PAPER No. 95.

TRANSPORTATION OF SILT BY FLOWING WATER.

By P. Claxton.

To discuss the transportation of silt by flowing water, the writer proposes to review the work of a few experimenters progressively, beginning with Chezy's investigation of flow in streams without load, passing on to Kennedy's practical law for streams carrying constant load, and finishing up with Gilbert's experiments on transportation of varying load. The useful application of these investigations is the final goal, and progressive treatment is admittedly right in direction, but how far towards finality we shall have advanced, it will be for others to say.

C_a, C_b, C_c indicate coefficients applied in Chezy's equations, $C_{,}$, $C_{,,}$ in Kennedy's and C' C'' in the writers.

The method by which Chezy's formula $V = C\sqrt{R.S.}$ has been derived is familiar to all, but has been reproduced in the appendix. This formula deals with loadless streams, but, if it be applied to streams carrying load, the coefficient is modified, not the exponent. This treatment is questionable, for it is here contended that the exponent not the coefficient should be the varying quantity.

Later, Kennedy conducted another series of experiments on canals in the Punjab which were in perfect regime. The canals were, in other words, passing constant load of silt. Though temporary variations did occur, these were not dealt with. From these experiments Kennedy found a relation between depth d and mean velocity V which is expressed by the equation :—

$$V = C_{,}d^{\cdot 64} \text{ or } V^{1 \cdot 56} = C_{,,}d$$

Gilbert has finally given us experiments which deal with varying load. In the appendix a brief resume is given of his procedure. We here simply note that his "synthetic mean ratio" $\frac{I\,v\,q}{I\,w}$, expressing variation of slope to mean velocity, for 8 grades of silt, works out to 2·6, and ranges from 1·9 for fine to 3·8 for coarse silt. From these experiments it will be shewn that the exponent of V is varying with load. Not only so, but it also varies with nature of boundary. Hitherto, as we have noted, the exponent has been taken to be a constant and variations have been made in the coefficient.

Before we can compare these results, it will be necessary to find some means of correlating them. To do so we must examine the nature of the problem which flowing water and its power to transport silt present. For this reason, attention is drawn to yet another set of laboratory experiments by Osborne Reynolds on the features of flow when examined by means of tubes.

Reynolds recognizes two leading features of motion in fluids, direct and sinuous, and proceeds to find a connection between them. This connection he finds, lies in a critical velocity at which direct flow becomes sinuous. As long as the velocity is low a streak of colour introduced into the experimenting tube is drawn out in a straight thread, but at the critical velocity eddies burst throughout the mass. Internal resistance, which varied simply as V, at the same time changes to obey another law, and thenceforward for all higher velocities varies as V^2.

The reasons for this behaviour Reynolds could not for long account, but finally he discovered that it lay in the presence of boundaries. So long as these were absent, as in a free fall, flow was direct, but the moment the slightest lateral restraint by boundaries was impressed upon the filaments, eddies burst into being. The remarkable phenomenon by which a mere film of oil is able to calm a troubled sea shows how slight the restraint may be. Reynolds has shewn that the film produces eddies below the surface, and the form of energy is merely changed not destroyed. There are a great variety of boundaries having varying degrees of lateral restraint. For instance, we know that air is a boundary, depressing the line of maximum velocity which would under circumstances be otherwise at the surface. From a vacuum, which is the conception for absence of boundary, to the absolutely unyielding boundary we may have a variety of degrees of conversion, the exponent rising in value till it is 2 at the unyielding boundary.

Thus we see that it is not friction, but lateral restraint which brings about eddying resistance. Water may be said to be frictionless, its filaments sliding freely over each other. Admitting that a film clings to the boundary, the other filaments over this are freely sliding. Therefore the effect of friction cannot be conveyed into the mass. It is lateral restraint which is the cause, and by it, the boundary, whether rough or smooth, appears to exert its influence.

But how is it possible for smooth boundaries to create eddies? Reynolds has shewn that water in flow is highly unstable, steady motion evincing a tendency to break down for the slightest cause. This, and the fact that eddies burst throughout the mass at one time, give us the impression that it is for reasons inherent in flow itself that eddies appear. We have hitherto talked of the direct flow of filaments as also straight, but it is more correct to conceive that each is in reality in the form of a wave with a regular period. In nature we seldom, if ever, get strictly straight motion. There is always the period or rhythm to be considered. It would appearthen that the filaments of water when free to move develop such motion. The moment, however, a boundary is applied the regular motions at the sides of contact are thrown out of step. Water being highly elastic, though incompressible, these impressions are immediately communicated from the boundary throughout the mass. This, Reynolds shows us actually happens.

To further support this conception, compare the method recently adopted by Gibson at Niagara for measuring discharges in pipes. He does so by noting the periodicity of pressure at the walls of the pipe. If the boundary itself be regular, as a pipe, the readjustments of free periodicity to the boundary would themselves become regular and would be characteristic for each separate condition, thus making the computation of discharges by observation of periodicity possible.

We therefore conclude that there is no distinction between smooth and rough boundaries where only lateral restraint is in question, but while both exercise restraint equally, the rough boundary also offers irregularities of surface as direct obstructions. We shall need to differentiate between the two boundaries for this reason.

