A Computer Program for Raft Analysis with Specific Reference To Wapda House Raft Foundation

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SYNOPSIS

The analysis of raft foundations by conventional manual methods is laborious and time consuming, involving a number of compelling assumptions. At the same time, the use of raft foundation is becoming more and more desirable from the point of view of soil conditions, load density and basement requirements etc. For an optimum design and to cover variation in soil parameters, the analysis has to be repeated a number of times. This becomes prohibitive in the conventional manual methods. It is therefore necessary that the alternate computer method of analysis may be resorted to. A number of computers are available in Pakistan which have certain storage restrictions. In the present exercise, a computer program has been developed which analyses raft foundations using finite difference method. The program accounts for the storage restrications of the available computers. A special compressed storage mode is used for this purpose. The program can handle a larger number of column loads placed in any arrangement and gives the resulting moments, torsions, shears, deflections, and bearing pressures at a number of node points for a given size and stiffness of the raft. The existing

WAPDA House raft foundation is an interesting example. At the time of its design, the analysis was manually performed. This raft has now been analysed on the computer program and the results are reported. It is interesting to note that an analysis which requires a manual work of 3 engineers for more than 2 months costing over Rs. 10,000/-, does only require about 5 minutes on the computer costing less than Rs. 100/-

INTRODUCTION

A raft foundation differs from the conventional rigid spread foundation in a way that it is essentially a two dimensional slab carrying a large number of column loads which makes its deflection pattern not easily comprehensible. A more predominent distinction arises from its usually enormous size which renders it flexible, causing its deflections to influence upon the soil pressures underneath. As illustrated in Fig. 1, the soil pressures under a rigid foundation are purely derived from statics, the deflections having no effect on pressures. However in a flexible foundation, the soil pressures are modified by the deflections following a logic that the pressures should increase at points where deflections are more. For simplicity, it is frequently assumed that the soil is elastic, that the pressures vary proportionately with the deflections and the constant of proportionately is termed as the coefficient of subgrede reaction (CSR). The value of this coefficient depends on the size of the raft and the nature of soil, but there exist a number of methods, as described by Bowles(1) Teng(2) and others, from which a range of expected values for a particular case can be determined.

A peculiar problem of raft foundation is the choice of its stiffness. An increase of stiffness beyond an optimum value results in increase of moments and consequently reinforcement which adds to the cost. Wherees, decrease of stiffness results in undesirable total and differential settlements. Thererore it is necessary to work out a compromising value of the stiffness of the raft.

A number of methods exist by which a raft foundation can be manually analysed. All these manual methods require a set of approximations and assumptions to make the analysis possible. One method is to divide the foundation in a number of independent strips, thus reducing the problem from two dimensions to one only. These strips may be treated as beams supported at appropriate column points, and acted upon by soil pressures including rest of the columns loads. By trial and error, deflections and soil pressures are made compatible. Another method, which is usually known as 'Soil Line' method, as described by Baker (3) assumes a certain soil line. The corresponding pressures are compared with the column load above, and the unbalanced load is distributed as shears in the slab. Still another method, as described by Timoshenko (4), uses the energy criterion. The deflections are described by an infinite series of a harmonic function whose terms are usually convergent. Except for very simple cases, these manual methods require laborious calculations which consume appreciable time. Their repititions, which are required either to cover a range of soil parameters or to optimise the design or even to check the original calculations become prohibitive.

Inspite of these complexities, raft foundations are becoming increasingly popular. These are specially desirable where soil strata is compressible and the column loads are intensive and closely spaced. Frequently where basement is already a planning requirement and excavations have to be done In any case, a raft foundation yields a cheap and convenient solution.

There are a number of examples in Pakistan where raft foundation is employed. These are WAPDA Housa, Lahore, Tibet Centre Karachi, Mercantile Mutual Insurance Building Karachi, Kuwait Embassy Building Islamabad, etc. However because of non-availability of computers the analyses have been done manually.

DEVELOPMENT OF THE PROGRAM

A raft foundation can be conveniently analysed on computer using first order thin plate theory. The general differential equation in terms of deflections W, in a combined equilibrium and compatibility form, can be written as :

$$\frac{\partial^4 W}{\partial x^4} + 2\,\frac{\partial^4 W}{\partial x^2\,\partial y^2} + \frac{\partial^4 W}{\partial y^4} = \frac{P}{D} \qquad \text{........ (1)}$$

where P is the intensity of lateral pressures acting in the same direction as W. An applied celumn load V distributed on an area $\partial x.\ \partial y$ produces an equivalent lateral pressure of $V/\ x.\ \partial y$. Futher the soil action can be idealised as linear springs with stiffness same as the coefficient of subgrade reaction CSR. A deflection W at any point causes an additional lateral pressure of - CSR. W, making the r-h-s. of equation (1) as :

$$\frac{P}{D} = \frac{V}{D\,\partial x.\ \partial y} - \frac{CSR.\ W}{D} \qquad \text{........ (2)}$$

The foundation is then dividied into a grid selecting the number of grid lines in each direction in such a way that the grid spacing H, is equal in both directions, so that it is easy to write finite difference equaiions. The grid spacing should be so small that each column is located near a grid point, though not necessarily exactly at it.

The finite difference equation at each grid point is then written, which takes the following form :

$$20W_o - 8\,(W_T + W_B + W_R + W_L) + 2\,(W_{TL} + W_{TR} + W_{BL} + W_{BR})$$

$$+ (W_{TT} + W_{BB} + W_{LL} + W_{RR}) = \frac{VH^2}{D} - \frac{CSR.\,H^4.w}{D} \qquad \text{.........} \; (3)$$

At free boundaries, the boundary conditions are ; moment across the boundary is zero, and shear across the boundary is zero :

$$\frac{\partial^2 W}{\partial x^2} + EMU\,\frac{\partial^2 W}{\partial y^2} = 0 \qquad \text{.........} \; (4)$$

$$\frac{\partial^3 W}{\partial x^3} + (2 - EMU)\,\frac{\partial^3 W}{\partial x\,\partial y^2} = 0 \qquad \text{.........} \; (5)$$

The finite difference equations and boundary conditions are diagramatically illustrated in Fig. 2.

