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DESIGN OF STEEL FIBRE REINFORCED CONCRETE BEAMS USING THE NEW FIB MODEL CODE 2010 – AN EVALUATIVE

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DESIGN OF STEEL FIBRE REINFORCED CONCRETE BEAMS USING THE NEW FIB MODEL CODE 2010 – AN EVALUATIVE

By

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Abstract

Concrete is a material that needs strengthening in tension in order to meet the structural requirements. New techniques of strengthening concrete, besides the usual ordinary reinforcement bars, are developing, creating a need for new design methods. Fibre reinforcement is a method that has been in use over the last 30 years, yet it is unfamiliar to some and there is no common guideline for design using this method.

This paper evaluates the new FIB (Fédération Internationale du béton - International Federation for Structural Concrete) Model code regarding design of steel fibre reinforced concrete, aiming at studying the applicability and accuracy of the code. Design calculations, regarding moment and shear resistance in ultimate limit state and crack width calculations in serviceability limit state, were carried out in Mathcad for simply supported beams, with different combinations of ordinary reinforcement and fibre dosages. The design results were then compared with existing experimental results to assess the accuracy of the design codes. The simply supported slabs were also designed in Mathcad, where two reference slabs with ordinary reinforcement were compared to concrete slabs only reinforced with fibres.

Regarding accuracy, when compared to the experimental results, underestimations were revealed yet the FIB model code results proved to be close to the experimental findings. The FIB model code was also found to be applicable due the fact that it was complete and clear in most regards.

The design of the simply supported slabs revealed that, it is possible to replace ordinary reinforcement with steel fibres but requires large fibre fractions, as those used in this study were not enough.

Key words: concrete, steel fibres, fibre reinforced concrete, moment resistance, shear resistance, crack width calculations, fibre fractions.

1. Introduction

Concrete is a material with varying material behaviour, exhibiting high strength in compression but poor in tension. This has led to a need for reinforcement in the tensile parts of the structures. Traditionally this has been done using ordinary reinforcing bars. However, the need for designing structures with more complex geometries has led to the development of relatively new reinforcement materials such as steel fibres, which have further raised the potential of designing such geometries. Steel fibres can partly or entirely replace conventional reinforcement owing to the fact that steel fibres increase the moment capacity of structures and improve crack control.

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Development of new reinforcing methods has left a need for the development of new design methods. Today, there are a number of different national guidelines and design codes for designing steel fibre reinforced concrete, but no general design code exists.

In this paper, results from experimental tests found in literature, on beams with varying fibre contents, performed by Gustafsson and Karlsson in 2006, were used as reference values and their material data and properties were used as input data for the design calculations. These design calculations were then compared with the results from the experimental tests to check the accuracy of the method. Before the explanation of the design method, a brief introduction of the experimental setup/program is given.

2. Experiments

The four point beam bending tests reviewed here were carried out by Gustafsson and Karlsson (2006), also Jansson (2008). The study contained 5 series with 3 beams tested in each series, (Table-1). The first series of beams contained only conventional reinforcement, while the remaining series (2-5) contained different amounts of fibres as shown in Table 1 and Table 2. All tested beams had three reinforcing bars with a diameter of either 6mm or 8mm. The concrete composition used in the bending tests had a post crack softening behaviour.

Table-1: Details of test specimen reinforced with 8mm reinforcement bars

Series	Fibre Content %/[kg/m^3]	Reinforcement number and diameter [mm]	Number of beams	Number of WST[1] cubes	Number of Compression cubes
1	-	3ø8	3	9	6
2	0.5/39.3	3ø8	3	9	6

Table-2: Details of test specimen reinforced with 6mm reinforcement bars

Series	Fibre Content %/[kg/m^3]	Reinforcement number and diameter [mm]	Number of beams	Number of WST cubes	Number of Compression cubes
3	0.5/39.3	3ø6	3	9	6
4	0.25/19.6	3ø6	3	9	6
5	0.75/58.9	3ø6	3	9	6

A self-compacting concrete, with w/b-ratio 0.55, was used in the experiments. For more information see Gustafsson and Karlsson (2006).

3. Compressive strength

In each series a total of 6 compression cubes were tested in order to determine the compressive strength. The strength achieved for the concrete with only conventional reinforcing

1 WST stands for Wedge Splitting Test, which is a test performed to determine the toughness of steel fibres

bars was 47 MPa while it varied between 36 MPa and 40 MPa for the fibre reinforced concrete, see Table 3. For equivalent cylindrical compression strength used in design, the cube strength is multiplied by a factor 0.8 derived from FIB model code 2010, (Table-7.2-1)(with both the strengths given and where, the cylinder strengths, f_{ck} are 80% of the cube strengths, $f_{ck,cube}$.

Table-3: Average values of cube compression strength and equivalent cylinder compression strength from the tests on beams with 8mm reinforcement bars.

Series	Reinforcement bars [mm]	Fibre content [%]	Compression cube strength [MPa]	Equivalent cylinder strength [MPa]
1	3ø8	0	47.0	37.6
2	3ø8	0.5	38.2	30.6

Table-4: Average values of cube compression strength and equivalent cylinder compression strength from the tests on beams with 6mm reinforcement bars.

Series	Reinforcement bars [mm]	Fibre content [%]	Compression cube strength [MPa]	Equivalent cylinder strength [MPa]
4	3ø6	0.25	39.2	31.4
3	3ø6	0.5	37.7	30.2
5	3ø6	0.75	36.8	29.4

From the test results in Table 3 and Table 4, it is noted that fibres had a negative impact on the compression strength, as it was reduced with higher fibre content. It should however be mentioned, that the concrete composition had little variation.

2.2 Tensile behaviour

Nine wedge splitting tests (WST), on small cubes with a volume of 0.1x0.1x0.1 m^3, were conducted for each series to determine the toughness of the steel fibres. For more details, also see Gustafsson and Karlsson (2006).