To do so we may have recourse to a very simple experiment, taking first the side effect of a rough boundary into account. A vertical plane is anchored to the bank of a stream and the other end is thrown out at an angle into the current. What is the result? Head is raised along the upstream face of the plane in a ridge as velocity is converted into potential head. Released at the end of the plane, the acquired head and resulting high velocities react in violent eddies. This gives us the formation of an eddy in a striking manner, and shows us how it consists of two distinct processes, *viz.*, tying up and release. The first converts velocity into head of potential, the second gives us the eddy in reaction. Now a bank is made up of a succession of such planes, though of minute dimensions.

To examine the bed we have only to look into a clear shallow stream. There it will be seen that the bed is laid out in a succession of dunes and hollows. Particles of silt are seen rolling up the face of the dune, and are caught up into the mass, action being severest at the crest, where also the residue drops into the hollow beyond. What is striking is the intensity of differences in velocity, and their local eddying effect. We cannot measure head of potential as we may at the ridge along the vertical plane, but we conclude that it is there because of differences in velocity. In the one instance we argue from cause to effect, *viz.*, from the ridge, or potential, to increased local velocity. In the other we argue from effect to cause, *viz.*, from increased velocity to potential.

Now it is this potential, evinced by differences in velocity to which the writer draws particular attention. Where such differences of velocity occur in a stream, there must be their immediate adjustment by means of the eddy. We have traced the process from cause to effect, and back from effect to cause, and we conclude that irregularities of surface create potential, and that potential in release gives us the eddy.

But this process is different to the first, *viz.*, that of lateral restraint. Eddies there arise, not from the irregularities of boundary, but from the reactions of periodicity or rhythm when thrown out of step. The cause is from within and the form of energy is merely converted from momentum of direct flow to momentum of sinuous flow. In the second potential is borrowed from without, and results in eddies giving us accession of energy which is also varying. The outside source is provided by reason of position.

To explain what position means a diagram will be helpful. A. B. C. is the longitudinal section of a stream originating at section A and flowing down to sections B and C. At A the stream has no outside source from which to borrow potential, while at B and C it has, the source increasing all the way down. At B it is represented by head which may rise to h_1, and at C to h_2. An obstruction at A, such as a shoal, could not raise head as it has none to borrow from. It would therefore in the ordinary silting up process remain and increase, extending as a wedge downstream. At B and C, however, immediate reactions would set in; in other words, the obstruction would borrow head or potential, which would react in eddies and these would remove the obstruction if moveable, otherwise if not moveable, the eddies would continue to rise in excess at the obstruction.

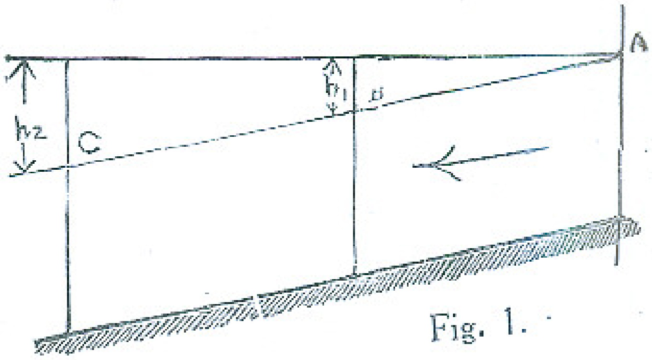


Fig. 1.

The source to which we are here alluding does not mean that the whole of the head, h_1 or h_2 is used. Reactions are innumerable and minute, and a very small portion of the head needs to be utilised. By constant recurring increments of potential which merge into continuity, flow becomes regular over a uniformly rough boundary, and the accession of potential is more or less constant. To realize the combined effect we need the analysis. It is, however, the continuity of uniform result to which the writer alludes when he talks of potential.

It might be convenient to call the source on which potential borrows, velocity head, since at each obstruction velocity is converted into head to give us the potential.

The eddies produced by potential, it will be easy to understand, are not distributed evenly throughout the mass as are the eddies of restraint. Intensity must fall off with distance from boundary, and we thus get our differences of velocity, as represented by the vertical and horizontal velocity curves.

Now, since the eddies of restraint represent energy which has merely been converted from direct motion into sinuous motion, we should expect the energy to be unchanged. In conversion, the acceleration of direct motion has been absorbed by sinuous motion till uniform velocity is attained. This means that acceleration is eliminated and the equation for work becomes $Mas=O$, when M is mass, a, acceleration and s, space.

But by a dynamical principle, work is also half, vis viva of momentum i. e. $\frac{MV^2}{2}$.

To measure this, we have to destroy it by writing $\frac{MV^2}{2}=0$.

We therefore have $Mas=\frac{MV^2}{2}$ or, $V^2=2as$.

We thus get back to the original dynamical law of motion in its unchanged form. In other words, if the energy of eddies varies as V^2, we merely have conversion of energy without any loss. This is the fundamental law at the basis of equations of flow, and we shall always refer to it as such.