The problem is thus linearised and is reduced to writing the simultaneous equations and then solving them. The computer program reads the geometry and material properties. In a subroutine, the coefficients of the simultaneous equations are generated in the form of a matrix $| A |$. The coefficients of the equations at the boundaries are generated taking the relevant boundary conditions into account. The loads are read along with their relative location corresponding to their nearest grid point and are then statically transformed into equivalent loads at the grid points. The right hand side load vector IRI is therefore generated.

A second subroutine solves the simultaneous epuations of the form :

$$| A | . | X | = | R | \qquad \text{.........} \; (6)$$

where the deflections W are resulted in $| X |$. Gauss' elimination method is adopted for the solution. From the deflections the values of moments MXX, MYY, torsions MXY, shears QXX, QYY and soil pressures BP are derived at each grid point using the following equations :

$$MXX = -D\left(\frac{\partial^2 W}{\partial X^2} + EMU\frac{\partial^2 W}{\partial Y^2}\right)$$

$$MXY = \frac{-D}{(1-EMU^2)}\cdot\frac{\partial^2 W}{\partial X\,\partial Y}$$

$$QXX = -D\left(\frac{\partial^3 W}{\partial X^3} + \frac{\partial^3 W}{\partial X\,\partial Y^2}\right)$$

$$BP = CSR.W$$

The flow chart of the program is illustrated in Fig. 3.

RESTRICTIONS OF AVAILABLE COMPUTERS IN PAKISTAN

Until recently computers were virtually not available in Pakistan. Consequently all structural calculations had to be either done manually or had to be sent abroad to the available computers in foreign countries. A number of rafts analysed in the past, which have been mentioned before, were designed on the basis of manual calculations.

A number of computers have now been installed, and are in working order. There is one at the University of Engineering and Technology, Lahore. It is an IBM-1300 machine with 16 K capacity of storage. The second computer is installed in the WAPDA House which is an IBM-360 machine with a storage capacity of 64 K. Two mere computers are available, one in Karachi and the other at the University of Islamabad. Both of them are IBM-370 and have storage capacities of 128 K. In addition of these storages, the latter three computers have disc output recording arrangement which can be utilised if storage problem arises.

It can be observed from the development of simultaneous equations for the raft foundation, that there are as many number of equations as the grid points are. The storage requirement goes approximately with the square of the number of equations. In addition extra storage is required during execution and also for determination of auxiliary results such as moments, shears etc. Thus for a problem of medium size quite extensive storage is expected to be required.

USE OF SPECIAL STORAGE MODE

The available storage capacities of the computers in Pakistan are described previously. It is evident that in order to run an analysis of the raft foundation of a reasonable size, it is necessary that the storage requirements are somehow reduced. In the present exercise, this was achieved by utilising a special storage mode for the coefficient matrix IAI since this occupies the largest storage area. It may be noticed, that since an equation at a grid point involves the terms of the surrounding points only to the second grid line, the resulting set of simultaneous equations appears in the form of a thin band. The terms outside this band are invariably zero. This is illustrated in Fig. 4. In Guass' elimination process the zero terms do not play any role, therefore it is attempted that no storage locations may be provided for these terms. The coefficient matrix IAI is stored in such a way that immediately after storage of a row in a band the second row is stored giving no allowance for the zero elements. The rest of the process is programmed in such a way as to take this modification into account.

In this process, the band width is important In order to further reduce the storage requirement, the grid points should be so chosen that a minimum band width is resulted. If however, the problem is so large that storage has to be further curtailed, then the disc output record has to be utilised. In the present exercise, however this was not required.

WAPDA HOUSE FOUNDATIONS

WAPDA House designed by the Italian architect Edward D. Stone in Lahore is an important building. It is used for semi-commercial and offices purposes. On two basements and a spacious ground floor it rises eight stories high. Plan dimensions are approximately 225 × 225 ft. There is an outer periphery of relatively nominaly loaded verandah columns and two inner peripheries of heavily loaded columns. Further inside, there are two concentric circle lines of lightly loaded columns except those next to inner corners which are again severely loaded. Thus, the pattern of column loads is unique, in a sense that firstly no three columns are in a straight line and secondly great differences in magnitudes of column loads occur quite near to each other. The columns rest on a solid 5 ft. thick reinforced concrete raft overlying soft to medium silty sand.

The choice of a raft foundation has probably been made from the conception of the basement in excavation, the particular pattern of column loads and the soil conditions. The coefficient of subgrade reaction has been suggested to be in the range of 15 to 40 Kips/Cuft. For this soil and the size of raft, it has not been possible to provide a rigid foundation. A diagramatic illustration of the plan area is given in Fig. 5.