According to Löfgren (2005), it is necessary to consider fibre orientation and the number of fibres crossing a cracked section. This is normally done by defining a fibre efficiency factor, η_b, see equation (1).

$$\eta_b = \frac{A_f}{V_f} N_b \tag{1}$$

Where,

A_f is the area of the fibre cross section

V_f is the fibre volume fraction

N_b is the number of fibres per unit area

The fibre efficiency factor obtained from the wedge splitting tests used in this report varied, according to Gustafsson and Karlsson (2006), between 0.49 and 0.56. The fibre efficiency factor for the beams was also calculated theoretically, according to Dupont and Vandewalle (2005), and the value obtained by Gustafsson and Karlsson was equal to 0.54. According to Löfgren (2005), experiments have shown that it is reasonable to assume a linear relationship between number of fibres and fibre bridging stresses as seen in equation (2).

$$\sigma_{b,beam} = \sigma_{b,exp}\frac{\eta_{b,beam}}{\eta_{b,exp}} \qquad (2)$$

Where,

$\sigma_{b,beam}$ is the bridging stress applicable for the beam elements

$\sigma_{b,exp}$ is the experimental bridging stress from the wedge splitting tests

$\eta_{b,beam}$ is the fibre efficiency factor for beam elements

$\eta_{b,exp}$ is the fibre efficiency factor obtained from the wedge splitting tests

This method was used to transform the fibre bridging stresses obtained in the WSTs to beam stresses.

2.3 Conventional reinforcement

The reinforcement used in the beams consisted of 6mm and 8mm diameter bars. The measured yield stress and ultimate stress capacities are shown in Table 5.

Table-5: Yield and ultimate stress capacities from tests on reinforcing bars done by Gustafsson and Karlsson (2006)

Reinforcement Bars	**Yield Stress Capacity, f_{sy} [MPa]**	**Ultimate Stress Capacity, f_{su} [MPa]**
6 mm	660	784
8 mm	590	746

2.4 Results

Four point bending tests were performed on series of simply supported beams with loading conditions and dimensions as shown in Figure-1. The tests were performed using load control.

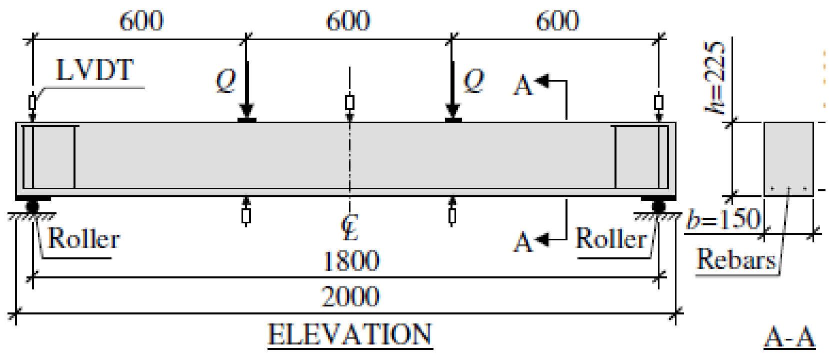

Figure-1: Dimensions and loading conditions for the beam tests, from Jansson (2008)

From the test results, values of loads, deflections at mid-span, support settlements and crack widths were obtained. For more details, see Gustafsson and Karlsson (2006). The results from the three beams in each series were presented as average moment-curvature curves and are shown in the Figure 2 and Figure 3.

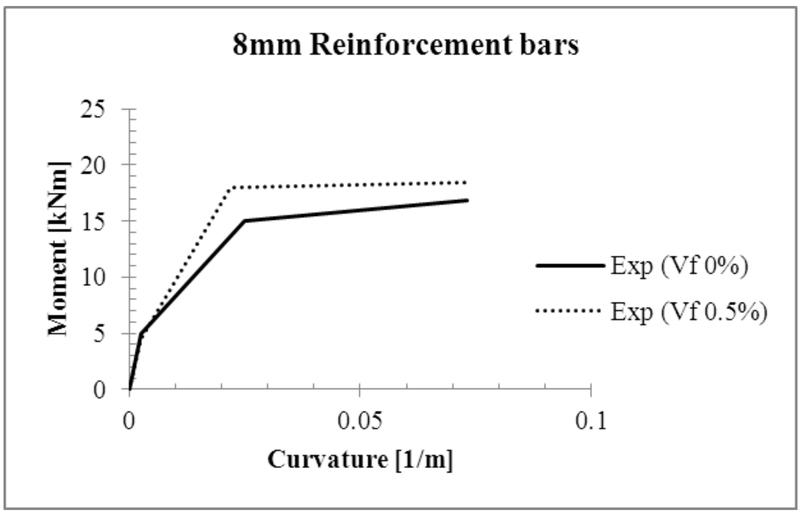

Figure-2: Moment versus curvature diagrams from the beam tests with reinforcement bar ø8 mm

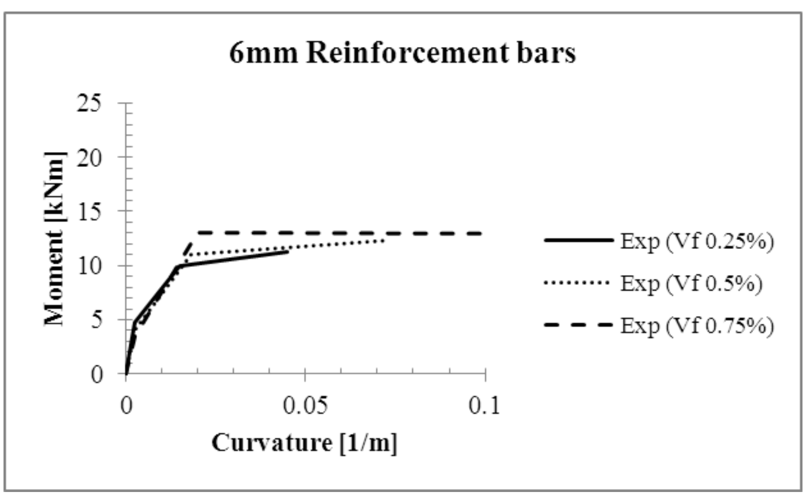

Figure-3: Moment versus curvature diagrams from the beam tests with reinforcement bar ø6 mm

Table-6: Ultimate moment capacities from experiments, for beam series with 8 mm reinforcement bars

Series	Vf (%)	Reinforcement	Moment Capacity (kNm)	Increase of capacity due to addition of fibres (%)
1	0	3ø8	16.8	-
2	0.5	3ø8	18.9	12.5