Though there is a discrepancy, as will be shown later on, it will still be convenient to account that energy of eddies due to restraint varies as V^2, the exponent remaining constant.

The energy of eddies, due to potential, on the other hand, has no fixed law, except that it is some constant function of V when the nature of the boundary is regular. Its distribution throughout the mass is always varying. Now the effect of potential is to reduce velocity, but this velocity in absorbing potential has a higher exponent, *i.e.,* it has greater eddying energy. Just as uniform velocity, when eddying energy of restraint balances acceleration, absorbs acceleration, so modified velocity, when roughness causes obstruction, absorbs accession of energy due to potential. We shall always have to deal with modified velocities, and it is well to remember wherein the forces lie.

Let us then again review our position. We have a stream with internal forces created by eddies which come into existence because of lateral restraint and potential. Lateral restraint gives us eddies, which have evenly distributed energy, expressed by the function V^2, the exponent remaining constant. Potential gives us eddies with energy distributed unevenly through the mass, falling off in intensity with distance from the boundary. Otherwise, the energy becomes constant when boundaries are uniform. The combined energy of eddies gives us the total resistance within flow with which we have to deal.

Now acceleration is balanced by this resistance when uniform velocity is attained. Just as for the mass, so with each filament. Thus velocity curves give us varying velocity throughout the mass, since energy of eddies by potential is varying. But since we express our laws by the mean velocity of flow we are not at present concerned with varying distribution. The all important condition which enables us to find a law is the balance at uniform mean velocity. By it we obtain our equation, thus :—

Acceleration $\propto$ Resistance.

Of these two quantities we know all about acceleration, but resistance needs to be still more carefully considered. We have already considered it, but we may not have exhausted all the conceivable forces which influence it. There is, for instance, viscosity. This, Reynolds shows, tends to maintain direct flow and opposes sinuosity, till broken down at a critical velocity. If this be so, some energy must be lost in converting direct into sinuous flow. As a matter of fact it is, for Reynolds shows, how glass, which for our purposes may be regarded to be the limit of smooth unyielding boundaries, gives us eddying energy not as V^2 but as $V^{1 \cdot 83}$. Viscosity therefore lowers the exponent, but, when considering the law of restraint, we may for practical purposes ignore it, since there are also other discrepancies for which we do not account.

Having then considered all the forces which give us the eddies of resistance, let us take any cross section for our investigation. Over this cross section we have acceleration of gravity opposed and balanced by resistance of eddies. Since a section, and not a reach is being treated, acceleration is constant throughout the section. Call it f.

Then treating unit width, the work the force f does through depth d=fd, for since it is constant, it is the same as if we moved the force f through distance d.

Opposing this force we have resistance of eddies, expressed as some function of V as V^n.

Therefore $fd \propto V^n$,
or writing the expression as an equation,

$$fd = CV^n, \text{ or } f = \frac{CV^n}{d}$$

Writing n=2 for the fundamental equation, $f = \frac{CV^2}{d}$.

In this last equation we recognize much that has been familiar but hitherto elusive.

The above formula deals with streams without load, but we have now to consider streams with load and to enquire into the forces which enable streams to transport silt.

To be transported, silt must be held in suspension, velocity merely giving it its forward motion. But we have only two forces present to do so, those of acceleration and eddies. Of these, acceleration has no vertical lifting component, as its action may be taken to be horizontal. It cannot therefore help the silt which must be supported vertically to move forward. If not supported it would fall to, and rest on the bed.

We conclude therefore that eddies supply the lifting force, for they only have vertical components capable of doing so. Over the cross section we have chosen, particles of silt are distributed in varying density, having been brought into those positions, measured on the vertical, by the energy of eddies. Part of the energy has therefore been absorbed by lifting silt in suspension. The resistance of eddies to oppose acceleration is therefore reduced; acceleration in consequence should increase, till a balance is again established at a higher velocity. This makes it possible for streams with increasing load to quicken their velocity.

Eddies have therefore a two-fold work to do. Part of their energy is spent in resisting and eliminating acceleration and part in supporting silt. Then considering V, which is the measurable quantity, we have to determine what fraction of V^n goes to resist acceleration, and what to support silt. The fraction is changing with the nature, that is the specific gravity of load, but enters the problem as a coefficient, and not as a function affecting the exponent. In other words, the capacity for resisting acceleration and the capacity for supporting silt both vary as V^n. The amount of silt in suspension is therefore also a measure of the energy of eddies. This is important to remember.

The forces of gravity along the slope of the channel, for convenience considered to be horizontal, can be requisitioned by causing debris to roll or slide along the bed. If, however, we examine a vertical curve, drawn to illustrate its theoretic character near the bed, we shall find that the velocity at contact with the bed is either zero or indefinitely small, and rolling and sliding along the bed cannot be due to actual bed velocities. As these, however, rapidly increase from the bed upward, and as particles have some depth, the changing forces bear more on the upper part of the particle than on the lower. The resistance of bed with this difference constitutes a strong couple which may set the particle in motion. Add to this the fact that the nature of the dune induces potential, and hence velocity, on its upper face, and we see that the forces for rolling are really present. Such action would have the tendency to reduce the forward velocity, since it would absorb acceleration. Rolling, however, except for the very slow progression of dunes in most natural streams is not a large consideration.