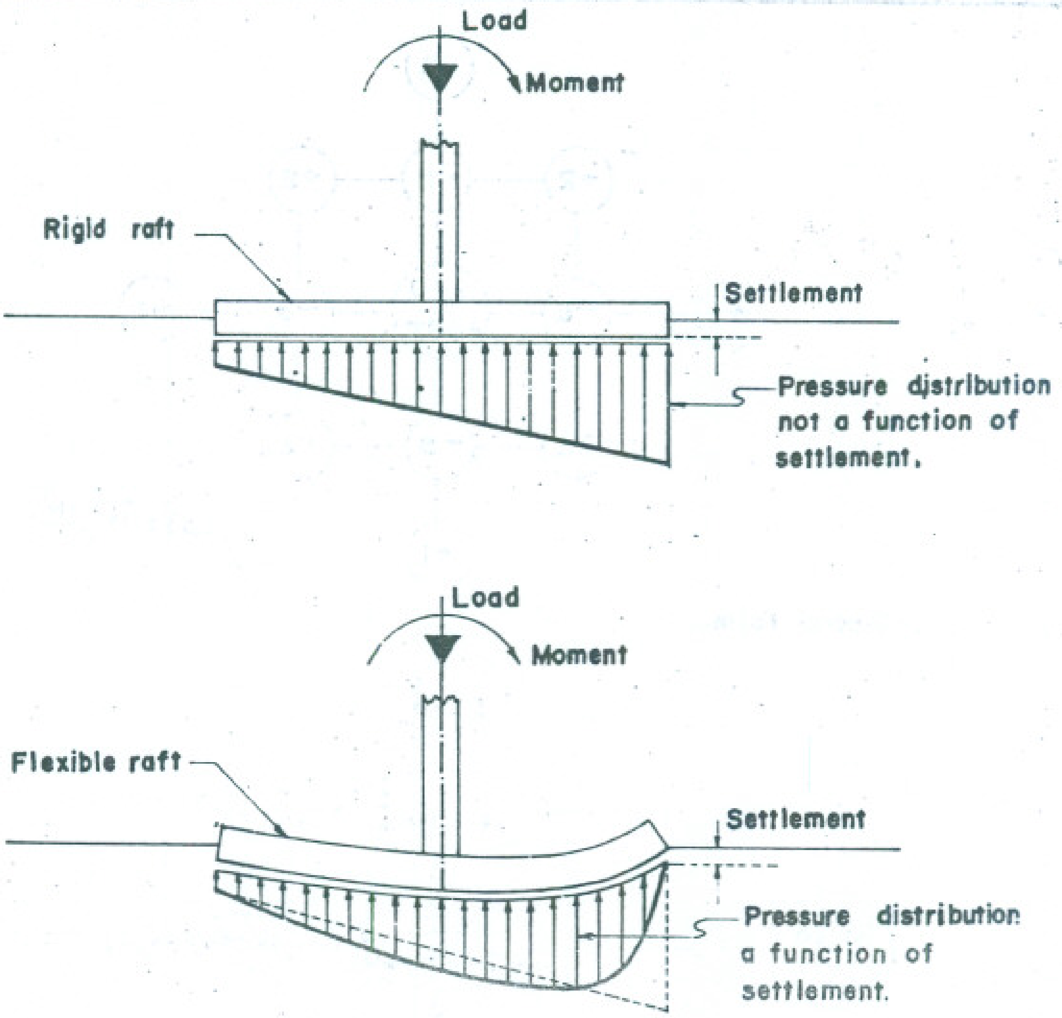
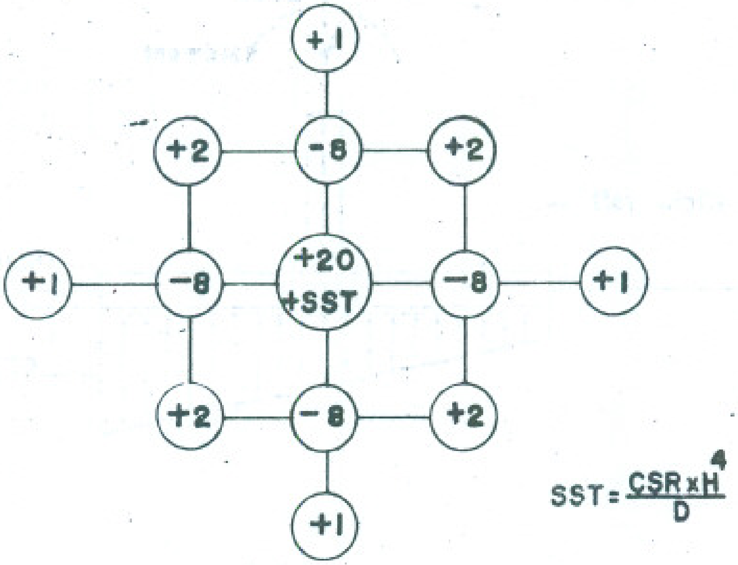
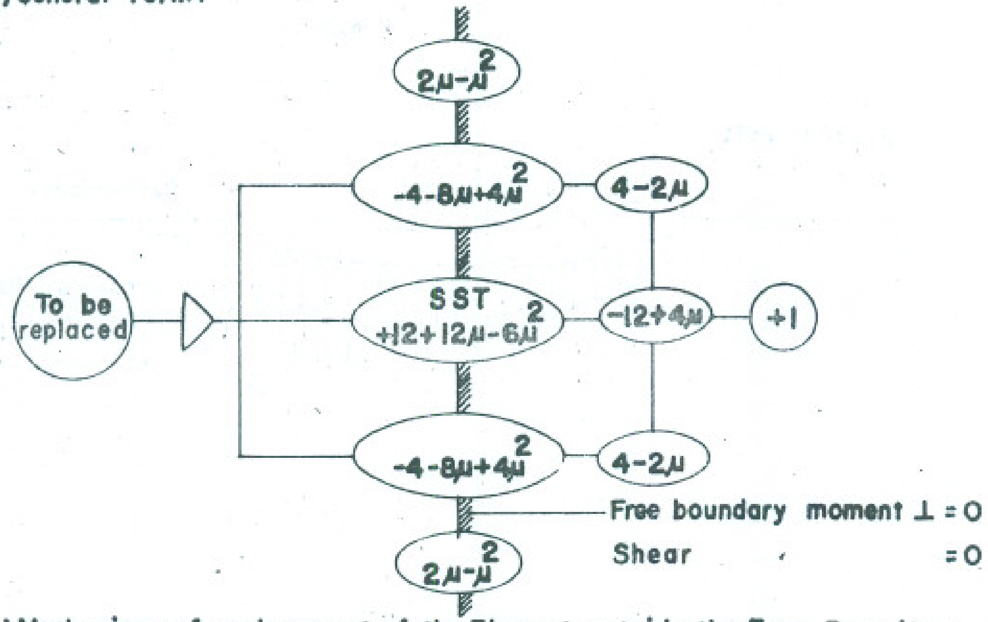


