case of the simple floor would rarely be exactly fulfilled in practice. In the first place any plain floor would have an appreciable thickness, which would lower its under surface below the bed. The effect of this may be seen in Fig. I. From this it is evident that while the gradient retains a form approximating to the sinusoidal, the range of pressure is reduced, being about 85% of the total head at the upstream end and 15% at the downstream end. These figures would of course only apply to a floor with the same thickness to length ratio as the model.

Examples of the pressures obtaining in combined forms may be seen in Figs. III and IV. Fig. III shows that with the pile upstream, the pressure varies from 10 to 35% of the total head, figures that are considerably less than in the case of the plain floor. If the pile be placed downstream, however, the pressure ranges from 65 to 90%, requiring a very heavy floor for safety. In Fig. IV the pressure is from 40 to 50% or 50 to 60% of the total head according as the longer pile is at the upstream or downstream end. It will be noted that the uplift pressures are generally greater in the case of the combined types which are most effective from the point of view of resisting sub-soil flow.

It is evident that uplift pressure will be affected by variation in the permeability of the sub-foundation. If this is considerable, the pressure distribution calculated or obtained from model experiments for uniform material will not be directly applicable. If, however, the variation is not considerable, or the zones of variation approximately coincide with the pressure contours, great error will not be involved by assuming that the streamline conformation is not changed, and recalculating the pressures on this basis. If these conditions are not satisfied, as will usually be the case with floors, the only reliable way of obtaining the pressure distribution is by experiment on a model placed in sand the permeability of which is varied in a manner similar to that which will obtain in the prototype.

In designing against uplift it is customary to provide only a very small margin of safety. In view of this, and since the average pressures calculated in accordance with the old theory were not unduly high, while locally excessive pressures resulting from causes not provided for by the theory must have occurred, it is surprising that more failures in the past have not been traced to insufficient provision against uplift.

In view of this some engineers have attempted to demonstrate that the actual pressures acting on the underside of a floor are less than the theoretical. The explanation of this, it is alleged, is that pressure is reduced, or non-existent, over the actual area of the sand grains in contact with the floor. The writer is unable to accept this theory. He considers that the low uplift pressures recorded can generally be explained by variation of permeability. In the upstream sub-foundation of a work permeability generally falls with time owing to the tamping of the interstices by fine material brought down by the river and forced in under the action of the seepage flow. On the other hand at the downstream end erosion generally increases the permeability. The natural bed is washed away and replaced with coarser material, and the effect of this may be considerable. Low uplift pressures also can frequently be traced to flow divergent in plan.

The writer considers it necessary to provide for uplift pressure at the full calculated value, plus a margin of about 15 per cent for safety.

In general the necessity of restraining uplift under impervious floors adds considerably to the cost of a work involving this form of construction.

**Effect of Downstream Erosion.**

However carefully a work be designed from the point of view of stilling the turbulence created by the passage of the water over it, there is a probability of erosion occurring downstream of it when it is brought into use, and the effect of this on the seepage flow must be provided for. In the case of river weirs, the downstream floor is generally placed at the lowest level consistent with reasonable expense in dewatering. In some cases higher levels are used with a view to reducing seepage uplift, but this can only be done while effecting a reduction in the efficiency of the design from the point of view of the dispersion of flow energy. When the lowest level practicable is used, it is generally higher than the depth of probable scour, and protection for the additional depth is usually provided in the form of a flexible apron of stone or blocks, intended to settle from the downstream end and prevent deep scour in the vicinity of the inflexible floor. This form of protection is notoriously unreliable, stone and blocks are frequently washed out in bulk and it is not unusual to have the scour extending up to the edge of the permanent work.

Any such scour, of course, reduces the length of seepage flow. In design we must either prevent scour occurring in areas where its effect on the seepage gradient would be harmful, or else take into consideration the maximum scour possible when calculating the length of subsoil flow.

Considering the latter case, we know that sand under the influence of seepage flow can only stand at a very slight inclination, consequently scour of a given depth at any point will extend horizontally for a distance considerably in excess of this depth even in areas not directly subject to the scouring action. This action can take place to an extent which is only slightly moderated through a screen, such as a layer of stone, or a line of wells, as is provided by the numerous cases of cavitation which have occurred under weirs equipped with this type of protection. The only effective way of preventing horizontal spread of scour is a solid curtain wall, the depth of which is generally limited by constructional difficulties or a pile line.

The chief point the writer wishes to make in connection with downstream erosion, is that scour is effective from a given centre to a considerably greater distance horizontally than vertically. A seepage defence which depends on horizontal flow is, therefore, less effective, from the point of view of resistance to scour, than one embodying vertical flow.

## Quantitative Design.

It is obvious from the foregoing that, to resist seepage flow, a work must have depth below the bed of the stream at the downstream end; in this way only can a stable gradient be secured in the downstream subfoundation. This depth may be only that due to the thickness of the floor, or a shallow curtain wall since, though such forms are not desirable, stability is possible with them so long as bed erosion does not reduce the depth covered to a dangerous extent.

Given this depth the only conditions essential for safety are:

(*a*) The gradient in the downstream sub-foundation must be safe.

(*b*) The weight of the pervious structure must be sufficient to resist uplift.

As regards (*a*), the gradient required to produce failure in the material of the sub-foundation should be determined. If this material varies, that occurring in the vicinity of the surface of exit should be used for this purpose. Preferably the bursting gradient for conditions of standard flow should be measured by actual experiment, arranging a specimen of the material in a vertical column, and measuring the head required to produce failure. In doing this care should be taken that the area of the column is large in proportion to its height, in order to reduce the effect of friction on the sides of the container and to avoid arching from side to side. Alternately, the bursting gradient may be estimated from the physical properties of the material, using the formula,

$$B=(d-1)(1-n), \text{ or } B=\frac{W}{w}-1+n,$$ *vide p.* 81, *supra.*

Having determined the bursting gradient, a safety factor, F, has to be applied to obtain the permissible working gradient.

The proportion of the total head which will occur downstream of the toe of the work must also be determined, either by experiment directly or by estimation from the recorded data of similar forms. In estimating this proportion, allowance must be made for any variation which may have been determined in the permeability of the sub-foundation, also for any possible scour of the bed which may occur. As regards the latter, we have seen that in the case of a pile line any scour that occurs will produce a new surface of exit which will correspond approximately with one of the original pressure contours, and it is evident that the same will be true of scour in the downstream sub-foundation of any work terminating in a vertical face. Great error would not, therefore, be be involved by assuming that, for moderate depths of scour, the stream-lines would be unaltered, and recalculating the pressures on this basis. In practice the necessity for providing a reasonable margin for possible scour makes any form involving shallow depth at the downstream end undesirable and suggests the use of piles at this point.

If the proportion of the total head finally allocated to the downstream sub-foundation be '$p$', the average gradient in this critical locality will be $\frac{p\,.\,H}{b}$, where '$b$' is the depth of the vertical end of the work. We know also that the average gradient to any depth less than $b$, will be less than $\frac{p.\,H}{b}$, since flow is diverging in his region. Condition (*a*) above may, therefore, be expressed mathematically as

$$\frac{B}{F} = \frac{p.H}{b}.$$

As regards the safety factor, the writer suggests the use of the figure 3·0 pending further investigation. He considers that a series of tests to destruction on models of the simpler forms of construction should be carried out. The effect of the resistance of the material surrounding the highly stressed region at the base of the cut-off in providing a margin of safety has been referred to previously. It is possible that the suggested experiments may show that this factor is so potent that the safety factor might be considerably reduced. This safety factor is based on the assumption that the permeability of the sub-foundation will be reasonably investigated and any considerable variations allowed for in calculating '$p$.'

It is not intended to cover the variations which may occur in an unexplored river bed.

As regards (*b*), the uplift pressures would be calculated and allowed for as explained under that heading. The pressure under an impervious floor for a given head will obviously be smaller if a pile line is used at the upstream end of it, and will vary inversely with the length of the pile. Design, therefore, becomes a matter of proportioning the lengths of the pile lines to give reasonable pressures under the floors, utilizing any weight that may be available in piers and gates. To facilitate this, further model experiments, in which the length of the pile lines are varied with reference to the length of the floor and to each other, are desirable.

In designing the floor thickness, of course, the uplift under different flow conditions must be investigated and the worst case provided for. In this connection particular attention must be paid to the pressures under the trough of the standing waves which may occur.