Table-7: Ultimate moment capacities from experiments, for beam series with 6 mm reinforcement bars

Series	Vf (%)	Reinforcement	Moment Capacity (kNm)	Increase of capacity due to varying fibre volume (%)
4	0.25	3ø6	11.3	-
3	0.50	3ø6	12.3	8.8
5	0.75	3ø6	12.8	13.3

3. Design of Beams according to the new FIB model code 2010

FIB (fédération Internationale du béton) is an international federation for structural concrete which was formed when the Euro-International concrete committee (CEB) and the International federation for pre-stressing (FIP) merged. (FIB bulletin 1, volume 1)

According to FIB model code, bulletin 56, volume 2, the following assumptions, while determining the ultimate limit moment resistance of reinforced or prestressed concrete sections are made;

- Plane sections remain plane
- The strain in bonded reinforcement or bonded prestressing tendons, whether in tension or in compression, is the same as that in the surrounding concrete
- The tensile strength of the concrete is ignored
- The stresses in the concrete are derived from stress-strain relations for the design of cross-sections.
- The stresses in the reinforcing and prestressing steel are derived from design curves given in subclause 7.2.3.2 and 7.2.3.3 in the FIB model code.
- The initial strain in the prestressing tendons is taken into account when assessing the stresses in the tendons.

When designing fibre reinforced concrete sections, all the above assumptions are valid except the third one, where the concrete tensile strength is ignored.

Equation (3) to equation (6), taken from FIB model code, were used to derive the concrete tensile stresses, f_{ctm}, and modulus of elasticity, E_{cm}, with all the stresses in MPa.

$$f_{cm} = f_{ck} + 8 \tag{3}$$

$$f_{ctm} = 0.30 f_{ck}^{2/3} \tag{4}$$

With f_{ck} being the cylindrical compressive fibre reinforced concrete strength.

$$E_{cm} = \left(\frac{f_{cm}}{10}\right)^{0.3} \tag{5}$$

$$\varepsilon_{cu} = 3.5 \times 10^{-3} \tag{6}$$

It should however, be noted that equation (5) is incomplete, as the mean concrete modulus of elasticity, E_{cm}, cannot be smaller than the mean compressive strength. So equation (7), given by RILEM TC-162-TDF (2003)[1], was used.

$$E_{cm} = 9500 (f_{fcm})^{\frac{1}{3}} \tag{7}$$

3.1 Residual flexural tensile strength

According to the FIB model code, the strength of fibres is measured as a residual flexural tensile strength, $f_{R,j}$. This can be done by performing crack mouth opening displacement (CMOD) tests. A CMOD test is a deformation controlled loading test, where the crack opening is measured as a horizontal deflection. The test setup requires a beam, notched to prevent horizontal cracking, and devices for recording the applied load and the crack opening, which is referred to as CMOD. The FIB model code proposes that it is to be done in accordance with EN 14651 (2005). The CMOD, for the experimental data used in this report, was, however, from the wedge splitting tests which are basically the same tests but performed on small cubes, for more details see Gustafsson and Karlsson (2006), see also Jansson (2008).

$$f_{R,j} = 3 \frac{F_j}{2 b h_{sp}^2} l \qquad \text{(MPa)} \tag{8}$$

Where,

$f_{R,j}$ is the residual flexural tensile strength corresponding to *CMODj*, with [j=1,2,3,4]

F_j is the load corresponding to $\mathrm{CMOD_j}$

$CMOD_j$ is the crack mouth opening displacement

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[1] Rilem is an international committee of experts which aims at advancing the scientific knowledge in structures, systems and construction materials. Among their aims, Rilem is to assess scientific research data and publish their recommendations as guidelines.

l is the span of the specimen

b is the width of the specimen

h_{sp} is the distance between the notch tip and the top of the specimen

The values f_{R1} and f_{R3} are obtained from the corresponding F_{R1}- $CMOD_1$ and F_{R3}-$CMOD_3$ values as shown in Figure 4.

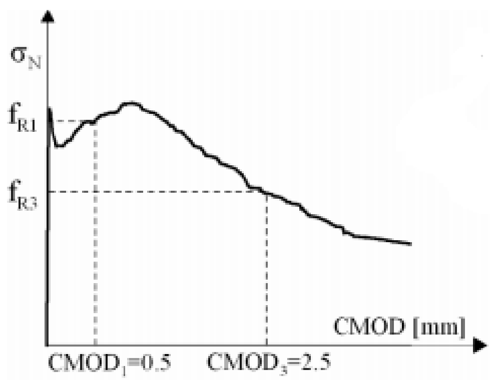


Figure-4: An example of typical results from a bending test with a softening material behaviour. From FIB model code, bulletin 55, vol. 1

The FIB model code simplifies the real response in tension, as shown in Figure (4), into two stress-crack opening constitutive laws, a linear post crack softening or hardening behaviour, see Figure (5), and a plastic rigid behaviour, see Figure (6).

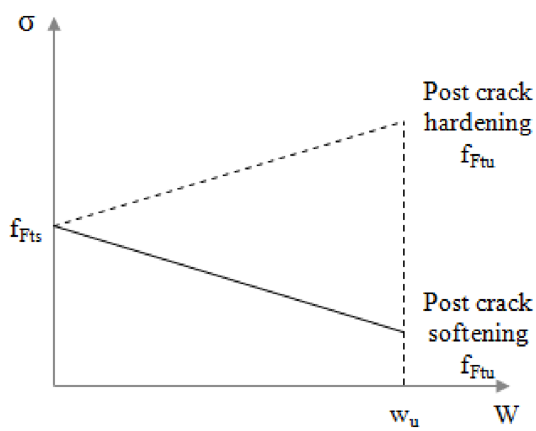


Figure-5: Simplified post-crack constitutive laws; linear post cracking tress-crack opening. From FIB model code, bulletin 55, vol. 1