Another more important effect needs to be considered, and that is, the obstruction which the suspended load itself presents to flow. This must also reduce velocity, but we should be careful how we understand it. A floating body does not oppose acceleration or alter velocity. It is simply borne forward. The moment, however, that its direction is not the same as that of the current, opposition sets in. Now, the particles of silt are shot up with the impulse of eddies. It is their angle of trajectory which is opposed to the direction of the current. They therefore oppose the stream, just as the eddy which gave them being would have done. There is however this difference. Part of the energy of the eddy is absorbed by supporting the weight of the particle and the effect is not the same.

Thus while we have absorption of eddies by suspended load increasing velocity, we have also the obstruction of that load reducing it. As weight has to be supported by the eddies we may conclude that part of their energy is absorbed, the amount being represented by the specific gravity of the silt and the vertical distance it is raised from the bed. The above are therefore tendencies which still show that streams receiving increasing load are able to quicken their velocity but it modifies our previous impression.

Hitherto we have been considering the forces of only one cross section, but now consider a defined reach of channel with uniform slope. For all cross sections in the reach, f becomes constant and we have :

$$V^n = Cd.$$

$$\text{or } V = C^{\frac{1}{n}} d^{\frac{1}{n}}$$

In a channel transporting silt the tendency is for the bed to assume uniformity of slope. For channels in regime this equation would therefore express the true relations though with modifications of the exponent as we shall see.

Now, we have the general expression :

Energy of eddies $\propto V^n$,

this energy being measured at any cross section.

But the forces at the cross section are being renewed at the rate of V.

Therefore the power, or capacity of eddying energy, to transport silt in suspension varies as V^{n+1}

For the fundamental equation $n+1=3$ and transportation $\propto V^3$.

We may now return to the three sets of experiments mentioned at the beginning.

To first compare Chezy's results with the writer's theory, we have

$$\text{Resistance of eddies} = \frac{CV^n}{d}$$

This is balanced by the force of gravity acting parallel to the stream's bed, as F in Chezy's formula.

If W=weight of water and h= fall in length L

$$\text{Then } \frac{F}{W} = \frac{h}{L}$$

$$\text{or } F = W.s.$$

From which we derive $V^n = C' s.R$ (Chezy)

But $F = \frac{CV^n}{d}$ (writer)

$$\therefore \frac{CV^n}{d} = Ws$$

and for any one site Ws is constant $\therefore V^n = C'' d.$ (writer).

Chezy makes $n=2$.

The writer does this only for the fundamental equation. The exponent he shows is altered when roughness or silt load are introduced into the equation. For regularity of surface, or constant silt load, it again becomes constant, but when these conditions vary it also varies.

Reynolds and Gilbert both express the equation for such conditions by a variable exponent.

Actually the exponent of V for the fundamental equation, when only the law of restraint is considered, is lower than 2, and should be 1·83 because of viscosity.

Now, the fundamental equation applies to pipes where the full effect of lateral restraint is obtained by a complete boundary. A stream on the other hand is made up of a boundary partly in soil, which is comparatively unyielding, and partly in air, which is extremely flexible. The value of the exponent should therefore be appreciably lowered.

Against this we have added energy of eddies due to potential of roughness which all channels possess. This will again raise the value of the exponent.

We see then that the actual exponent is very uncertain, and by theory is in fact indeterminate. To overcome this difficulty Kennedy evolved the method by which an empirical exponent is found giving us conditions as they actually occur on streams in regime. For this reason his experiments have been invaluable.

By Kennedy's plan observed velocities and depths (the only quantities which he found affected the results) were written down. He then found that these were consistently correlated by the equation.

$$V = C_{\prime}\, d^{.64}.$$

Now, since Kennedy treats streams in regime the quantity f of the writer's equation

$$f = \frac{C\,V^{n}}{d}$$

becomes constant, *i.e.*,

$$C' = \frac{V^{n}}{d}$$

To reduce Kennedy's equation then to a form similar to that used by the writer we have merely to write his equation as

$$V^{1\cdot56} = C_{\prime\prime}\, d$$

$$\text{or } C_{\prime\prime} = \frac{V^{1\cdot56}}{d}$$

i.e., we express the exponent of V when that of d is unity.

It will be seen that this exponent for Kennedy's channels is lower than 2. To explain this Kennedy advanced the following theory, taking both suspended and bed load into account. Suspended load varies as V^{n} which expresses eddying energy. But the whole of this load is being carried forward at a velocity V. Therefore the load transported varies as V^{n+1} Add to this bed load which is rolling forward simply as V. Its effect will be to lower the power and make it something less than n + 1. Assuming n to be 2, as Chezy does, we get the final result something less than 3, and this Kennedy finds to be 2·56. Reconverting back from transportation to suspension, we get the exponent 1·56. The reasoning is clear, but it is certain, as we have shown, that the load does not roll at simply a velocity V.