**Pressure Relief.**

It has been realized for a considerable time that if a reliable system of pressure relief can be found, economy can be effected in weir design. An inflexible masonry floor is invariably necessary for some distance below a weir to resist the turbulent flow which occurs there. If this floor is impervious it has to be made heavier than is necessary for resistance to turbulence to be safe against uplift. Again, if an adequate outlet for seepage flow can be secured under the inflexible floor, the gradient in the downstream sub-foundation will be reduced, since water emerging here will come from a considerable distance above the weir, *i.e.*, upstream of the area from which the water discharging under the floor comes. The additional length of path here simplifies provision against erosion and secures a low gradient where local steepening may be expected from the presence of the blocks or stone of the protection apron. Again, the critical shortest path of seepage flow under the weir will be secured from disturbance by scour.

Past attempts at pressure relief have frequently been unsuccessful with the result that the device is now looked upon with suspicion by many engineers. Some of the methods used will now be described, and the reasons for their failure considered.

The principal methods employed consisted of the use of pressure relief pipes or unprotected inverted filters. Pressure relief pipes are small diameter pipes passing through the impervious floor and terminating in small pockets of ballast or in tube well strainers. Relief from pipes of this description can only be very local. Any one acquainted with tube wells knows how steep is the slope of the infiltration cone in the vicinity of the tube. The gradients in the vicinity of a pocket are liable to be worse, since in this case flow is converging on a point, instead of on a line, as in the case of the well. The liability of tube well strainers to choke is well known. Pockets of ballast probably choke up in a very short time owing to the penetration of the surrounding material, which cannot stand the high pressure gradients on the inclined surface of the pocket. The permeability of a choked pocket will be less than that of the surrounding material.

Inverted filters require very careful design and construction. They must consist of layers of material so graded that each is impenetrable by the underlying one, a condition difficult to obtain in practice. Moreover, the gradient in the material underlying the filter may be such as to give pressure which will overcome the total overlying weight a short depth below the filter bottom.

Again, both inverted filters and pressure relief pipes are liable to be choked by material deposited from above. They usually discharge into the open stream, and when this is in flood the seepage gradient through them is reduced, allowing silt to deposit and penetrate into them. It should not, however, be a matter of great difficulty to guard against this trouble. In the case of relief pipes, it would probably suffice to bend over the discharge end of the pipe, and in the case of a filter, the top could be covered over and a similar precaution taken with the discharge.

A pressure relief measure which has been uniformly successful is the so-called semi-impervious floor commonly found in the downstream section of the impervious floor of the older Punjab weirs. This merely involved constructing the lower layers of the floor in dry masonry.

The conditions necessary for efficient and permanent pressure relief appear to be the following :

(*a*) The pressure relief surface should be horizontal and should extend to the whole area to be relieved.

(*b*) The gradient in the underlying material should be safe.

(*c*) The filter and its outlet should be protected from any danger of choking from downstream.

In the writer's opinion the best form of filter is a layer of ballast on a layer of sand the permeability of which is about ten times that of the sub-foundation, *i.e.*, the average grain size about three times as large. The purpose of the sand layer is to provide for any locally excessive gradients which might result from the placing of the ballast directly on the sub-foundation and might cause the latter to penetrate the ballast. The ballast merely serves as a French drain to permit the escape of the seepage flow. Only one exit should be provided : more than one orifice permits flow from outside through the filter with consequent silting. The orifice should be arranged to discharge vertically downwards at the site of lowest pressure (if there is any variation in the external pressure) and might be provided with a silt trap in the thickness of the floor. Means for observing the pressure in the filter should also be provided.

In the writer's opinion a filter constructed in this manner could be relied upon to work efficiently and not to deteriorate appreciably with time.

**Application of Pressure Relief to Design.**

Assuming that an efficient and reliable system of pressure relief can be devised, it is obvious that the best means of defence is a simple line of piles. This should be placed under the main crest or gate line. If it were placed upstream of this we would have to provide a length of impervious floor between the pile line and the crest, and we have seen that a floor downstream of the pile line is disadvantageous. Apart from this, this floor, the integrity of which would be essential to the safety of the work, would be very badly placed for the purposes of inspection and maintenance.

Reliance being placed on the pile line for resistance to seepage, the floor of the work would be designed mainly from the point of view of resistance to the dynamic action of the water passing over it. While inflexibility and impermeability would not be essential for this purpose, there would be a short length of floor upstream of the pile line, and a longer length downstream of it, which, in order to secure the necessary solidity and strength, would be constructed in a form which would entail these qualities. The downstream portion would, however, be provided with a pressure relief system of the form indicated above.

A point requiring attention in connection with the design of this system is that water in contact with the sub-foundation must not be allowed to move at high velocities. The seepage discharge can be calculated given the streamline conformation and the permeability of the sub-foundation. Normally, of course, it will be very small : of the order of 0·02 cusec per 100 s. ft. of area of surface of exit. We have, however, specified that there shall be only one outlet through the overlying floor, and in leading the water to the entrance to this, there will be a concentration of flow which may involve high velocities. The writer would limit the velocity where flow is in contact with the sub-foundation to 0·2 foot per second. This figure is based on a non-scouring velocity of 0·6 foot per second and a safety factor of 3. Where velocities higher than this are necessary, the sub-foundation should be shielded, *i.e.*, the water should be carried in ducts within the floor designed for normal velocities.

The arrangement of the outlet of the pressure relief system is a matter requiring consideration. As stated above it must be designed to avoid choking by material deposited from above. It must also be placed at such a level that, when the floor is unsubmerged, the pressure under it will not be more than its weight can hold down. This, in effect, means that it cannot be placed at a height much greater than the thickness of the floor itself above its upper surface.

An interesting application of pressure relief is to use it to provide for uplift under the troughs of standing waves. As the necessity of this is not very widely realised a short explanation of the phenomenon will first be given.

As we have seen above, we can calculate or determine from model experiment the pressure under the impervious floor of a weir of given form resulting from any head. These calculations will give us the percentage of the total head across the work occurring at any point. Applying these percentages to the heads likely to occur in practice gives us the uplift pressure which will act under the floor. This uplift is, however, to some extent dependent on the level of the downstream water relative to the floor. If this level is below the bottom of the floor there may be departure of the streamline conformation of the work from the standard form owing to the surface of emergence being displaced. This would only happen when the floor is left high and dry and in this case the sub-soil flow would have a free surface on its upper side under the work. If this condition is likely to occur the corresponding streamline formation should be investigated specially, but is not likely to be dangerous owing to the additional length of flow which would be involved.

If the downstream water level is at or above the floor level the uplift will correspond with that calculated by the method given at the beginning of the last paragraph provided that the downstream water level has access to the upper side of the floor. The actual pressure under the floor will, of course, be greater than the uplift by the amount of the static head measured from the downstream level. If, for any reason, this static head is removed or reduced over any area, the unbalanced uplift under this area will be correspondingly increased. One occasion on which this may occur is when local areas of the floor are bunded off and dewatered for inspection or repairs. The necessity of guarding against excessive uplift when this is being done is, however, well known. That danger may occur when flow over the weir produces a standing wave is not so widely recognized. In this case the uplift is increased over the area covered by the trough of the wave by the amount that the surface is depressed below the downstream water level. This amount, which is a maximum at the foot of the wave, added to the normal uplift, may give local pressures considerably greater than those occurring when the weir is closed though the head across the weir is a maximum for the latter condition.

Some cases of failure on the older Punjab weirs have definitely been traced to this cause, and in recent designs it has been customary to provide for it by thickening the glacis over the area where the wave trough is likely to occur. This is an expensive business, as the area may be large and the amount of thickening required is approximately equal to the height of the wave.