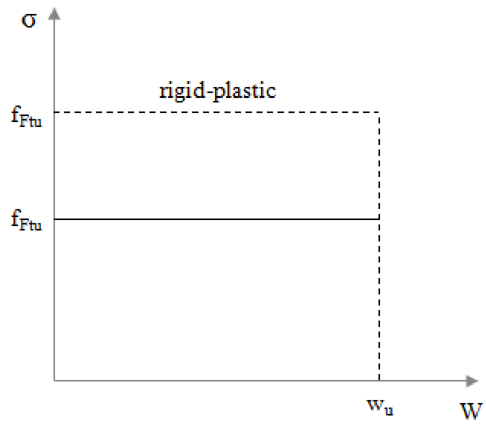


Figure-6: Simplified post-crack constitutive laws; plastic-rigid behaviour.
From FIB model code, bulletin 55, vol. 1

Two reference values are introduced, f_{Fts} representing the serviceability residual strength and f_{Ftu} representing the ultimate residual strength. See equation (9) and equation (10). No partial safety factors were used, due to comparison with experiments.

$$f_{Fts} = 0.45 f_{R1} \tag{9}$$

where

f_{Fts} is the serviceability residual strength

f_{R1} is the residual flexural tensile strength corresponding to $CMOD_1$

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3}(f_{Fts} - 0.5 f_{R3} + 0.2 f_{R1}) \geq 0 \tag{10}$$

where

f_{Ftu} is the ultimate residual strength

w_u is the ultimate crack opening accepted in structural design, see equation (11)

f_{R3} is the residual flexural tensile strength corresponding to $CMOD_3$

$CMOD_1$ is the crack mouth opening displacement and is equal to 0.5mm

$CMOD_3$ is the crack mouth opening displacement and is equal to 2.5mm

Equation (10) gives the values of f_{Ftu} where, $w_u \neq CMOD_3$. Using a linear constitutive law between $CMOD_1$ corresponding to serviceability limit state and $CMOD_3$ corresponding to the crack opening of 2.5mm, any value up to w_u can be obtained, see Figure (4). The crack width, w_u, is the maximum crack opening accepted in structural design, where its value depends on the required ductility, and therefore should not exceed 2.5mm, according to the FIB model code. w_u is calculated as equation (11).

$$w_u = \varepsilon_{Fu} l_{cs} \tag{11}$$

where

ε_{Fu} is assumed to be equal to 2% for variable strain distribution in cross section and 1% for only tensile strain distribution along the cross section

l_{cs} is the structural characteristic length, calculated in equation (12).

$$l_{cs} = min\{s_{rm}, y\} \tag{12}$$

Where,

s_{rm} is the mean crack spacing

y is the distance between the neutral axis and the tensile side of the cross section

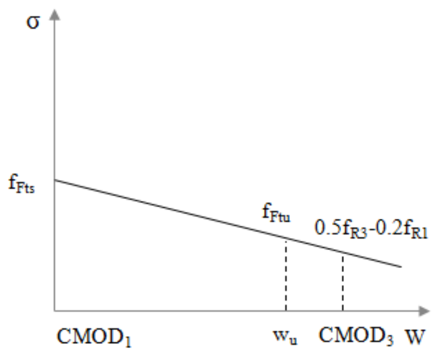


Figure-7: Simplified linear post-cracking constitutive law.
From FIB model code, bulletin 55, vol.1

The requirements in equation (13) and equation (14) need to be fulfilled, according to FIB model code, if fibre reinforcement is to partially or entirely substitute the ordinary reinforcement in ultimate limit state.

$$f_{R1k}/f_{Lk} > 0.4 \tag{13}$$

$$f_{R3k}/f_{R1k} > 0.5 \tag{14}$$

where

f_{Lk} is the limit of proportionality

f_{R1k} is the flexural tensile strength corresponding to $CMOD_1$

f_{R3k} is the flexural tensile strength corresponding to $CMOD_3$

3.2 Moment resistance

The residual flexural tensile strength of the fibres is added as a stress block as seen in Figure-8. For bending moment and axial force in the ultimate limit state, a simplified stress/strain relationship is given by the FIB model code. The simplified stress distributions can be seen in Figure-8 where the linear post cracking stress distribution is to the left and the rigid plastic stress distribution is to the right, with η = 1 and λ = 0.8 for concrete with compressive strength below or equal to 50MPa. However, it should be noticed that the safety factor, γ_F, has been removed for the reason that, the moment resistance is compared with experimental results. The linear stress distribution to the left was used for design in this paper.

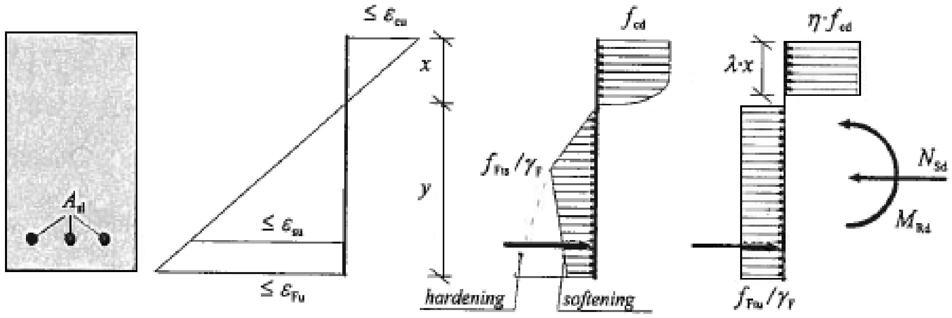


Figure-8: Simplified stress/strain relationship including the residual flexural tensile strength of fibres, from FIB model code, bulletin 56

Moments at cracking, yielding and ultimate stage were calculated for all beam series using the FIB model code. The flexural cracking moment for all the series was calculated as:

$$M_{cr} = W_1 f_{ctm} \tag{15}$$

Where,

M_{cr} is the cracking moment resistance

f_{ctm} is the mean tensile strength of the concrete mix

W_1 is the sectional modulus calculated as equation (16)

$$W_1 = \frac{bh^2}{6} \tag{16}$$

With b being the width of the cross section, and h the height of the cross section

The moments at yielding and ultimate stage were calculated using the simplified stress-strain relationship, in accordance with FIB model code, see Figure 8.