But more than this, the nature of the boundary must be taken into account. Chezy experimented on artificial flumes with practically solid boundaries, whereas Kennedy's channels had a soft bed composed of silt which presents a comparatively yielding boundary. For this reason Kennedy's exponent would be lower. Gilbert shows that solid flumes have notably higher capacity to transport silt than channels in soil.

Now, as we have said, Kennedy's results apply to one grade of silt (though mixed) and further, to constant load. To extend them so as to include other grades it is necessary to pass on to Gilbert's experiments. As these involve an entirely new method of observation, they are given at some length in the appendix. Gilbert experimented with varying grades and loads of silt and found that the exponent of V is a varying quantity for both. Hitherto we have applied Kennedy's results to varying grades by simply taking a larger or smaller proportion of V_o at random. By doing so we modify the co-efficient of Kennedy's equation, not the exponent of V. It is correct, as Gilbert shows, to modify the exponent for each grade.

The useful results of Gilbert's experiments are represented by a table in the appendix and this abstracts the means of 8 grades of silt. The ratio $\frac{I\,v\,q}{I\,w}$ gives us the relation of slope to mean velocity, the same as Chezy's formula does. We may therefore treat it as we have Chezy's formula, by reducing it to a form similar to that given by the writer. We thus see that the exponents of V vary from 1·9 to 3·8 according as the silt is fine or coarse. The varying nature of the exponent is important. Reynolds also in his experiments on tubes found the same. Why the exponent should vary the writer has tried to show in his theory, for while lateral restraint gives us uniformly distributed and constant resistance on any cross section, potential gives unevenly distributed and varying resistance.

The three experiments therefore which we set out to examine have been compared by one standard, but what is the useful application? Kennedy has again given it to us by his critical velocity V_o at which the relation between V and d subsists. By plotting the locus of this equation on his discharge curves he makes it conveniently applicable.

Now, Kennedy's equation as reduced to the form

$$V^{1\cdot 56} = C_{\prime\prime}\, d$$

may be also written

$$\frac{C_{\prime\prime\prime}\; V^{1\cdot 56}}{d} = 1$$

In this form the expression on the left may be called a silt index, and may be defined as the capacity of a stream to hold silt in suspension.

Comparing it with the writer's equation :

$$f = \frac{C V^n}{d}$$

we see that $\frac{C V^n}{d}$ is equivalent to the constant force of acceleration across any section of a stream. If then we wish to measure suspension by this force we have merely to write f as unity.

Kennedy's V_o then is therefore a silt index. But n is varying for each grade of silt and we should therefore have distinct curves for different grades of silt. This distinction is given to us by Gilbert who finds the curve for the varying exponent n. We have in this table 8 values for 8 distinct grades.

But before attempting to apply Gilbert's results we should examine whether his conception of terms is similar. One very notable difference occurs in his accepting capacity at the maximum load which a stream is able to carry. None of Kennedy's streams can be said to be carrying maximum load, since during freshets they are able to pass on loads many times in excess by drawing more largely on potential. Gilbert's exponents are therefore notably high.

Another deficiency may be said to exist in the load, for Gilbert deals with the load of traction, *i.e.*, with the load near the bed. This may, however, be assumed to be proportionate to the total load, and the error may thereby be partly eliminated.

Accepting then Gilbert's results as being uniformly higher than Kennedy's, we have yet to correlate the grade of silt in both. Unfortunately Kennedy graded his silt by the rate of subsidence in still water, whereas Gilbert graded his by means of sieves. Kennedy's grade is also mixed, whereas Gilbert's grades are distinct. The nearest approach to comparison therefore to which we can come is to accept a distinction for Kennedy's sample between his grades $\frac{\cdot 05}{\cdot 10}$ and $\frac{\cdot 10}{\cdot 15}$ which represent 68 per cent. of his sample, and to find the corresponding grade in Gilbert's samples. The discrepancy between the two equations would then approximately represent that due to average load of Kennedy's channels and maximum load of Gilbert's. Further, Kennedy's exponent for V could be extended on both sides of the correlated sample by completing the curve parallel to that given by Gilbert. We thus should have two similar curves, one for average load, and one for maximum load, enabling us to apply the results of experiments on a wider scale.

The above represents work which might be taken up by the Research Department. The correlation of the two experiments can but be an approximation, but other experiments may be instituted after Gilbert's plan to obtain the extended exponent for conditions suited to the Punjab. Not only do grades of silt vary gradually according to locality, but very often they change rapidly due to temporary floods and freshets. These last are the greatest danger. But though these investigations are for the future, we have their general principles for immediate instruction. Gilbert's experiments show us where the limitations of Kennedy's equation lie, and also how modifications when applied should be made. The choosing of greater or less proportions of V_o is plainly wrong. Each grade of silt has its separate equation. Each grade of silt has also a separate set of velocities, as shown by Gilbert and as explained by the writer's theory.

The work involved in finding the variable exponent is formidable, and we should therefore be well advised to meanwhile gratefully accept Kennedy's single set of results, being content to know where the limitations and errors of application lie, and using our judgment accordingly. This conclusion is rather disappointing, but happily relieves us of much labour.