It is readily seen, however, that if the floor be provided with a pressure relief system, the outlet of which is placed at the lowest point of the trough, all uplift is avoided. Similarly in the case of dewatering; the outlet can be temporarily connected up to the area within the bund. In the case of a gated weir with a horizontal floor, the standing wave will always form immediately downstream of the gates and the position in which the outlet should be placed is obvious. For a glacis weir the position of the wave will vary with the discharge and downstream level. (It is assumed that the weir is semi-modular, *i.e.*, that for a given upstream level, the discharge will be fixed.) The downstream level for a given discharge will, however, vary with regime changes. The outlet must be placed upstream of the position nearest to the crest in which a dangerous wave is likely to form. Fortunately for regime conditions the horizontal displacement of the wave with varying discharges is not great. A rise in level, tending to move the wave upstream, is counteracted by the greater discharge, tending to move it downstream. It is not difficult, therefore, to place the outlet for a settled regime. Changes in regime occur slowly, and it is possible to design the outlet so that it can be moved at low cost when such a change occurs. The weight of the floor itself would give a margin for error and this might be increased by providing the outlet with one or more pitot orifices, directed downstream, by means of which the pressure under the floor might be considerably reduced under the high velocities of the jet.

With the pressure under the floor reduced below downstream, static sub-soil flow would take place from the downstream to the low pressure area under the floor, and a cut off at the downstream end of the floor would be necessary to provide for this. The streamline conformation would be different from that of the simple pile line and would have to be specially investigated, but no objectionable form of flow is likely to arise.

If a standing wave occurs on a floor provided with a pressure relief system the outlet of which discharges downstream of the wave, pressures under the floor due to sub-soil flow will be obviated, but the uplift of the wave will remain. In this case the floor must be thickened to take these pressures.

## Flow Converging in plan.

An interesting case of sub-soil flow in conjunction with works on sand foundations is that of flow converging in plan. Flow of this type will occur at the ends of weirs when the spring level in the adjacent bank is high. In addition to the flow under the work there will also be flow from the banks towards the river bed, the effect of which will be to crowd the streamlines together. This condition may be accentuated in the case of a drain passing under a canal or any ground to which sub-soil water has access. In this case we have flow from the two sides and the end converging into what may be a very small area. Flow will obviously be converging in section also, and hence flow converging on to a point wil be approximated to. It has been shown, *vide* Case II of Appendix A, that with this type of flow the gradient at exit will be $m.\frac{H}{L}$, and since the area varies as the square of the distance, $m$ will be large and the gradient very steep.

The best form of defence against these conditions appears to be a cut off of piles arranged across the lines of flow, *i.e.*, in a semicircle about the end of the drain, so that seepage flow will have to pass under them to emerge in the bed of the drain. The exact flow conformation pertaining to this arrangement cannot be calculated, but it is evident that flow must be approximately vertically upwards and parallel downstream of the piles : hence the gradient in this section will be nearly uniform and no great error will be involved in assuming that gradients in excess of the average do not occur.

The proportion of the total head occurring downstream of the pile line will depend chiefly on the relation between the depth of the pile line and the radius of the circle in which they are arranged. The greater this ratio, the greater will be '$p$'. We can determine $p$ by experiment for standardized cases, for which it would be necessary to assume that the head is constant around the work. In this case it can be shown theoretically, *vide* Appendix C, that $p$ is greater than $\frac{b}{b+1\cdot17r}$ where $b$ is the depth of the piles and $r$ the radius of the circle in which they are arranged. This formula in only useful, however, when $r$ is small compared to $b$, say less than $b/2$. When $r$ is large compared to $b$, and hence to H, $p$ will approximate to 0·5.

The application of the theoretical case to practice is modified by the consideration that spring level may vary considerably within a short distance, also convergence of flow on a point is not complete since it merges into flow parallel in plan as we proceed down the drain.

In the absence of any experimental determination of the flow conformation under these conditions, the writer must content himself with pointing out the dangerous conditions which may arise with this form of flow, and indicating the best line of defence. The determination of $p$ for any particular case is a matter for judgment when all the conditions are known, with the aid of such special experiments as the importance of the case justifies.

Having determined $p$, the depth of the pile line would, of course, be obtained from

$$\frac{B}{F}=\frac{p.H}{b}$$

**Summary.**

The conclusions arrived at in this paper may be summarized as follows:

(1) In order to be in a position to design a weir on sand foundations which is safe against seepage flow we must know—

(*a*) the flow conformation resulting from different forms of defence, and

(*b*) the conditions of flow which are dangerous and hence to be avoided.

(2) As the basis for (*a*), a generalization of Darcy's Law is accepted. This basis assumes that at any point flow will take place in the direction of, and will be proportional to, the maximum rate of change of pressure.

(3) On this basis the flow conformation for various standard conditions is obtained mathematically in certain cases.

(4) Confirmation of the theory is found in the agreement between the conformation as thus determined and as given in the recently published results of experiments by Dr. McKenzie Taylor.

(5) Flow conformations for cases which cannot be solved mathematically are also obtained from these results.

(6) As regards (1) (*b*), the basis is advanced that failure can take place at the surface of exit only, and will occur when the pressure resulting from seepage flow exceeds the containing forces due to the weight and frictional resistance of the sub-foundation.

(7) It is shown that under certain standardised conditions of flow, the condition of failure can be related to the physical properties of the material of the sub-foundation by the formula

$$B = (d-1)\ (1-n)$$

where B is the gradient necessary for failure, $d$ the grain specific gravity, and $n$ the void percentage.

The standard conditions are—

(*a*) Flow must be parallel.

(*b*) Flow must be vertically upwards.

(*c*) The medium must be uniform.

Variation of these conditions is considered and it is shown that—

(8) Converging flow increases the pressure gradient at the downstream end, while diverging flow decreases it.

(9) For inclined flow failure will occur when the pressure gradient exceeds

$$B.\ (\cos A - \sin A \cot. C),$$

where A is the angle of inclination of the surface to the horizontal and C is the angle of repose.

(10) If the permeability varies, the gradient will be increased in regions of low permeability and *vice versa.*

(11) Variation of permeability is generally accompanied by a change in the streamline conformation.

(12) Examined by these standards it is shown that a pile line is greatly to be preferred to a floor as a defence against seepage flow, and that no work that does not terminate in a vertical defence can stand.

(13) For the quantitative design of a work the formula

$$\frac{B}{F} = \frac{p.H}{b}$$

is advanced. In this, F is a safety factor for which the value 3·0 is suggested, $b$ is the depth of the downstream vertical defence, and $p$ is the proportion of the total head supported by this portion of the work.

(14) The uplift acting under floors in different combined forms is considered, and it is shown that large uplift pressures are generally associated with the most efficient forms.

(15) The possibilities of pressure relief as a means of avoiding uplift are considered.

**Conclusion.**

The writer claims no perfection or finality for the theory put forward in this paper. He believes that the fundamental bases he has adopted for flow conformation and safety are justified by experience and trusts that engineers interested in the subject will endorse this view.

As stated earlier in the paper, the theory developed from these bases is very incomplete. On the mathematical side it is likely that cases, the solution of which has been impracticable to the writer may be solved by others whose mathematical knowledge is greater.

On the experimental side, obviously, considerably more work is indicated. This should include the determination of flow conformation for the forms embodying a floor and a pile at one or both ends, in which the lengths of the piles are varied both with reference to the length of the floor and to each other ; the effect of variation or permeability on the flow conformation for the simpler forms ; determination of the effect of the frictional resistance of surrounding material on the average gradient of failure from tests to destruction ; and determination of the pressure distribution for simple cases of defence against flow converging in plan. The Irrigation Research Laboratory is continuing its research on this subject and the writer hopes that the work indicated above will eventually be covered by their experiments.

Lastly the writer hopes that engineers may find the theory of use in the practical design of works and that it may thus be tested in the only really satisfactory manner, *i.e.*, in the light of actual experience in practice

## APPENDIX A.

## DESIGN OF WEIRS ON SAND FOUNDATIONS. MATHEMATICAL TREATMENT OF SIMPLE CASES OF SUB-SOIL FLOW.

*Notation.*

$a$—cross sectional area.

$s$—distance from origin.

$p$—excess pressure.

$q$—discharge.

$k$—transmission constant.

Suffixes '$_1$' and '$_0$' indicate the beginning and end of any section.

$L$—length of section, $=s_1-s_0$.

$h$—head across tube, $=p_1-p_0$.

Case I. Converging flow, area varying as distance.