The yield moment was calculated using the linear post cracking constitutive law; see stress distribution to the left in Figure-8. The total contribution of fibres to the moment resistance was referred to as f_{Ft} and used in calculations.

$$M_{Rd} = f_{sy} A_s (d - \beta x) + f_{Ft}(h - x)b[\beta x + x_{tot} y] \tag{17}$$

Where,

f_{sy} is the yield strength of the ordinary reinforcement

β is the distance from the top of the beam to the center of the concrete compressive zone

A_s is the area of the ordinary reinforcement bars

d is the effective depth

f_{Ft} is the total stress of the tensile stress block from the fibre contribution

h is the height of the beam

x is the distance from top of the beam to the neutral axis

x_{tot} is the centre of gravity for the tensile zone of fibre stress, given as a percentage of the total height

y is the height of the tensile stress block

The ultimate moment resistance was also calculated using the simplified linear post-cracking stress distribution in Figure-8.

$$M_{Rdu} = f_{sy}A_s(d - \beta x) + f_{Ft}(h - x)b[\beta x + x_{tot}y] \quad (18)$$

For the definition of the variables, see equation (17).

The corresponding curvatures were calculated according to equation (19) to equation (23).

$$k_{cr} = \frac{\varepsilon_r}{h/2} \quad (19)$$

Where,

k_{cr} is the curvature at cracking

ε_r is the elastic strain in the concrete calculated as equation (20)

$$\varepsilon_r = \frac{f_{ctm}}{E_{cm}} \quad (20)$$

$$k_y = \frac{\varepsilon_{c2}}{x} \quad (21)$$

Where,

k_y is the curvature at yielding

ε_{c2} is the strain in the concrete when the ordinary reinforcement reaches yielding and calculated as equation (22)

$$\varepsilon_{c2} = \frac{\varepsilon_{sy}}{\left(\frac{d-x}{x}\right)} \quad (22)$$

Where,

ε_{sy} is the yield strain of the ordinary reinforcement

$$k_u = \frac{\varepsilon_{cu}}{x} \quad (23)$$

With,

k_u being the ultimate curvature; and

ε_{cu} is the ultimate strain in the concrete equal to 3.5×10^{-3}

The moment-curvature relationships for the different beam series obtained when designed using the FIB model code are given in Figure-9 for beams with 8mm ordinary reinforcement bars and in Figure 10 for beams with 6mm ordinary reinforcement bars. In both Figure-9 and Figure-10, it can be seen that the moment resistance slightly increases with increased fibre volume; it is however, evident from these figures that, the moment resistance does not significantly increase with the addition of fibres.

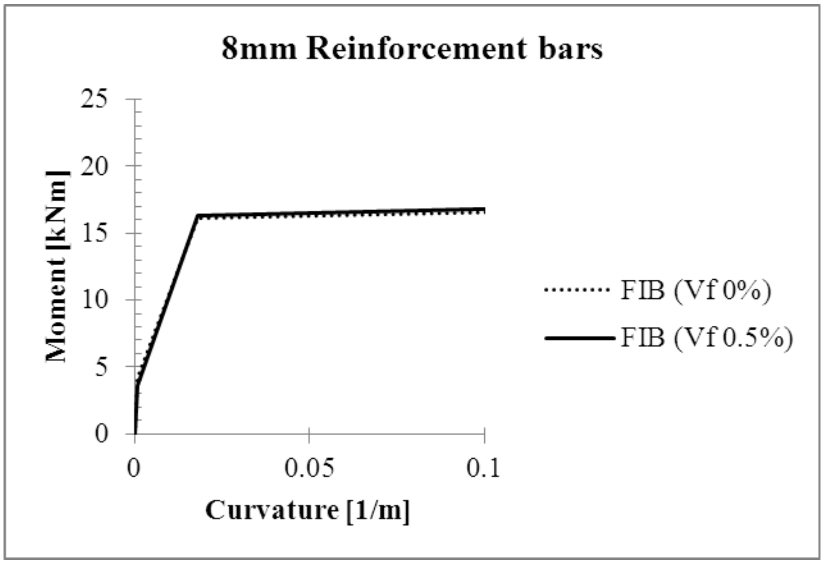


Figure-9: Moment versus curvature diagrams for beams with reinforcement bar ø8 mm, designed according to FIB model code

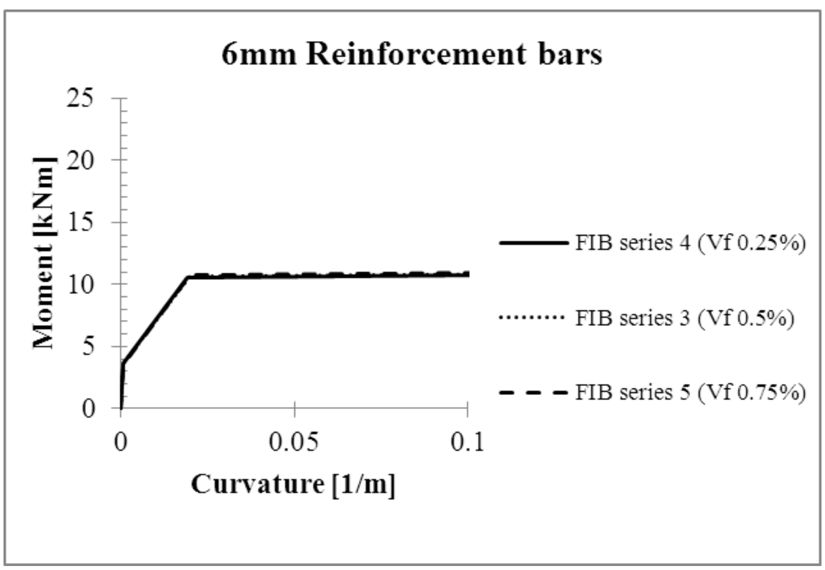


Figure-10: Moment versus curvature diagrams for beams with reinforcement bar ø6 mm, designed according to FIB model code

In Table-8, it can be seen that addition of 0.5% fibre volume in a beam reinforced with 8mm diameter ordinary reinforcement bars increases the moment capacity by 0.6%.

Table-8: Moment capacities for beam series with 8 mm reinforcement bars, designed according to FIB model code.

Series	V_f (%)	Reinforcement	Moment Capacity (kNm)	Increase of capacity due to addition of fibres (%)
1	0	3ø8	16.9	-
2	0.5	3ø8	17.0	0.6

Decreasing the diameter of ordinary reinforcement bars from 8mm to 6mm significantly reduces the moment resistance. This reduction can however be complemented by addition of sufficient amount of fibres. Table-9 shows how the moment capacity increases with variation of fibre volume from 0.25% to 0.75%. Here it is clear that much more fibre fractions are needed in order to compensate for this reduction.