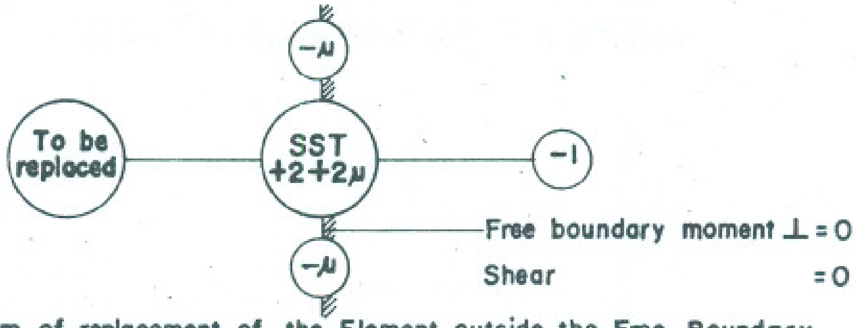
Fig. I: Comparison of Conventional Rigid Foundation with Flexible Raft on Elastic Subgrade.



(a) General Form.



(b) Mechanism of replacement of the Element outside the Free Boundary on Second Grid Line.



(c) Mechanism of replacement of the Element outside the Free Boundary on First Grid Line.

Fig. 2: Diagramatic Illustration of General Finite Difference Equations and the Boundary Conditions.