Yet some will ask how is it possible for Kennedy's results to work so efficiently? The writer ventures to reply that the possibility lies in the variable potential.

A stream, as pointed out, is able to borrow potential at all points of its course except at the offtake. Thus it carries on silt or drops it, eliminating the errors of design and giving the application of the rule great latitude. It is this also which forces down large excesses which appear during freshets.

If this be so, it is necessary to pay far great attention to the offtakes where adjustments by potential are not available and which are therefore the critical points of our canal systems. It is indeed the offtake which governs not only the efficiency of our canals but the courses of all natural channels which make up rivers. The writer has now observed the courses of river channels for years, and has no hesitation in giving the offtake the first place of importance. Some time ago he was engaged in a controversy in "Indian Engineering" over the Sukkur Barrage, and he then argued from the outfall. The argument in effect defends the offtake, for unless there is the outfall, and only when its influences reaches as far as the offtake, will the latter probably cease to silt up the line of a threatened break away, such as critics had so prominently in mind at Sukkur. In other words, while the outfall favours a breakaway, the offtake chokes it. It is only when the latter overcomes the former that it will occur. The offtake, however, often silts up a channel which already has an outfall, and that in the face of strong forces tending to keep the head clear. The offtake in fact is more the changing factor than the outfall. The greatest danger to our canal heads therefore lies here, and we may also say that their efficiency is determined here.

But it is just at the offtake where Kennedy's equation fails. The reason undoubtedly lies in potential which also fails to function. The writer has shown that at such points no reserve of potential can be borrowed from, and as some potential is also lost in effecting change of direction, the conditions become still worse. Moreover when freshets come down the capacity of streams to borrow on potential needs to be much enhanced, but this increased capacity is denied the offtake.

For all these reasons it is particularly necessary to pay attention to the offtake and the question arises what are the remedies? Since potential cannot be enhanced one remedy is to increase velocity by giving greater slope to the branch at head. The capacity of the head to carry on silt is thus raised. Over and above this some excess head should also be allowed to pass over the critical periods during freshets.

But this means loss of command which cannot always be spared. Is there yet then another remedy? Many have been advanced by specially designed heads, but the simplest and best in the writer's opinion is the groyne. A figure indicates how it may be applied.

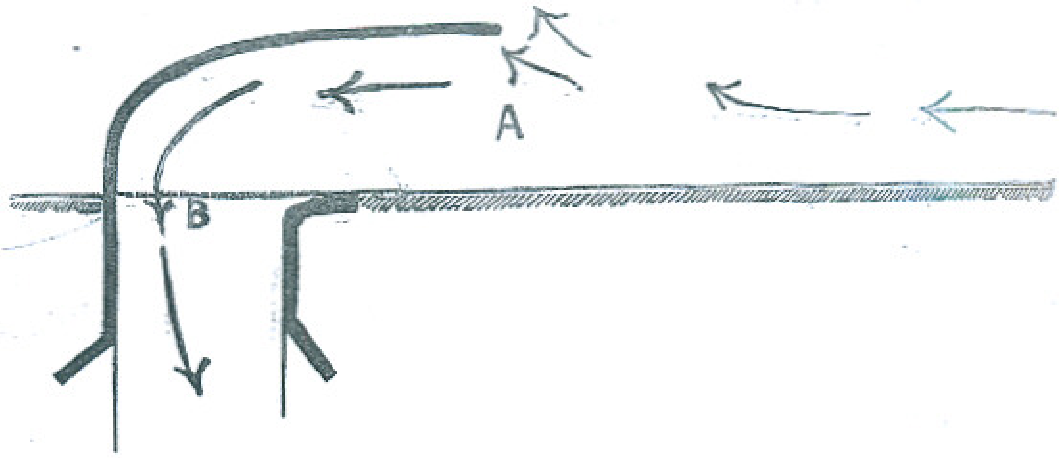


Fig 2.

The direct effect of such a groyne is to move the offtake higher upstream from B at the branch head, to A, a point in the feeder itself. The loss of head at B, due to change of direction is then eliminated, being conserved as potential in the groyne pocket, and redelivered to the branch as velocity after the change has been effected.

Other advantages not connected with the shifting of the offtake are: the exclusion of backwater, and in fact of all direct flow from in front of the branch head; the indraw is limited only to that section of the stream marked out by the dividing wall, and this side section we know is less heavily charged than other sections further out; by having a cut water at the nose of the divide wall, the action of eddies, indicating loss of potential, is almost eliminated.

Returning now again to the effect of the offtake, the efficiency of the groyne lies in the creation of head in the pocket. The greater this head the more effectually will the offtake be carried upstream. By reaction velocities are checked, and silt is at once dropped, beginning at the point to which the afflux reaches. Our object is to cause the silt to drop outside the groyne pocket. It is then carried away again by reaction which sets in as cross flow due to head at the bank. Not only bed but higher particles of silt may in this way be diverted away from the groyne pocket.

But the moment a canal is added, the efficiency of the groyne is lowered, because of the indraw thereby created. Some efficiency still remains, but it depends largely on the difference in velocity, inside and outside the groyne pocket, which again is proportional to the difference of capacities of groyne pocket and head of canal. For small channels we may proportion our design to give the desired effect, *i. e.*, to gain the potential necessary to carry on silt, but for larger channels the writer has a suggestion to offer which is a departure from past practice.