For beams reinforced with 6mm diameter reinforcement bars, no reference beam without fibres was tested, but it can still be noted that increase in fibre volume increases the moment capacity.

Table-9: Moment capacities for beam series with 6 mm reinforcement bars, designed according to FIB model code.

Series	V_f (%)	Reinforcement	Moment Capacity (kNm)	Increase of capacity due to varying fibre volume (%)
4	0.25	3ø6	11.0	-
3	0.50	3ø6	11.1	0.9
5	0.75	3ø6	11.2	1.8

When designing fibre reinforced concrete beams without the presence of ordinary reinforcement, the FIB model code proposes that the same stress strain relationship in section 3.2.2 applies, excluding the contribution of the steel reinforcement. The equation for moment resistance for beams without ordinary reinforcement can be seen in equation (24).

$$M_{Rdu} = f_{Ftm}(h - x)b[\beta x + x_{tot}y] \quad (24)$$

For definitions of variables, see equation (17).

The results of the ultimate moment capacities from design of beams without ordinary reinforcement, designed using the FIB model code, are presented in Table-10. The results revealed that the moment capacity increased with increasing fibre volumes, the results however showed very low moment capacities for the chosen fibre fractions.

Table-10: Ultimate moment resistances for beams without ordinary reinforcement bars designed according to FIB model code

Series	Fibre Volume (%)	$M_{ultimate}$ (kNm)	Percentage increase (%)
4	0.25	0.29	-
3	0.5	0.43	48
5	0.75	0.60	107

When designing in ultimate limit state, ductility requirements need to be fulfilled. FIB takes this into account by implying that ductility requirements are fulfilled when the need for minimum ordinary reinforcement amount is satisfied. The minimum reinforcement is calculated as equation (25).

$$A_{s,min} = k_c k (f_{ctm} - f_{Ftsm}) \frac{A_{ct}}{\sigma_s} \qquad (25)$$

where

f_{ctm} is the mean concrete tensile strength

f_{Ftsm} is the residual tensile strength of fibre reinforced concrete

A_{ct} is the tensile part of the concrete cross section

σ_s is the maximum tensile reinforcement at cracking stage

k_c is the coefficient taking into account the stress distribution in the cross section just before cracking and the change of inner lever arm

k is the coefficient taking into account non-uniform self-equilibrating stresses leading to reduction of cracking force

Table-11 illustrates that the ductility requirements were fulfilled as the steel reinforcement, A_s, was larger than the minimum required reinforcement, $A_{s,min}$.

Table-11: Results of the ductility requirements for beam series with 8 mm reinforcement bars, designed according to FIB model code

Series	Reinforcement	Fibre Volume (%)	A_s (mm^2)	$A_{s,min}$ (mm^2)	Ductility
2	3ø8	0.5	150.8	134.9	Fulfilled

For the beams with a smaller amount of reinforcement, the ductility requirements were not fulfilled for the used fibre content, see Table-12. Thus, less ordinary reinforcement can be compensated by adding more fibres. Table-12 illustrates that more than 0.75% fibre content is needed in order to fulfil the ductility requirements.

Table-12: Results of the ductility requirements for beam series with 6 mm reinforcement bars, designed according to FIB model code

Series	Reinforcement	Fibre Volume (%)	A_s (mm^2)	$A_{s,min}$ (mm^2)	Ductility
4	3ø6	0.25	84.8	131.0	Not Fulfilled
3	3ø6	0.5	84.8	123.7	Not Fulfilled
5	3ø6	0.75	84.8	117.4	Not Fulfilled

According to FIB model code, ductility requirements can be satisfied in fibre reinforced concrete structures without minimum ordinary reinforcement if one of the conditions in equations (26) and (27) are fulfilled.

$$\delta_u \geq 20\delta_{SLS} \tag{26}$$

$$\delta_{peak} \geq 5\delta_{SLS} \tag{27}$$

where

δ_u is the ultimate displacement

δ_{peak} is the displacement at the maximum load

δ_{SLS} is the displacement at service load computed by performing a linear elastic analysis with the assumptions of uncracked condition and initial elastic Young's modulus.

The values in equation (26) and equation (27) are obtained from experiments.

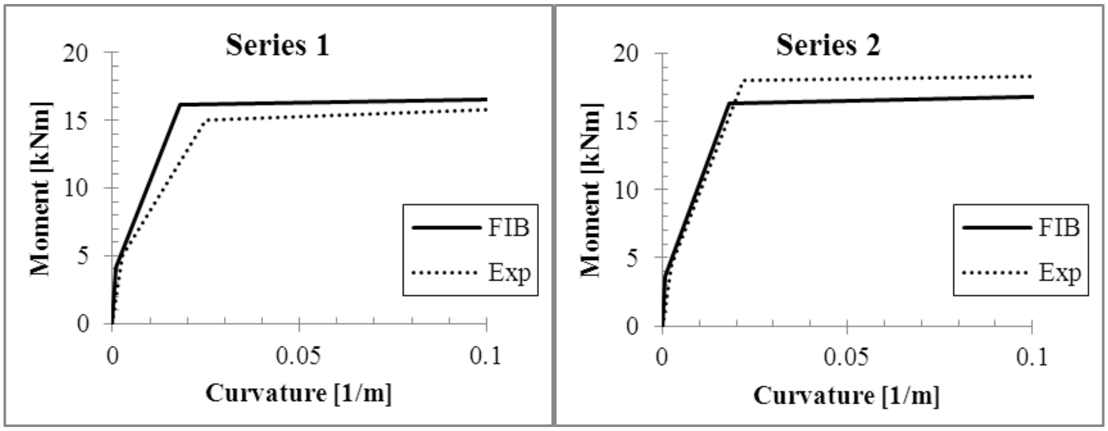
The ductility requirements were fulfilled for all the series, but for beams without ordinary reinforcement, no experimental data on load-deformation conditions is available.

The ductility requirements in equation (26) and equation (27) are valid for design of fibre reinforced concrete without ordinary reinforcement if ultimate load is higher than the cracking load.