In this case, $a=c.\ s$, where '$c$' is a constant.

$$q = k.\ a.\ \frac{dp}{ds} = k.c.s.\ \frac{dp}{ds}$$

$$dp=\frac{q}{k.c}.\ \frac{ds}{s}$$

$$h=\frac{q}{k.c}.\ \log_e\frac{s_1}{s_0}$$

Let '$m$' $= \frac{a_1}{a_0} = \frac{s_1}{s_0}$, then $s_1 = \frac{m.L}{m-1}$ and $s_0 = \frac{L}{m-1}$

Also $a_1 = c.s_1 = \frac{c.m.L}{m-1}$, and $a_0 = c.s_0 = \frac{c.L}{m-1}$

Whence $h=\frac{q}{k.c}.\ \log_e m=\frac{q.L}{k.a_1}.\ \frac{m}{m-1}\log_e m=\frac{q.L}{k.a_0}.\ \frac{1}{m-1}\log_e m.$

At exit, $\frac{dp}{ds} = \frac{q}{k.a_0}=h.\frac{m-1}{L}.\ \frac{1}{\log_e m}$

Let 'G' be the average gradient across the section, $= \frac{h}{L}$,

then $\frac{dp}{ds} = \frac{m-l}{\log_e m} . G$

**Case II**. Converging flow, area varying as the square of the distance. In this case, $a=c.s^2$

$$q=k.a. \frac{dp}{ds} = k.c.s^2. \frac{dp}{ds}$$

$$dp = \frac{q}{k.c} \cdot \frac{ds}{s^2}$$

$$h=\frac{q}{k.c.}\left(\frac{1}{s_0}-\frac{1}{s_1}\right)=\frac{q}{k.c} \cdot \frac{L}{s_0 s_1}$$

In this case, $\frac{s_1}{s_0}=\sqrt{m}$, and $s_1 = s_0 = L$

$$s_1=\frac{\sqrt{m}.L}{\sqrt{m}-1} \text{ and } s_0 = \frac{L}{\sqrt{m}-1}$$

Hence $h=\frac{q.L}{k.c.\sqrt{m}.s_0^2}=\frac{q.L.\sqrt{m}}{k.\ c.\ s_1^2}$

$$=\frac{q.L}{k.a_0.\sqrt{m}.}=\frac{q.L.\sqrt{m}}{k.\ a_1}$$

The gradient at exit will be

$$\frac{dp}{ds}=\frac{q}{k.c.s_0^2}=\sqrt{m}.\ \frac{h}{L}.$$

**Case III**. Parallel flow in two lengths of different area.

Let the lengths of the two parts be $nL$ and $\overline{1-n}.L$ and the corresponding areas $ma$ and $a$. Let $p_1$ and $p_2$ be the heads used in the two lengths, $h$ being the total head.

Then $p_1=\frac{q.n.L}{k.m.a.}$. and $p_2=\frac{q(1-n).L}{k.a.}$

$$h=p_1+p_2 =\frac{q.L}{k.a} \cdot \left(\frac{n}{m}+1-n\right)$$

or $q=k.a. \frac{h}{L} \cdot \left(\frac{m}{m.\ (1-n)+n}\right)$

In the second part the pressure gradient is constant and equals

$$\frac{p_2}{(1-n).L}=\frac{q}{k.a.}=\frac{m}{m.\ (1-n)+n} \cdot G$$

where G is the average gradient over the two parts $=\frac{h}{L}$.

The expression, $\frac{m}{m.(1-n)+n}$, approximates to $m$ as $n$ approaches unity. It falls off fairly rapidly as $n$ decreases. For $m=2$, with $n=0{\cdot}8$ and $0{\cdot}5$, its values are $1{\cdot}66$ and $1{\cdot}33$ or $0{\cdot}83m$ and $0{\cdot}67m$. For $m=5$, the respective values are $2{\cdot}77$ and $1{\cdot}67$, or $0{\cdot}55m$ and $0{\cdot}33m$.

**Case IV**. Converging flow (area varying directly) at the end of straight flow.

Let $nL$ and $(1-n).L$ be the lengths of parallel and converging flow, $a$ the area of the parallel portion, converging to $a/m$. If $p_1$ and $p_2$ are the heads absorbed in each part, then

$$p_1=\frac{q.n.L}{k.a}, \text{ and } p_2=\frac{q.(1-n)L}{k.a}.m.\frac{\log_e m.}{m-1}$$

$$h=p_1+p_2=\frac{q.L.}{k.a.}\left(n+(1-n).m.\frac{\log_e m}{m-1}\right)$$

If $n$ approaches unity, $q$ approximates to $k.a.\frac{h}{L}$.

The average gradient through the second part will be

$$\frac{p_2}{(1-n).L}=\frac{q.m}{k.a}.\frac{\log_e m}{m-1}=\frac{h}{L}.m.\frac{\log_e m}{m-1}=G$$

and the gradient at exit will be

$$\frac{m-1}{\log_e m}.G=m.\frac{h}{L}$$

**Case V.** Area varying as square at end of parallel flow. With the same notation

$$p_2=\frac{q.(1-n).L.}{k.a}\sqrt{m}$$

$$\text{and } h=\frac{q.L.}{k.a}\left(n+(1-n)\sqrt{m}\right)$$

As $n$ approaches unity, $q$ again approximates to $\frac{k.a.h}{L}$. Under this condition the average gradient through the second part will be

$$\frac{p_2}{(1-nL)L}=\frac{q}{k.a.}\sqrt{m}=\sqrt{m}.\frac{h}{L}$$

and the slope at exit will be $m.\frac{h}{L}$.

**Case VI.** A streamtube of uniform section, length, L, in length $nL$ of which the transmission constant is $m$ times that, $k$, in the remainder.

Here $q = \frac{m.k.a.p_1}{n.L} = \frac{k.a.p_2}{(1-n)L}$

$$p_1 = \frac{q.L.n}{k.a.m.} \qquad p_2 = \frac{q.L}{k.a}.(1-n)$$

$$H = \frac{q.L.}{k.a}\left(\frac{n}{m} + 1 - n\right)$$

Hence $q = k.a.\frac{H}{L}\left(\frac{m}{n+(1-n)\,m}\right)$

and the pressure gradients are

in the first part, $\frac{pl}{nL} = \frac{q}{k.a.m} = \frac{H}{L} \cdot \frac{1}{n+(1-n)\ m}$

and in the second part, $\frac{p_2}{(1-n)L} = \frac{q}{k.a} = \frac{H}{L} \cdot \frac{m}{n+(1-n)m}$

## APPENDIX B.

(1) To prove that any ellipse of the system

$$\frac{x^2}{a^2}+\frac{y^2}{a^2-b^2}=1\ldots\ldots\ldots.(i)$$

is cut orthogonally by any hyperbola of the same system.

The above equation may be written

$$a^4-a^2\,(x^2+y^2+b^2)+x^2b^2=0$$

which gives the values of $a$ for the particular ellipse and hyperbola which pass through the point $x, y$. If these are $\pm c$, and $\pm e$, respectively, then,

$$c^2+e^2=x^2+y^2+b^2$$

and $$c^2e^2=x^2b^2\ldots\ldots..(ii)$$

Differentiating $(i)$ above, we get

$$\frac{2x.dx}{a}+\frac{2y.\,dy}{a^2-b^2}=0$$

or $$\frac{dy}{dx}=-\frac{x}{y}\cdot\frac{a^2-b^2}{a^2}.$$

This is the slope of the tangent to the curve at the point $x, y$, and the ellipse and the hyperbola passing through this point cut orthogonally if

$$\frac{x^2}{y^2}\cdot\frac{c^2-b^2}{c^2}\cdot\frac{e^2-b^2}{e^2}=-1$$

*i.e.* if $$\frac{x^2}{y^2}\cdot\frac{c^2e^2-b^2(c^2+e^2)+b^2}{c^2e^2}+1=0.$$

Substituting from $(ii)$ *supra*, the right hand side of this equation

becomes $$\frac{x^2}{y^2}\cdot\frac{x^2b^2-b^2(x^2+y^2+b^2)+b^4}{b^2x^2}+1$$

which clearly equals nothing.

(2) To find the normal distance between the curves given by $a=c$ or $e$, and $a=c+dc$ or $e+de$ at the point of intersection of the ellipse and hyperbola.