Our object is to carry the offtake higher upstream. The further removed from B the more ability will the branch have to pick up potential at B, since potential has an increasing reserve the further downstream it is. To attain this object the intake at A may be narrowed as shown in the next figure :—

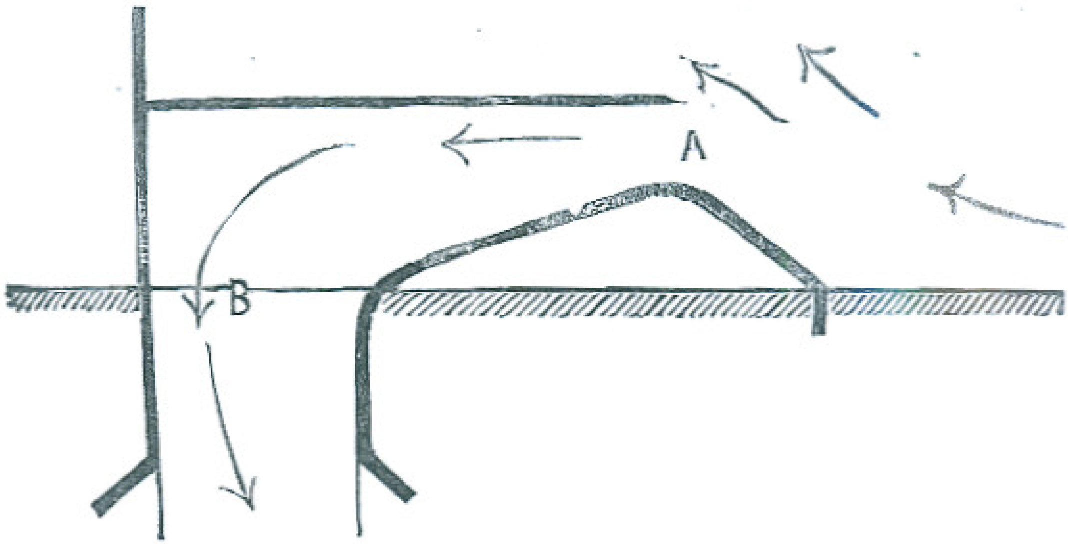


Fig 3.

The efficiency of groyne is thus transferred to the narrowed section at A, and is independent of the relative proportions of groyne and branch-head. Moreover silt is more effectively extended by being dropped outside the pocket. It will be advisable not to make the groyne pocket any wider than the branch itself, for the writer does not propose to work by means of under-sluices in the pocket at any time. By doing so the advantage of potential at B is lost. It will be much better to carry on regulation at A, and preferably partially by means of rising cills.

Thus, we have permanent control above our branch-head, thereby permanently transferring the offtake to a point above the branch-head. Let us recapitulate what the advantages are which are claimed for this design. By it we utilize the energy of the stream instead of letting it go waste. By doing so we gain potential at A, and by afflux divert silt away from the groyne pocket. Moreover we again conserve energy at the branch-head B, and deliver it usefully to the branch. (This energy with under-sluices would be lost).

Again, the creation of eddies, stirring up bottom silt is almost eliminated, the dangerous backwater is entirely eliminated, and indraw is limited to only the less heavily laden side section. These eliminations and limitations make a vast difference.

While some engineers may think that we beg these concessions for the groyne, may the writer assure them that he has given their claims a thorough trial at minor heads with satisfactory results. He can also give examples on inundation canals which prove the larger claims. Only this year by the addition of a groyne has one of the worst canal heads been converted into one of the best. A narrow cunette, 30 feet wide, involving but five lakhs of silt clearance was dug to give early supply, and this the groyne widened and deepened till 40 lakhs more silt had been carried down. If not credible to others, these results are at least familiar to us on these out-of-the-way canals. Not one but many examples may be quoted. It may only be noted that the pockets of these groynes are of large capacity which enhances efficiency.

The narrow section is however a departure which has to be tried; yet it hardly comes under an experiment, as we are familiar with the effects which such conditions secure. It only remains for the design to be applied.

Thus, while reviewing the experiments of the past, the writer has attempted to bring them into line with the present, and to lead them to practical result in a simple design.

APPENDIX.

Experiments by Grove Karl Gilbert.

The experimenters recognized that water carries forward debris in various ways. Sliding of particles on the bed they note rarely takes place. Pure rolling is also of small relative importance. If the bed is uneven the particle usually does not roll but makes leaps and the process is then called saltation. With swifter current the leaps are extended and the particle may be caught up by an ascending swirl, and its excursion may be indefinitely prolonged. Thus borne it is said to be suspended, and the process by which it is transported is called suspension. There is no sharp line between saltation and suspension, but the one grades into the other. The two processes, however, they consider to be distinct. In suspension the efficient factor is the upward component motion. In traction, a term which includes saltation, rolling and sliding, the efficient factor, they say, is the motion parallel with the bed, and close to it. Thus transportation is divided into suspension and traction, indicating two distinct processes.