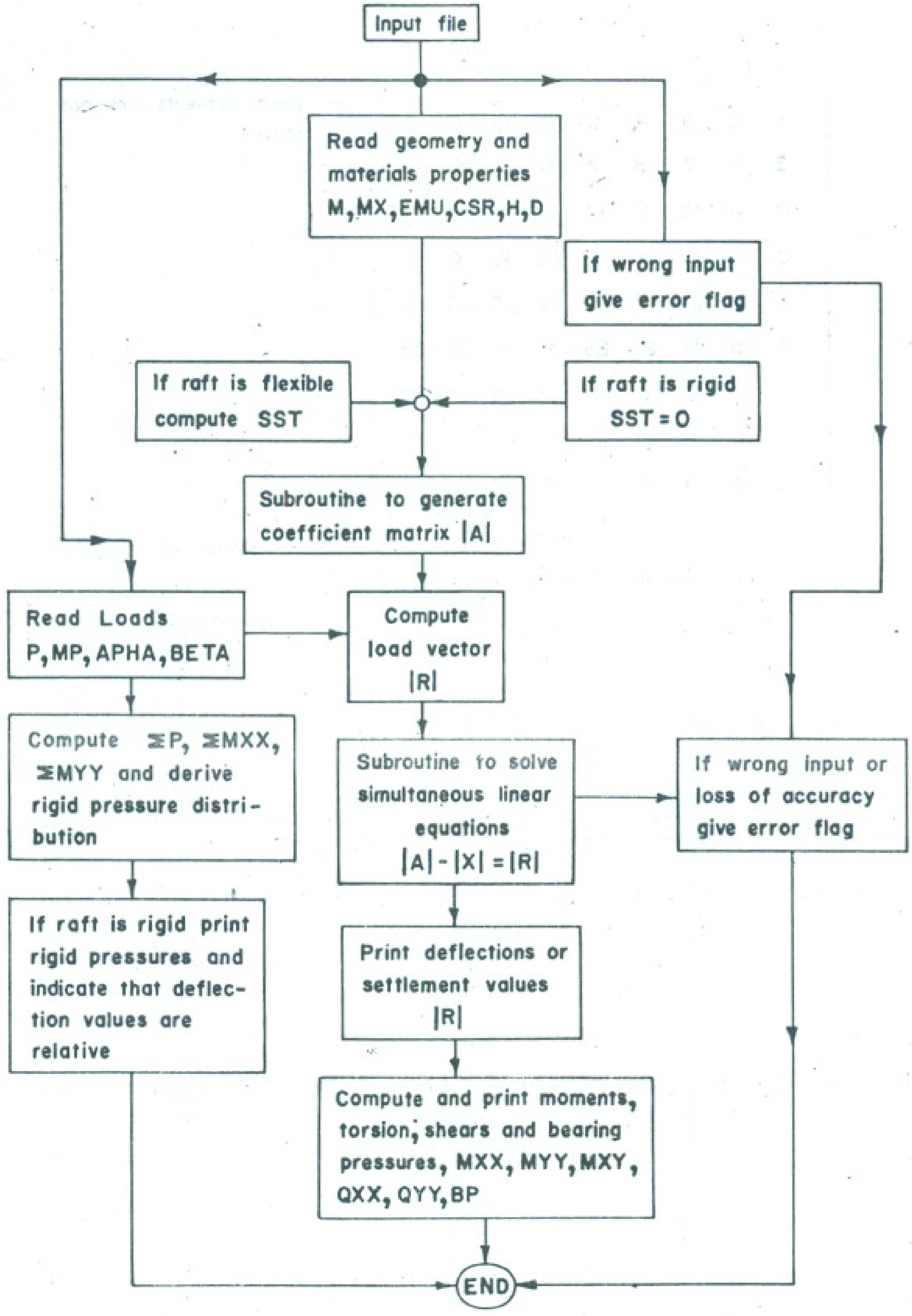


Fig. 3: Flow Chart of the Program for Raft Analysis