The curves

$$\frac{x^2}{c^2}+\frac{y^2}{c^2-b^2}=1$$

and

$$\frac{x^2}{e^2}+\frac{y^2}{e^2-b^2}=1$$

intersect in the points

$$x=\frac{\pm c.e}{b},\ y=\frac{\sqrt{c^2-b^2}.\ \sqrt{b^2-e^2}}{b}$$

Here $\frac{dx}{dc}=\frac{e}{b}$ and $\frac{dy}{dc}=\frac{c}{b}\cdot\frac{\sqrt{b^2-e^2}}{\sqrt{c^2-b^2}}$

The normal distance between the ellipses given by $a=c$ and $a=c+dc$, which we will denote by $m$ is given by

$$m=\sqrt{dx^2+dy^2}.\ dc$$

$$=\sqrt{\frac{e^2}{b^2}+\frac{c^2}{b^2}\cdot\frac{b^2-e^2}{c^2-b^2}}.\ dc$$

$$=\sqrt{\frac{c^2-e^2}{c^2-b^2}}.\ dc.$$

Similarly, the normal distance, $n$, between two adjacent hyperbolas is

$$n=\sqrt{\frac{c^2-e^2}{b^2-e^2}}.\ de$$

(3) To show that the normal distance between curves of one system representing a constant increment of discharge or pressure is proportional to the normal distance between curves of the other system representing a constant increment at any point.

The normal distances are proportional if

$$\frac{m}{n}=\sqrt{\frac{b^2-e^2}{c^2-b^2}}\cdot\frac{dc}{de}=\text{ a const.}$$

Let $\frac{e}{b}=\cosh.u$, and $\frac{c}{b}=\sin.v$

then $\frac{dc}{\sqrt{c^2-b^2}}=du$, and $\frac{de}{\sqrt{b^2-e^2}}=dv.$

also $\frac{m}{n}=\frac{du}{dv}$

The normal distances are proportionate therefore, if $u$ is proportional to the discharge and $v$ to the pressure or *vice versa.*

(4) To find the discharge and change of pressure between curves.

For any streamtube

$$dq=k.a.\frac{dp}{ds}=k.m.\frac{dp}{n}$$

$$k.\frac{dp}{dq}=\frac{n}{m}=\frac{\sqrt{c^2-b^2}}{\sqrt{b^2-e^2}}.\frac{de}{dc}$$

Integrating, $\frac{k.p}{dq}=\sqrt{\frac{c^2-b^2}{de}}.u$, where $u=\sin^{-1}\frac{e}{b}$

Between the surfaces of entry and exit, $p$=H, $\frac{e}{b}$ varies from +1 to −1, hence $u$ varies from $\frac{+TT}{2}$ to $\frac{-TT}{2}$ and the variation equals, TT,

$$\text{whence}\quad \frac{k.H}{dq}=\sqrt{\frac{c^2-b^2}{dc}}.\ TT$$

$$\text{or}\quad p=H\ .\ \frac{u}{TT}$$

The discharge of any streamline is given by

$$dq=\frac{H}{TT}.k.\frac{dc}{\sqrt{c^2-b^2}}$$

Integrating this we get

$$q=H.k.\frac{v}{TT},\ \text{where } v=\cosh^{-1}.\frac{c}{b}$$

Cosh. $v=\frac{1}{2}\ (E^{v}+E^{-v})$, where E=2·7183.

The following table gives corresponding values of $\frac{v}{TT}$ and $\frac{c}{b}$,

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{v}{TT}$ | 0·1 | 0·2 | 0·3 | 0·4 | 0·5 | 0·6 | 0·7 | 0·8 | 0·9 | 1·0 |
| $\frac{c}{b}$ | 1·05 | 1·20 | 1·47 | 1·89 | 2·5 | 3·3 | 4·6 | 6·1 | 8·4 | 11·5 |

(5) To find the pressure gradient and velocity at any point.

The pressure gradient at any point is

$$\frac{dp}{ds}=\frac{dp}{n}=\frac{dq}{k.m}$$

$$=\frac{H.k}{TT}.\frac{dc}{\sqrt{c^2-b^2}}.\frac{1}{k}.\frac{\sqrt{c^2-b^2}}{\sqrt{c^2-e^2}}$$

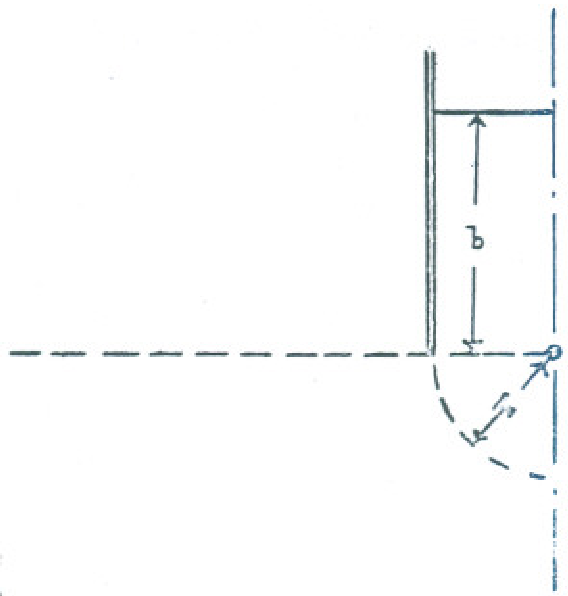
$$=\frac{H}{TT}.\frac{1}{\sqrt{c^2-e^2}}.$$

The velocity at any point is

$$K.\frac{dp}{ds}=\frac{H}{TT}\div\frac{k}{\sqrt{c^2-e^2}}.$$

## APPENDIX C.

To show that with flow converging in plan into a cylinder of piles of height $b$, and radius $r$, the proportion of the head occurring within the cylinder is not less than $\frac{b}{b+1{\cdot}17r}$.




Consider the path of flow as divided up into three portions, that within the cylinder, that between the bottom of the cylinder and the surface of a hemisphere or radius, $r$, suspended from it, and that outside the hemisphere.

A discharge, $q$, could be passed into the hemisphere from an infinite distance into the hemisphere under a head, $h_3 = \frac{q}{k.\ 2\ \pi.\ r}$, *vide* Case II of Appendix A, where it is shown that for this form of flow, $h = \frac{q}{k\ .\ c}\left(\frac{1}{s_0} - \frac{1}{s_1}\right)$. The head necessary to pass the discharge through the hemisphere must be less than $h_2 = \frac{2\ q\ .\ r}{3\ k.\ \pi.\ r^2}$, $2/3.r$, being the average length of flow.

The head necessary to pass the discharge through the cylinder cannot be less than $h_1 = \frac{q.\ b}{k.\ \pi.\ r^2}$.

Hence, $p$, the proportion of the total head used in the cylinder, must be greater than

$$\frac{h_1}{h_1+h_2+h_3} = \frac{b}{b+1{\cdot}17r}.$$

FIG. I

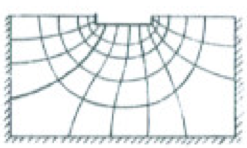


FIG. III

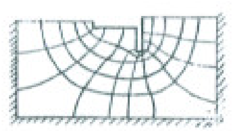


FIG. V

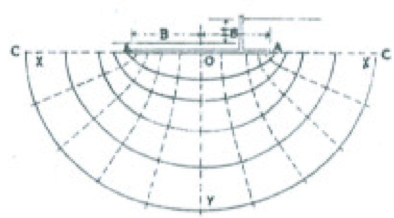



FIG. VII

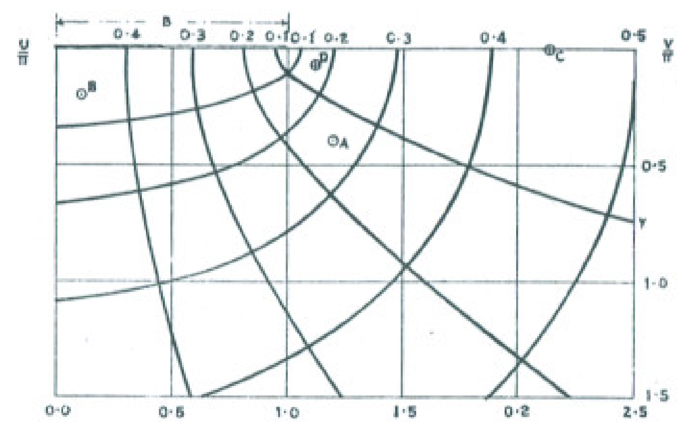



FIG. II

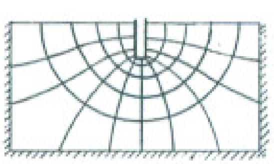


FIG. IV

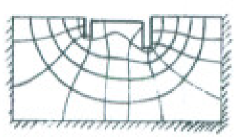


FIG. VI

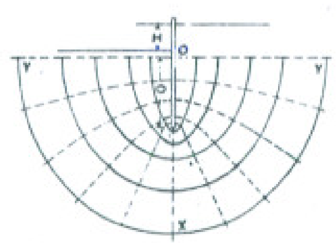
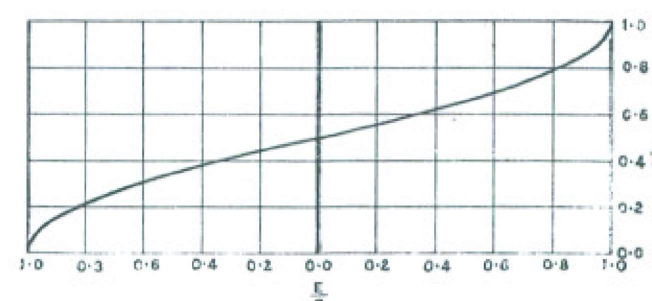



FIG. VIII



## DISCUSSION.

**The Author** first of all drew the attention of the members to one or two errors which had crept into the text of the paper.