3.3 Comparison with experimental results

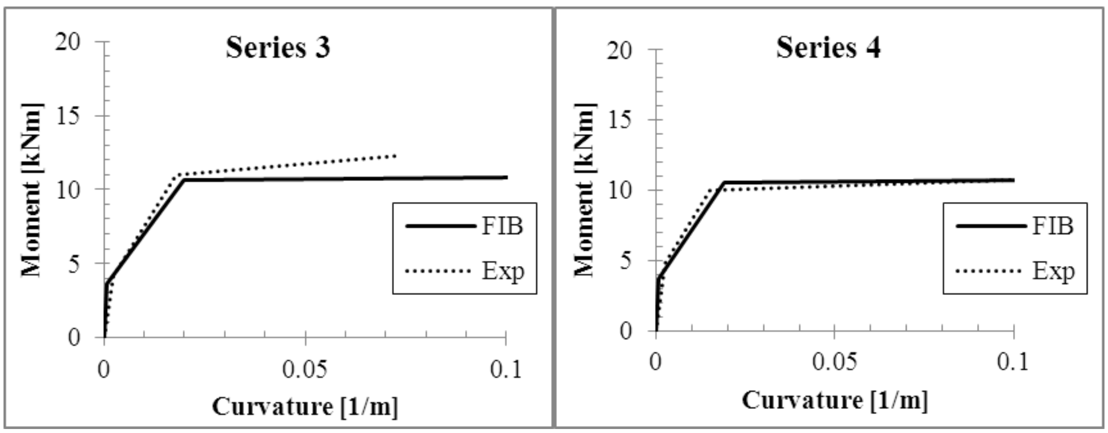
The moment resistance results were compared with the experimental results, for all the beam series, in order to identify the accuracy of the FIB model code.

Figure-11 and Figure-12 illustrate the comparison of the design results with experimental results and in all cases with fibres, it was noted that there was an underestimation of the moment resistance when designing according to FIB model code.

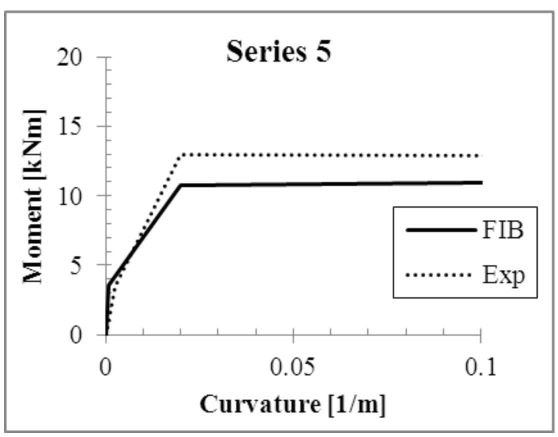


(a) (b)

Figure-11: Comparison of moment-curvature diagrams, according to FIB model code and the experimental results for (a) beams with V_f = 0 % and rebar ø8 mm, (b) beams with V_f = 0.5 % and rebar ø8 mm



(a) (b)



(c)

Figure-12: Comparison of moment-curvature diagrams, according to FIB model code and the experimental results for (a) beam with V_f = 0.5 % and rebar ø6 mm, (b) beam with V_f = 0.25 % and rebar ø6 mm, (c) beam with V_f = 0.75 % and rebar ø6 mm

The percentage over/under-estimation of the moment capacities for beams with 8 mm diameter ordinary reinforcement bars can be seen in Table-13 and for beams with 6 mm diameter bars in Table-14. The maximum underestimation goes up to 12.5%.

Table-13: Comparison of moment capacity for beams with 8mm reinforcement bars

$M_{ultimate}$		
Series	1	2
V_f (%) and reinforcement	0 3Ø8mm	0.5 3Ø8mm
FIB	16.9	17.0
Experimental	16.8	18.9
Difference (%)	0.6	-9.0

Table-14: Comparison of moment capacity for beams with 6mm reinforcement bars

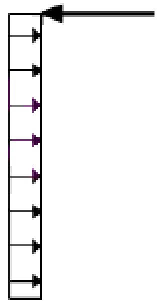
$M_{ultimate}$			
Series	4	3	5
V_f (%) and reinforcement	0.25 3Ø6mm	0.5 3Ø6mm	0.75 3Ø6mm
FIB	11.0	11.1	11.2
Experimental	11.3	12.3	12.8
Difference (%)	-2.6	-9.7	-12.5

4. Design of slab elements

In this section simply supported concrete slabs are designed, according to the FIB model code, in ultimate limit state. This is done for concrete slabs reinforced with ordinary reinforcement and for steel fibre reinforced concrete slabs.

4.1 The FIB model code 2010

For design of fibre reinforced concrete slab elements without ordinary reinforcement, subjected to bending actions, the FIB model code recommends the rigid plastic relationship, explained in Chapter 3 and in particular section 3.2.1 of the FIB model code bulletin 55. The rigid plastic model, proposed by the FIB model code, makes assumptions that the compressive force is concentrated in the top fibre of the section, see Figure 4.1. When using the rigid plastic model, the ultimate residual tensile strength of the fibres, f_{Ftu}, is obtained from the relationship in equation (4.1), suggested by the FIB model code.



f_{Ftu}

Figure-13: Simplified model to determine the ultimate tensile strength, f_{Ftu}, from the FIB model code

$$f_{Ftu} = \frac{f_{R3}}{3} \qquad (4.1)$$

The moment resistance, m_{Rd}, for slab elements without ordinary reinforcement is calculated using the formula given in equation (4.2)

$$m_{Rd} = \frac{f_{Ftu} h^2}{2} \qquad (4.2)$$

where

f_{Ftu} is the residual tensile strength of the fibres, calculated according to equation (4.1)

h is the height of the slab element.

It should be noted that the moment resistance, m_{Rd}, in equation (4.2), is given in kNm/m.

4.2 Compressive Strength

Due to difficulties in retrieving data on full scale fibre reinforced concrete slab experiments, regarding fibre content and residual tensile strength, the results of the tests performed on compression cubes with varying fibre contents, by Sanna Karlsson and Martin Gustafsson, at the Thomas Concrete Group, Sweden have been used. In each series a total of 6 compression cubes were tested in order to determine the compressive strength. The strength achieved for the concrete with no fibre content was 47 MPa while it varied between 36 MPa and 40 MPa for the fibre reinforced concrete, see Tab. 3. For equivalent cylindrical compression strength used in design, the cube strength is multiplied by a factor 0.8 derived from FIB model code 2010, Table 7.2-1 with both the strengths given and where, the cylinder strengths, f_{ck} are 80% of the cube strengths, $f_{ck,cube}$.