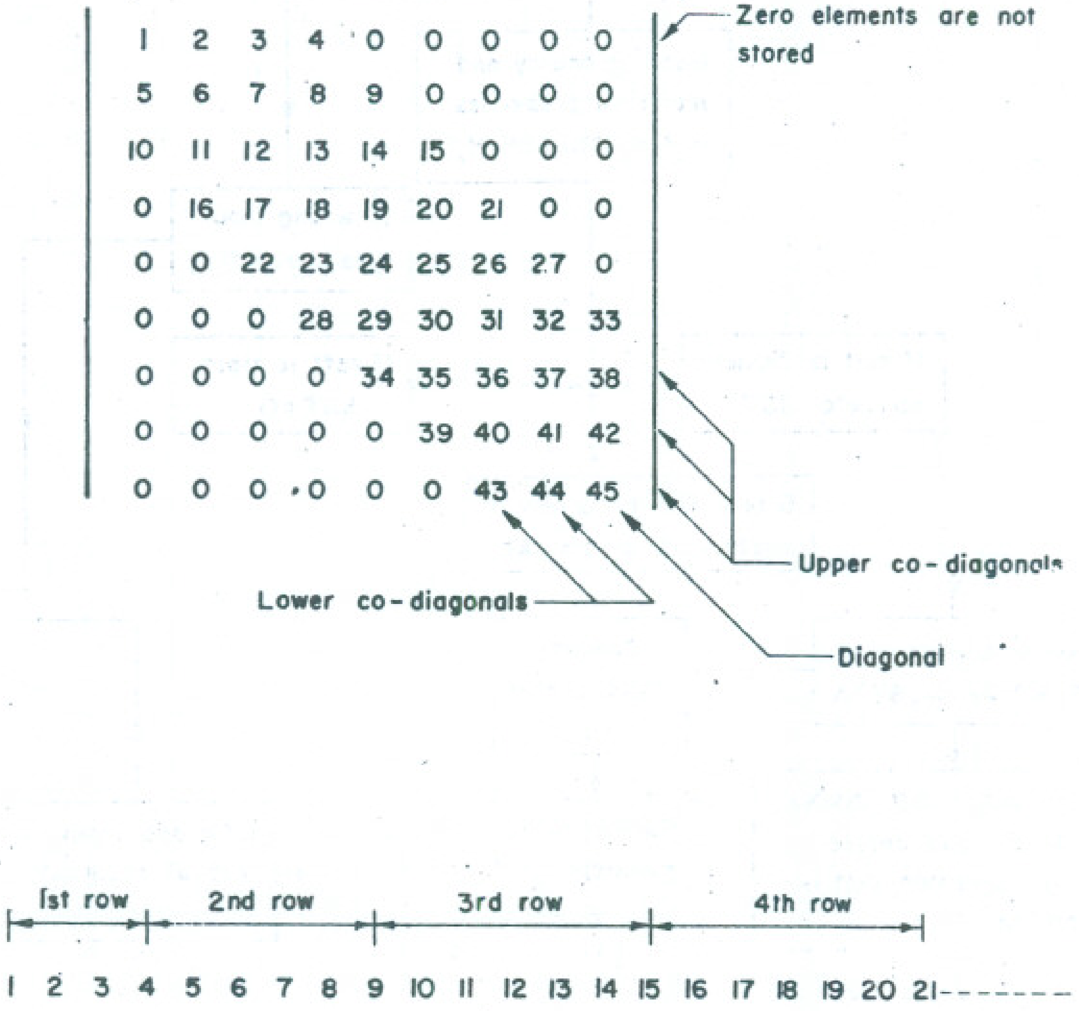
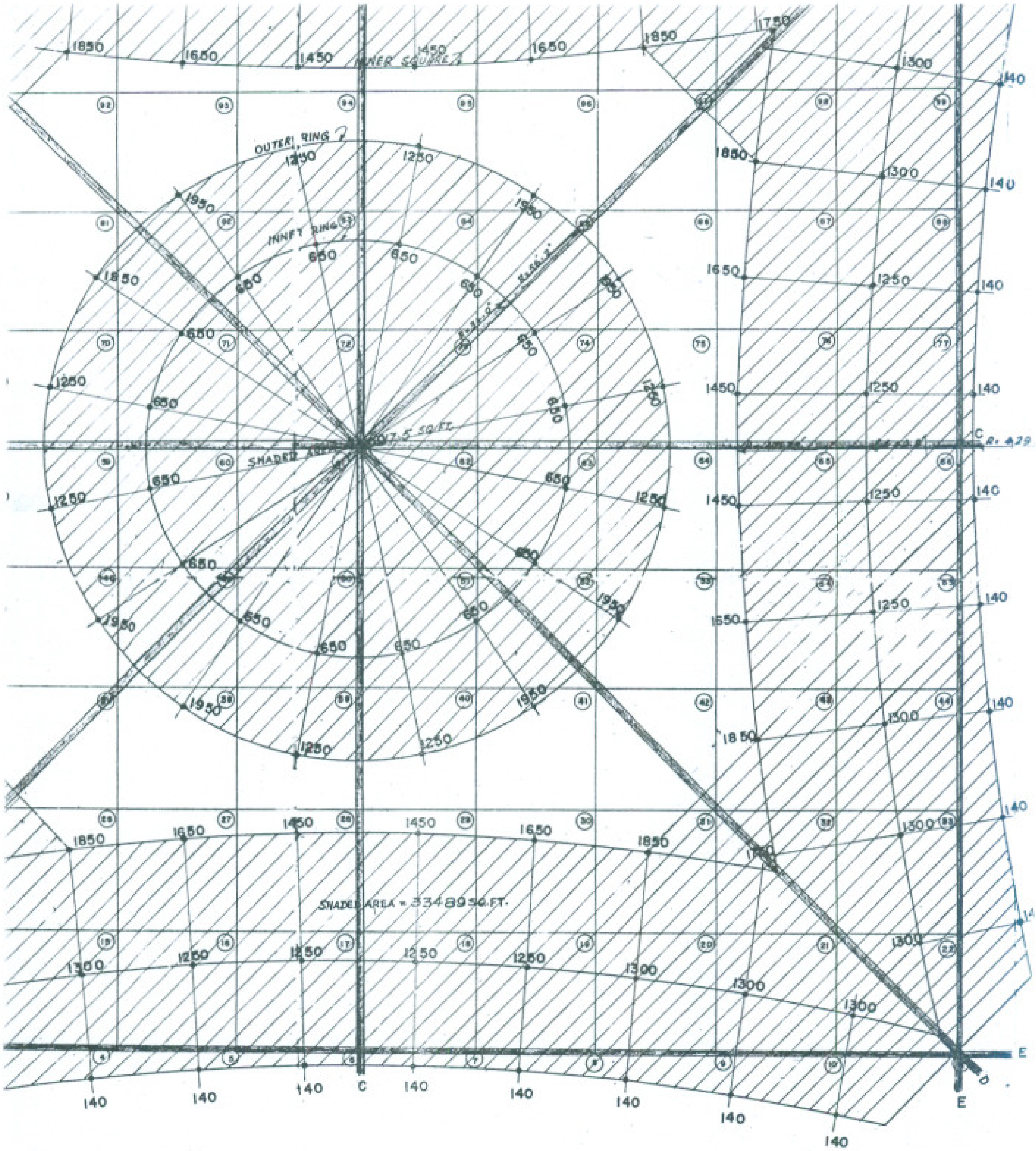