There was one error which might have made some of the members wonder if they had not picked up by mistake a temperance tract or a copy of a motoring journal. The author referred to the letters TT which the members would have found scattered through the mathematics of the paper. The explanation of that mistake was simple. The type-writer which the author used in typing out the manuscript had no symbol for $\pi$ on that and as a substitute he used two capital Ts overlapping.

On pages 75 and 76 where the results of the mathematics of the confocal conics were set forth for "Major axis" in each case read "Semi-major axis."

Referring to page 78 para 4, the author remarked as follows:—

In Fig. II the dotted external boundary of the original had been replaced in the reproduction by a full line.

There were, the author thought, the only errors of importance and said that he would like to give a little additional information on the subject of the paper.

This information was derived from two sources, the first being a very interesting paper by Mr. L. F. Harza, entitled "Uplift and Seepage under Dams on Sand," which appeared in the September proceedings of the American Society of Civil Engineers. It is somewhat disconcerting when anyone had written a paper of this kind to find that some one else had published a paper on very similar lines shorty before his appeared. At the same time the author said, it is very interesting to see the extent to which an independent mind working on the same subject arrived at similar conclusions to his own, and in this case the similarity was remarkable; about two-thirds of the matter in either paper being common to both.

The author remarked that making the rather doubtful assumption that most of the members had read his paper, and were therefore conversant with much of Mr. Harza's, he could proceed directly to the points of difference of which the following were the most important.

In the first place Mr. Harza's paper outstrips the author's in the mathematical development of the theory, in that it gave a solution of the case of the floor and pile line in combination. The mathematical solution Mr. Harza attributes to Weaver. They were apparently originally published in the American Journal of Mathematics and Physics, Vol. XI, No. 2, June 1932. Mr. Harza did not quote the mathematics at full length but gave firstly, a diagram showing streamlines as calculated therefrom, which the author need not reproduce as it was very similar to Fig. III of the paper, secondly a diagram, reproduced as Diagram IX, giving the pressure distribution under the floor for different ratios on lengths of floor to pile; and thirdly the formula, also reproduced, which gave the pressure at any point down the face of the pile. This, the author thought, was sufficient to enable an engineer to deal with this particular case.

On the experimental side Mr. Harza gave the result of various cases he had worked out by the Hydraulic Electric Analogy. Those included cases of piles of equal length at each end of a flush floor; a case of the depressed floor, and cases involving drainage or pressure relief, and varying permeability. Time would not permit him to reproduce those results which could be referred to in the original.

The author would like, however, to make a few remarks about the Hydraulic Electric Analogy.

That method was similar to a graphical solution in statics. It was a convenient way of obtaining results in a case which was beyond the mathematical treatment or in which the mathematical solution was laborious. Its results could not, however, be used as experimental confirmation of a theory of seepage flow. Its use involved the acceptance of what he had designated the "Streamline theory" or what Dr. McKenzie Taylor in replying to my remarks on his paper of last year, termed "Forchheimer's hypothesis." From the fact that the Research Laboratory had now adopted the use of the hydraulic-electric analogy the author trusted that they might deduce that the "hypothesis" had now been promoted to the rank of "theory".

To revert to Mr. Harza's paper, he and the author differ slightly in the method they propose for quantitative design. His treatment of that would be found on page 98 of the paper where, briefly, he suggested making the average gradient downstream of the pile line safe in terms of the bursting gradient of the material. Mr. Harza's proposal was very similar only instead of the average gradient, he used the gradient at exit, or "escape gradient" as he termed it. His method was undoubtedly the more conservative, as the gradient in the downstream sub-foundation increased with depth. In practice, however, as he would show later, the head necessary to cause failure corresponded more exactly with that estimated by Mr. Harza's method.

To make use of the author's method a knowledge was required of the factor P, representing the percentage of the head carried by the downstream sub-foundations. Similarly to use Mr. Harza's method they would require to know the escape gradient. That involved the use of a formula of the same form as his in which P was replaced by the factor *q*. For the case of the floor and single cut off, his *P* was given directly by Weaver's formula. For the escape gradient, *q* was given by the second formula of Diagram IX. In Diagram X were plotted values of *P* and *q* for ratios of floor length to pile length varying frcm 0 to 10. Incidently, Mr. Harza has given figures for *q* for that case which did not correspond with those of Diagram X. Their derivation was not clear and Mr. Harza was unable to confirm them by the method of hydraulic-electric analogy. The results he obtained by that method, however, agree exactly with Diagram X.

The second source of information referred to by him at the beginning of the remarks was a series of experiments which he recently carried out in order to try and obtain some practical confirmation of the power of resistance of various forms of defence as estimated by the theory. Time would not permit those experiments to be described in detail and he could do little more than to give the results.

The first series of experiments was designed to test the bursting gradient formula given on page 81 of the paper. The test covered four kinds of sands, varying from coarse river sand to fine distributary silt and the correspondence between theory and practice was remarkably good.

An interesting point noticed in connection with that formula was that there was very little difference in the power of resistance between coarse and fine material. In the four cases experimented cn, the value of the bursting gradient obtained varied between 0·9′ and 1·1′ only.

The second series was designed to test the formula given on page 85 for inclined flow. There no agreement was obtained. The sand successfully withstood a much higher gradient than that expected of it. It now seemed to him that the surface conditions on which the formula was based, had little relation to the failure of the main body of the sand. The results corresponded much more closely with those which would be expected if the surface condition was neglected. In that case they have:

$$B' = B \cos A,$$

where B′ was the gradient of failure, and B was the standard bursting gradient.

Typical results were given in Table A.

It was observed during these experiments that in every case failure occurred at the top edge of the specimen, indicating that some factor connected with the apparatus was involved; such as a slight settlement of the sand from the side of the container. Could that have been avoided possibly better agreement with the latter formula would have resulted.

The author's last experiment consisted of the test to failure of models of the simpler forms of construction, *i.e.*, the single pile line, the flush floor, and the combination of those forms. The actual models experimented with were a 2″ pile line, a 4″ flush floor and the 4″ floor with a $2\frac{1}{8}$″ and a 1″ cut off. For the cases involving a cut-off the results were best studied by calculating the coefficient that had to be applied to the theoretical failure head to give that determined experimentally. Those results were given as Table B where each of the figures was the average of a number of experiments varying from 5 to 14. Variation from the average of individual experiments did not exceed 15 p.c. A figure which was not very excessive considering that the sands used were natural ones.

It would be seen that in the case of Mr. Harza's formula close agreement was obtained while for his formula the results were about 50 p.c. high. The difference is undoubtedly due to the factor which he had drawn attention to on page 91 of the paper, *i.e.*, the influence of the surrounding material. In the case of Mr. Harza's formula that influence appeared to balance, over the range of the experiments, the factor he had just mentioned, *i.e.*, the increase of gradient with depth.

Which formula was used in practice was a matter of no great importance; when $P$ and $q$ and were not known $P$ was more easily found experimentally. Mr. Harza's formula might be used as it stood but the experiments showed that for equal safety the safety-factor might be reduced by $\frac{1}{3}$ if any formula was used.