Table-15: Average values of cube compression strength and equivalent cylinder compression strength from the tests on compression cubes.

Series	Fibre content [%]	Compression cube strength [MPa]	Equivalent cylinder strength [MPa]
1	0	47.0	37.6
2	0.5	38.2	30.6

Table-16: Average values of cube compression strength and equivalent cylinder compression strength from the tests on compression cubes

Series	Fibre content [%]	Compression cube strength [MPa]	Equivalent cylinder strength [MPa]
4	0.25	39.2	31.4
3	0.5	37.7	30.2
5	0.75	36.8	29.4

The series 1 and 2 are separated from the other three due to the fact that the concrete mixes in these two series were used in beams reinforced with 8mm diameter bars, during the 3 point bending tests, while in the other three series, 6mm diameter bars were used.

4.3 Moment resistance

Simply supported slabs with distributed load, reinforced with 8 mm and 6 mm ordinary reinforcement bars with spacing 250 mm, were used as reference slabs when design of fibre reinforced concrete slabs without ordinary reinforcement was carried out, see Figure-4.2 and Figure-4.3. The moment resistance for the slabs reinforced with various fibre fractions was compared to the moment resistance of the reference slabs in order to determine the effect of fibres.

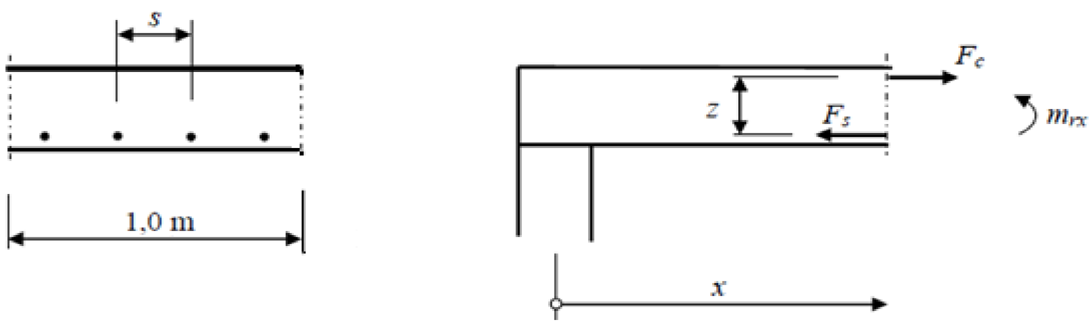


Figure-14: Slab cross-section in the x-direction, from Engström (2009)

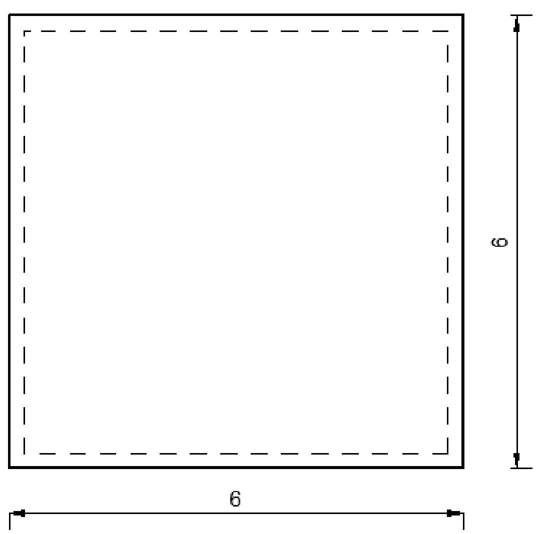


Figure-15: The designed simply supported slab, with dimensions in meters

4.4 Results

The results obtained from the design, using the rigid plastic model proposed by the FIB model code, can be seen in Table-4.1. From Table-4.1 it is clear that the moment resistance increases with increasing fibre volume. When compared with ordinary reinforced concrete slabs, it is clear that in order to substitute or replace ordinary reinforcement, large fibre fractions are needed. In Table-4.1, the moment resistances of the fibre reinforced concrete slabs are compared to the moment resistances of the reference slabs.

Table-17: Moment resistance results for slabs reinforced with fibres or ordinary reinforcement

V_f (%)	Reinforcement (mm)	Moment resistance (kNm/m)
0	ϕ8 s250	20.5
0	ϕ6 s250	12.9
0.25	-	3.0
0.50	-	4.5
0.75	-	6.4

No combination of fibres and ordinary reinforcement was done in this research, for the reason that, the FIB model code does not propose any methods for verification of moment resistance, for fibre reinforced concrete slabs with ordinary reinforcement. The FIB model code however states that it can be done using non-linear analysis.

Regarding shear resistance in slab members without ordinary reinforcement or prestressing, the FIB model code claims that the shear is not dominant unless there is a high load concentration close to the support.

5. Conclusion

The moment resistance obtained, when designing steel fibre reinforced beams using the FIB model code, confirmed the experimental results that the moment resistance increases with increased amount of fibres. There were however, slight underestimations of the ultimate bending moment capacities for all beams with fibres, designed according to FIB model code. This underestimation might be due to the variation in material properties for the different samples as, three experimental results from the same concrete mix varied significantly, where the mean value was used for comparison. Although, with the addition of increasing volumes of fibres, the moment capacity increases, it is, however, not sufficient enough to replace the ordinary reinforcing bars. The major magnitude of this capacity is still carried by the ordinary reinforcing bars. In order for the fibres to replace the ordinary reinforcement bars entirely, materials exhibiting strain-hardening behaviour need to be used.

Design of fibre reinforced concrete slab elements, revealed that fibres in low quantities have little influence on the moment resistance. It was more obvious in slab design than in beam design, that fibres are capable of replacing ordinary reinforcement entirely, if used in sufficient amounts as the increase of moment resistance with fibre volume was clearer. A benefit of using fibres in concrete slab elements is that only the direction with the maximum moment needs to be studied, since the moment resistance of fibre reinforced concrete is the same in all directions.

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