Fig. 4: Compressed Storage Mode of a Banded Matrix



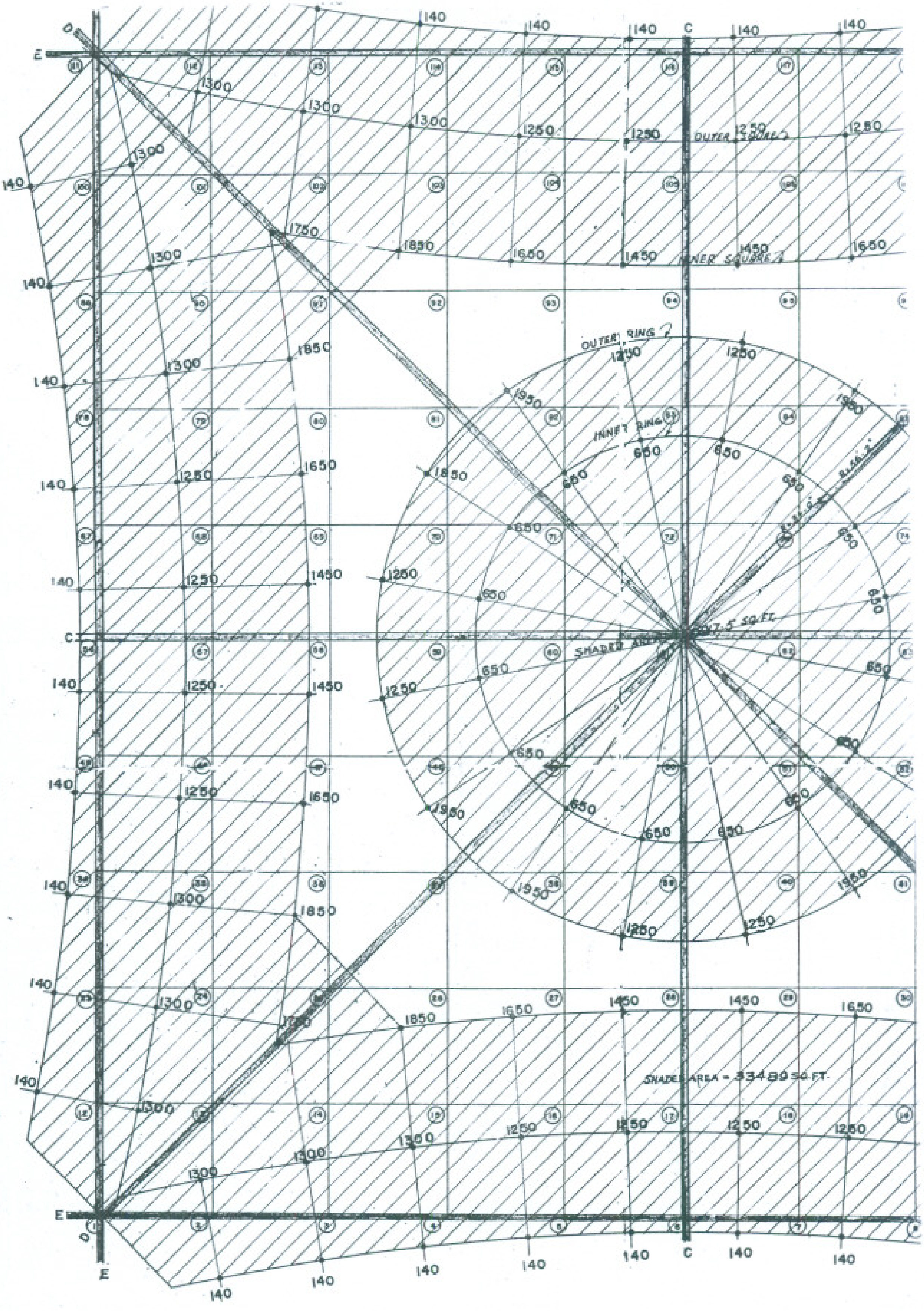
): Column loads in kips

Node points in computer analysis

▬ E Strips analysed in manual calculations

MX = 11
M = 121
H = 22·543 ft
CSR = 25·00 K /Cft
D = 4603580 Kft /ft
t = 5 ft

FIG. 5: PLAN OF WAPDA HOUSE RAFT FOUNDATION



1300 : Column loads in kips

②: Node points in computer analysis

E ▬ E Strips analysed in manual calculations

MX = 11

M = 121

H = 22·543 f

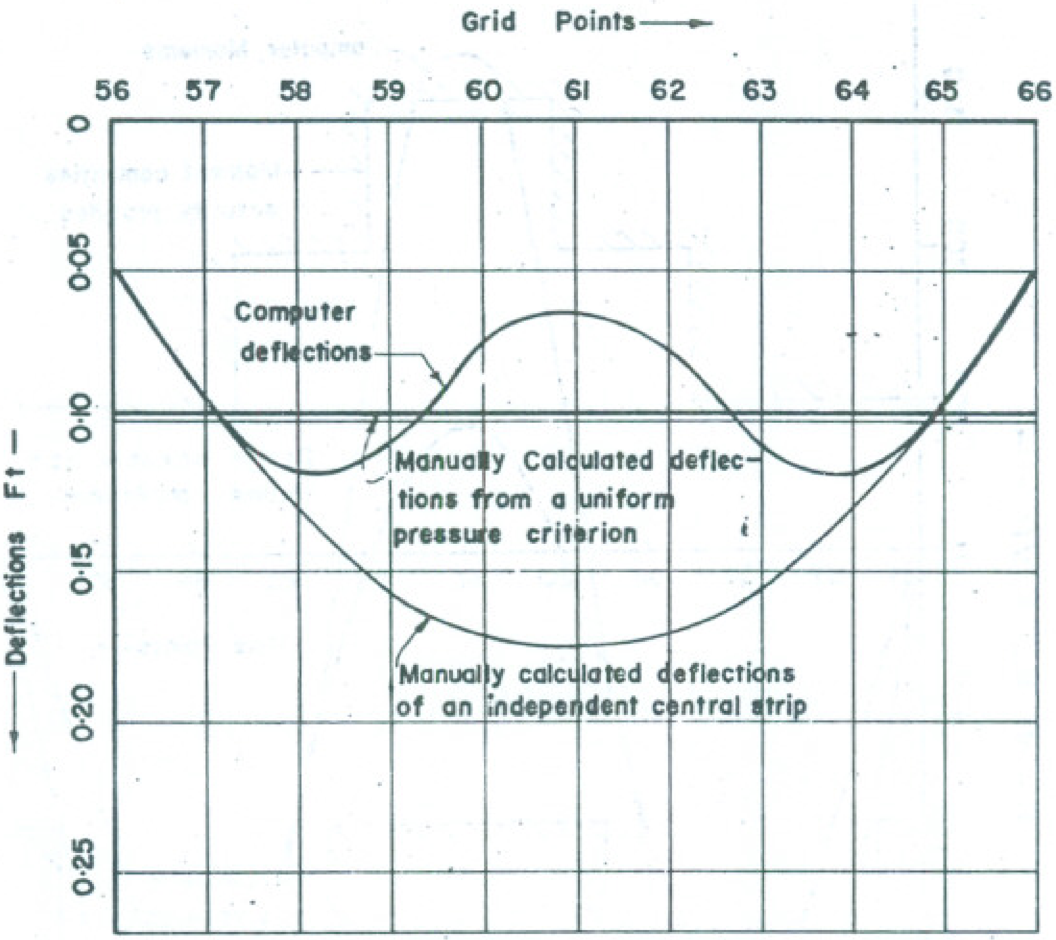


Fig. 7: Comparison of Computer Deflections with those Calculated Manually in the WAPDA House Raft Across a Central Strip.