Experiments were also made on the flush floor alone and on the cut-off models with flow reversed. Theoretically in all those cases, the gradient at the toe was infinite and consequently failure should occur with the slightest head. Theoretically also, as soon as the floor exerted any pressure on the sand, it should penetrate slightly and cease to be flush. The floor used in the models was a hollow one with a rubber base, water pressure being applied internally. Displacement following the application of the original pressure could not be observed, but any subsequent movement was evident. The critical point of failure was difficult to detect, but the figures of average failure head given in Table C referred to points of considerable movement.

The bursting gradient of the sand used was 1·055. Those figures indicated that if a floor was free to settle on its foundations while remaining impermeable a considerable head was necessary to cause total failure. In most cases, however, a slight movement was detected prior to the head given as that of failure and personally, notwithstanding those figures, he maintained the opinion that those forms were unsuited for use in practice.

Reviewing the experiments as a whole, they formed confirmation of the theory, except in the cases of inclined flow and escapage from a flush floor. The reason for the divergence appeared to be that in both cases the critical conditions considered were very local, and the theory neglected the influence of the surrounding material. That such influence might be considerable, the author pointed out on page 91 of the paper. The divergence was in each case on the side of safety and the cases were not of great practical importance.

**Dr. N. K. Bose** said that the author was to be congratulated on his excellent summary of the present position of the science of sub-soil hydraulics in relation to weir design. Though he claimed no originality for that, and admitted that his presentation was far from perfect and complete, he thought that was the first time that the subject had been dealt with so very exhaustively in English. Though Forchheimer in 1917 and Pavlovsky in 1921 dealt with that subject from the theoretical point of view and Terzaghi extended it in 1925 to the practical field of weir design, nothing appeared in English until the proceedings of the International Commission on High Dams in 1934 discussed the subject from all points of view and made it available for them.

The speaker said that the author referred to his paper No. 140 of the Congress in which he derived for the first time from strict hydrodynamical consideration the potential theory of subsoil flow which the author called the streamline theory. The two cases of pucca floor and single sheet pile treatment by him at length were really solutions of this "Potential Theory" and had been obtained by Forchheimer in his paper of 1917. More recently a solution of the combination of pucca floor and a sheet pile had been worked out as a solution of the Potential Theory by Prof. Weaver and had been experimentally verified in the Research Institute. So the Potential theory derived by him from purely hydrodynamical considerations, and by many others as a pure extension of Darcy's Law, had been verified by the experiments in the Research Institute. It must be admitted that theoretical solutions for all forms of weir design might not be possible and recourse must be taken to experiments. But the fact that experiments and theory agreed in those cases where it had been possible up till now to obtain theoretical solutions would add confidence in future experiments on models for weir design, mathematical solutions for which could not be obtained.

The author had solved a number of simple cases mathematically in Appendix A. Those solutions had been obtained under very restricted boundary conditions and the applications he had made of those solutions in the body of his paper to different hydraulic structures met with in weir construction were hardly justified as the boundaries in the practical cases were not restricted as they were assumed for theoretical treatment.

**Dr. Vaidhianathan** said that the author is to be congratulated in presenting a comprehensive summary of the various theories that existed regarding the sub-soil flow under weirs. He pointed out that the theory for the flush case of a masonry floor has been well known for a very long time and is given in standard treatises on electrostatics or hydrodynamics, the solution being identical with that of an electric or magnetic dipole. The steamline theory of sub-soil flow depends on Darcy's law which is analogous to Ohm's law in electricity.

Regarding the theoretical solution of the case with one sheet pile, he said that this has been dealt with by Weaver in the journal of Mathematics and Physics, January 1932. But these cases are not of practical importance.

He referred to the author's remarks on page 68 and explained with the help of diagrams the various investigations carried out in the institute to determine the floatation gradients.

Referring to the remark in the paper that a velocity of $\frac{1}{50}$ of a foot was not sufficient to move the sand particles, he said that the small intersticial velocity introduced a pressure difference and this caused the particles to be moved.

Referring to pages 80 and 81, he said that the expression for the uplift gradient given by the author has been derived by Terzaghi many years ago and is also given in a recent paper in the proceedings of the American Society of Civil Engineers by L. F. Harza.

He further pointed out that experiments recently conducted in the institute have shown that the whole question of the uplift pressures in practical cases could be solved by the electrical method.

**Mr. A. N. Khosla** while criticising Mr. Haigh's paper said that the author deserved to be thanked for having drawn pointed attention to the fact that it was the emergent gradient that mostly governed the stability of a work, and that while a good deal of research had been done on the distribution of pressure and streamlines, comparatively little had been done towards investigating the conditions of pressure and streamlines which were dangerous.

Dr. Vaidhianathan had done some research on the limiting gradient in the Lahore Research Laboratory and Dr. Terzaghi had done similar work in Vienna.

The speaker fully endorsed the author's view expressed on page 107 under conclusions, that considerably more research work was required in respect of various factors governing the flow of water through the subsoil. But research must be concentrated on the limiting exit gradients and the factors of safety required for various classes of soils and foundation profiles.

The speaker said that the author had dismissed the temperature effect as of no consequence but it had been proved by careful experimental work that temperature had a considerable effect. More research was in progress in that respect.

The speaker said that he could not agree with the author that a single sheet pile line, (which he believed was meant to be under the crest) and no downstream floor was the ideal design. It was yet to be proved mathematically that that was the best design. In actual practice it might be safely said that with an arrangement of relief holes downstream and a single pile line—however carefully planned and laid—the engineer-in-charge of such a work would not sleep in peace.

The speaker said that the paper was a very well-considered one and brought out clearly the various points at issue. It had given the engineers a new orientation in designs of works on sand foundations. He would, however, lay stress on the fact that the field work, model work and mathematics must all go together to produce the best result. Few engineers had much stomach for mathematics and for them the visual pictures on a model was more convincing. The aim of the field engineer, the research worker and the mathematician should be to work out simple rules easy of application and yet absolutely correct. The mathematicians had an important part to play and it was hoped that Dr. Bose would come to the rescue of the author in the more complicated cases as he came to the speaker's rescue in respect of Panjnad results. Whatever results Dr. Bose pronounced as eccentric from mathematical considerations had been proved so by later research.

**Mr. Haigh** replying to the discussion said that Mr. Khosla queried whether the single pile line was the best form of structure and remarked that taking all the factors into consideration, he was of the opinion that when dealing with subsoil flow, it was undoubtedly the best form.

Replying to Dr. Bose, who objected to the use of symptomatic cases in practice, the author informed him that he had used them in very few cases but that he had no idea of using them qualitatively.

Replying to Dr. Vaidhianathan, the author said that he did not claim originality in bringing out the problem and added that the matter had been investigated before by others.

Regarding the point whether low velocity could transport silt, the author said that Dr. Vaidhianathan seemed to think that this was the case. He said that what he meant was that velocity could not move fine particles in the same way as silt is carried in the open stream.

Replying to Mr. Khosla, the author said that it had been known for a long time that variations in temperature affected the velocity of a flow. In other words it affected the permeability but in the case of a model experiment the change of temperature could not produce any variation in pressure. He added that difference of temperature between two particles would surely produce such variation.

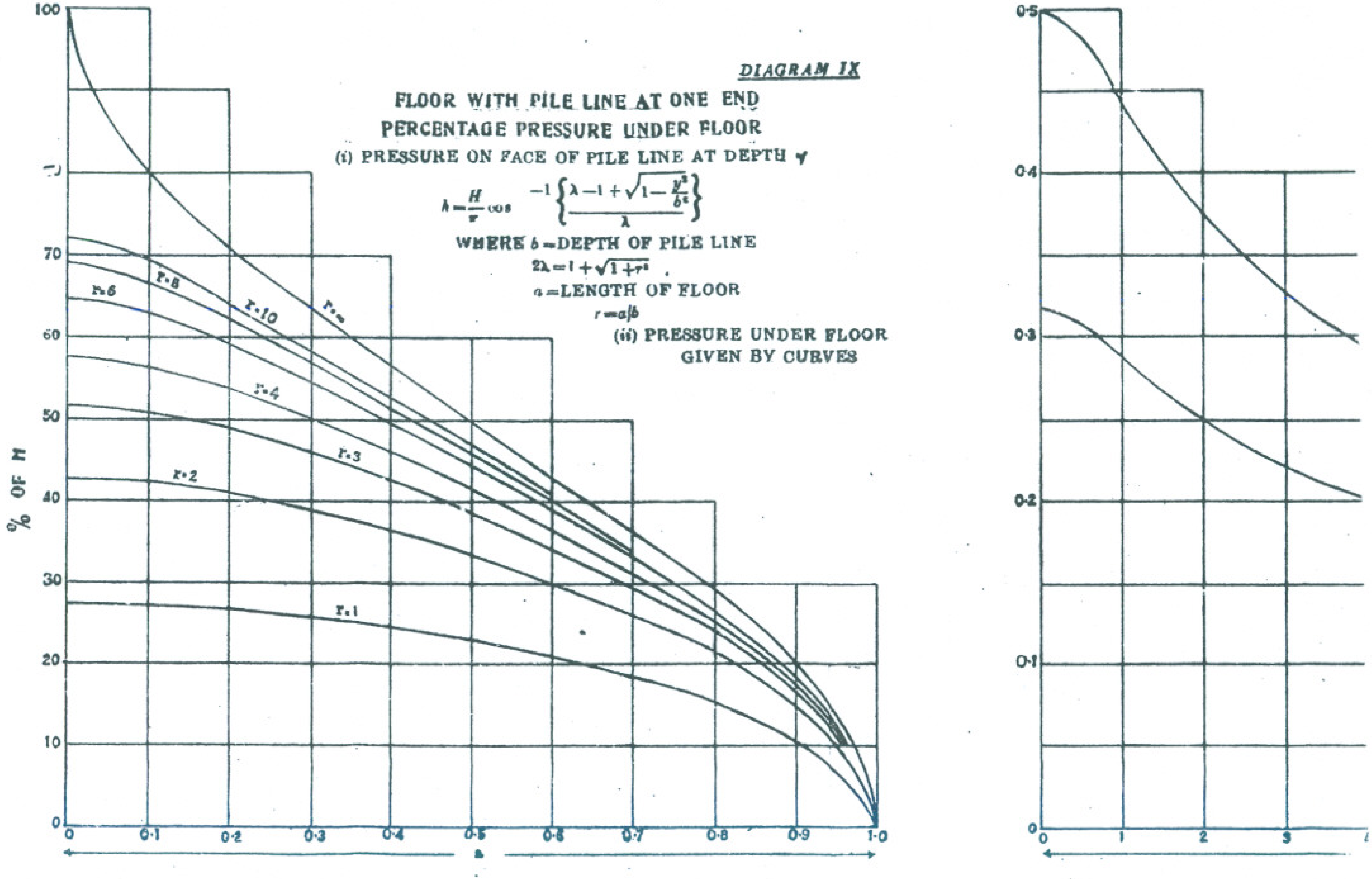


DIAGRAM IX
FLOOR WITH PILE LINE AT ONE END
PERCENTAGE PRESSURE UNDER FLOOR
(i) PRESSURE ON FACE OF PILE LINE AT DEPTH y
WHERE b = DEPTH OF PILE LINE
a = LENGTH OF FLOOR
r = a/b
(ii) PRESSURE UNDER FLOOR GIVEN BY CURVES
% OF H
r=1
r=2
r=3
r=4
r=6
r=8
r=10
100
70
60
50
40
30
20
10
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
0·5
0·4
0·3
0·2
0·1
1
2
3

DIAGRAM X

## FLOOR WITH PILE LINE AT END

i $p$ IN FORMULA $H = \frac{Bb}{Fp}$

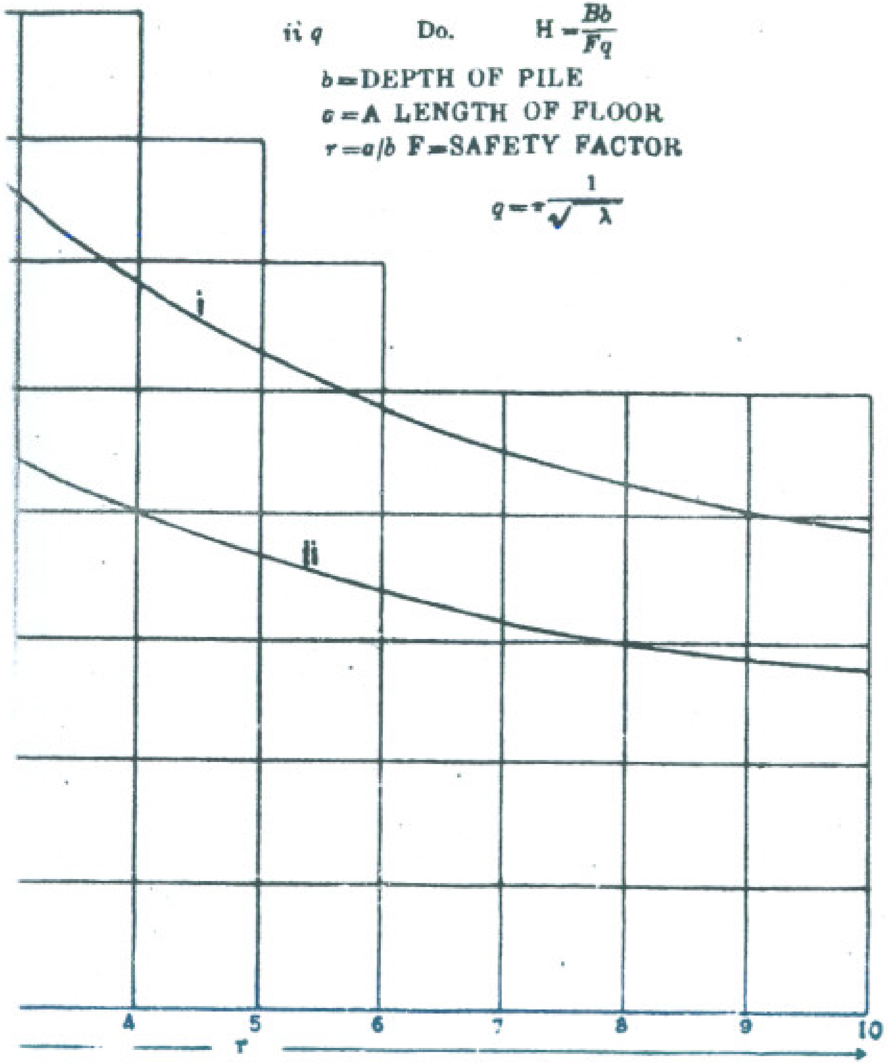
ii $q$ Do. $H = \frac{Bb}{Fq}$

$b$ = DEPTH OF PILE

$a$ = A LENGTH OF FLOOR

$r = a/b$ F = SAFETY FACTOR

$q = \frac{1}{\pi\sqrt{\lambda}}$




# TABLE A.

## EXPERIMENTS ON INCLINED FLOW

*FORMULAE*

(i) $B' = B(\cos A - \sin A \cot C)$

(ii) $B' = B \cos A$

| ANGLE A | | 0° | 10° | 15° | 20° | 25° | 30° | 35° |
|---|---|---|---|---|---|---|---|---|
| B | ACTUAL | 1·11 | 1·12 | 1·01 | 0·96 | 0·89 | 0·83 | 0·73 |
| | CALCULATED I | 1·14 | 0·87 | 0·72 | 0·58 | 0·91 | 0·26 | 0·10 |
| | Do. II | 1·14 | 1·12 | 1·09 | 1·07 | 1·03 | 0·99 | 0·93 |

# TABLE B.

## TES[illegible] [illegible]URE ON MODELS

HARZA'S [illegible]ETHOD $C_1 = \frac{qH}{Bb}$

WRITER'S Do. $C_2 = \frac{PH}{Bb}$

| METHOD | $C_1$ | $C_2$ |
|---|---|---|
| 2" PILE | 1·19 | 1·89 |
| 4" FLOOR WITH 2½" PILE | 1·04 | 1·54 |
| 4" Do. 1" do. | 1·20 | 1·77 |

# TABLE C.

## VALUE OF $\frac{H}{B}$ FOR FLUSH FLOOR DOWNSTREAM

| | |
|---|---|
| 4" FLOOR | 4·6 |
| FLOOR WITH 2½" CUT OFF | 4·9 |
| Do. 1" do. | 6·0 |