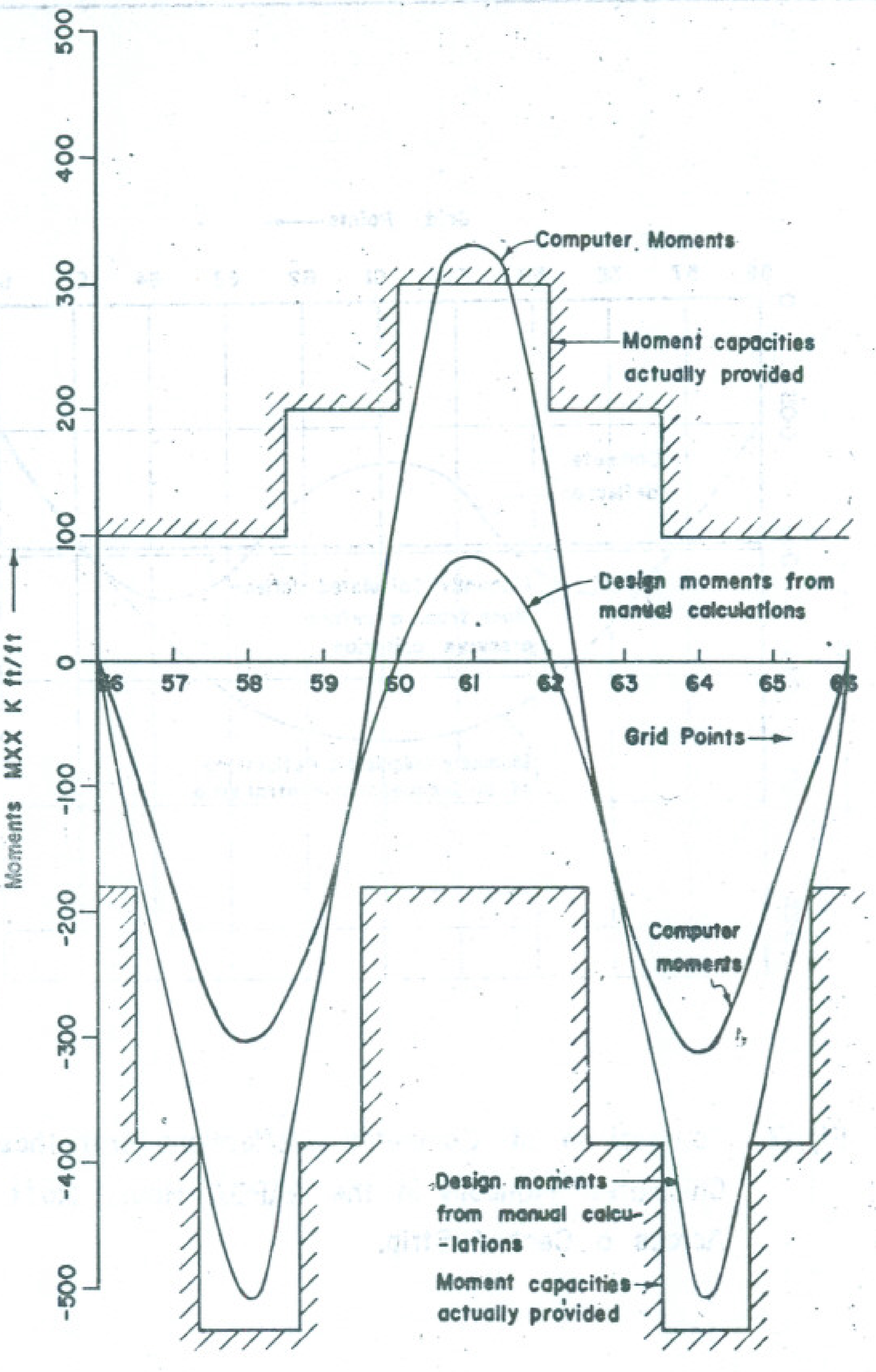


Fig. 6: Comparison of Computer Moments with those Actually Provided in the WAPDA House Raft Across a Central Strip.

MANUAL CALCULATIONS

In the manual calculations for the analysis of the WAPDA House raft foundation, firstly the soil line method has been attempted. It has not been possible to absorb the unbalanced shears conveniently and also considering the length of the calculations, this method has been abondoned. Then, a strip method has been adopted. For the purposes of calculating soil pressures, the raft has been assumed to be rigid. The soil pressures have been calculated from the free body statics.

A number of strips ; a central strip, a diagonal strip and an end strip, as shown in Fig. 5 have then been considered. These strips have been assumed to act independently as beams under the soil pressures and the relevant column loads. These beams are assumed to be supported at the heaviest column locations. Under these static configurations, deflection diagrams have been worked out, which have been found compatible.

However, the so calculated deflections have been excessively large as compared to those which could be estimated from the range of the modulus of subgrade reaction. The corresponding deflections and moments have been provided for those values.

In Fig. 6 the corrected deflections across a central strip is plotted for illustration. In Fig. 7 the corrected moments across the central strip is plotted for illustration. In the same figure, the moment capacities which have actually been provided in the raft are also plotted.

COMPARISON WITH COMPUTER RESULTS

The results of the computer program for WAPDA House raft for same loading condition and geometry of the raft as in the manual calculations, are shown in Fig. 6 and 7. The computer analysis has been performed by using a grid system as shown in Fig 5. Thickness of the raft has been assumed to be 5 ft. and the value of Poissons' ratio has been taken as 0.15. An average value of the coefficient of subgrade reaction equal to 25 Kips/Cuft has been used.

The deflection pattern, given by the computer, as shown in Fig. 7, fluctuates around the constant value of the uniform soil pressure. It correctly depicts increased deflections in the local regions of heaviest loaded column, and lesser deflections in the lightly loaded area. The manual method however fails to predict this effect.

The moments as determined by the computer are shown in Fig. 6. These moment are across a central strip, and can be compared with the manually calculated moments. It can be seen that the moment capacities actually provided in the raft are satisfactory in relation to the computer moments. However, the manually calculated moments are on the whole higher than the computer moments.

CONCLUSION

Manual methods of analysis of raft foundations are cumbersome and time consuming. Until recently, due to no-availability of computers of adequate storage capacity, a number of rafts existing in Pakistan have been analysed manually. A computer program has now been developed which analyses rafts using finite difference method and is capable of running on the available computers in Pakistan. A special storage mode has been used to minimise the storage requirement. The program can handle rafts of reasonable size with any number of column loads located in any arrangement.

WAPDA House raft foundation is an interesting example. In the original design, the analysis has been done manually. Strip method of analysis has been used. This raft has been re-analysed, now, on the computer program. The results show that though the provided reinforcement is satisfactory in relation to the moments determined by the computer, the manual method employed in the original design does not depict the true behaviour of the raft. The manual method fails to predict greater deflections under heavier loads, where as the computer program can reflect this behaviour. The deflections and moments in generel are also grossly over-estimated by the manual method.

REFERENCES

1. Bowles 'Foundation Analysis Design', 1971, New York.
2. Teng 'Foundation Design' 1961, New York.
3. Baker 'Raft Foundation' 1971, London.
4. Timoshenko 'Theory of Plates and Shells', 1961, London.
5. Hetenyi 'Beams on Elastic Foundation', 1961, Ann Arbor.

NOTATION

W	deflections normal to plane of the slab
x, y	coordinates
P	lateral pressure

D	stiffness of slab = $Et^3/12\ (1 - EMU^2)$
E	modulus of elasticity
t	thickness of slab
EMU	Poissons' ratio
CSR	coefficient of subgrade reaction
V	column load
H	grid spacing
| A |	coefficient matrix
| R |	load vector
| X |	deflection vector
MXX	slab moment in X direction
MXY	slab torsion
QXX	slab shear in X direction
BP	soil pressure
M	total number of grid points
MX	grid points along a line in X direction
SST	addition in diagonal elements of coefficient matrix due to subgrade reaction.