With this classification the writer does not agree. Omitting sliding and rolling, saltation and suspension are one process, the bed load being supported in just the same way, as the load of suspension, *viz.*, by the vertical components of eddies. It must be remembered that velocity is not a force capable of doing work. It is simply a condition balancing the internal forces. The intensity of eddies makes the only distinction, a distinction, however, with no line of division between traction and suspension.

The experimenters confined themselves to observation on traction, their primary purpose being to learn the laws which control the movement of bed load, and especially to determine how the quantity of load is related to the stream's slope and discharge, and to degrees of comminution of the debris.

To this end a laboratory was equipped at Berkeley, California, and experiments were performed in which each of the three conditions mentioned were separately varied, and the resulting variations of load were observed and measured. Sand and gravel were sorted by sieves into grades of uniform size. Determinate discharges were used. In each experiment a specific load was fed to a stream of specific width and discharge, and measurement was made of the slope to which the stream automatically adjusted its bed, so as to enable the current to transport load.

For each combination of discharge, width, and grades of debris, there is, they found, a slope, called the competent slope which limits transportation. With lower slopes there is no load, or the stream has no capacity for load. With higher slopes capacity exists.

Capacity is defined as the maximum load of a given kind of debris which a given stream can transport. When a fully loaded stream undergoes some change of condition, affecting its capacity, it becomes thereby overloaded or underloaded. If overloaded, it drops part of its load, making a deposit. If underloaded it takes on more load, thereby eroding its bed. Through these reactions the profiles of stream bed are adjusted. Load is introduced by adding debris at the head of the experimental trough and by trapping and measuring that borne close to the bed at the end of the trough. When the load is small the bed is moulded into hills, called dunes, which travel downstream, the current eroding their upstream faces and depositing debris on the downstream faces. With any progressive change of conditions tending to increase load, the dunes eventually disappear, and the debris surface becomes smooth. The smooth phase is in turn succeeded by a second rhythmic phase in which a system of hills, termed antidunes, travel upstream, by erosion on the downstream face and deposition on the upstream face. Both rhythms of debris movements are initiated by rhythms of water movement.

To these observations the writer offers the following remarks:—

These phases are important to remember as indicating how deposition and scour are initiated. Potential is the influence behind them. With the dune, potential is brought into existence at the base of the dune, and rising up its face, increases in intensity, till at the crest it reaches a maximum. Eddies are the direct result and at the crest become intense. Particles in consequence may be seen moving up the face and rolling into the hollow behind the crest, many leaping free, particularly at the crest into the body of water above, or being redeposited in the hollow again, or on the face of the next dune. Thus the dunes move forward. In the anti-dune the process is the same, except that action at the crest, becomes so severe that scour sets in and cuts the crest back, giving the appearance of a dune travelling upstream. It must be remembered that no dunes can form permanently in a stream at a point removed any distance from the head. Potential here has a big reserve to draw on and is capable of scouring out even the heaviest shoal. Every engineer will be aware of this. Thus streams are enabled to carry indefinite loads, once these have passed the head. We shall consider these phases again in relation to slope which they affect and determine.

Omitting the other observations of Gilbert, the relation of velocity to capacity, and slope as determined by him, will alone be considered, since it is the only part of the experiment useful for our present purpose. It is fortunate that Gilbert here deals with mean, not bed velocity. The other results are purely empiric involving the "competent" value which has to be deducted from the observed slope and discharge to express the exponent.

Examining then the experimenters' conception of capacity, we find that it expresses a maximum which is attained by increasing load till the limit of transportation is reached. In natural streams this maximum is seldom attained. A stream has usually capacity for more load which we have shown may be of unlimited quantity for any point removed some distance from the head. The results of the experiment should therefore be in excess of observations made in midstream, but will be very useful for streams at off-take. The excess values here referred to appear as larger exponents of the mean velocity in the results of the experiments.

Next it must be remembered that the experimenters' aim has been to represent observations of the bed load only.

They did so by means of an arresting trough across which the stream was made to leap, the bed load being trapped. This may also give higher values as exponents of V which cannot be correlated to results based on other experiments or expressed by the general laws of theory, though the bed load may be considered to be more or less a constant proportion of total load.

The experiments are still valuable, especially where the ratios of two indices of variation, such as the ratio Ivq/Iw (to be described below) make them comparable with the results of other experiments.

Other points of vital interest are also brought to light, principally the effect of slope on the redistribution of the internal forces on a vertical plane, and the effect of added load in modifying not only the slope but also the nature of the bed.

Turning then to the experiments themselves Gilbert conceived a process of comparing the rate of variation of one thing with the rate of variation of another by means of an algebriac form for which a rather lengthy digression will be needed. To this form the title of 'power-function' has been applied, the most common form being $y=ax^n$ (1), or if the co-efficient be suppressed $y \propto x^n$.

If we consider x and y simply as numbers, the rate of variation of y with respect to x is the differential ratio of y to x and is $\frac{dy}{dx}$, which equals anx^{n-1}.

If we consider x and y as powers of a common base, the equation becomes

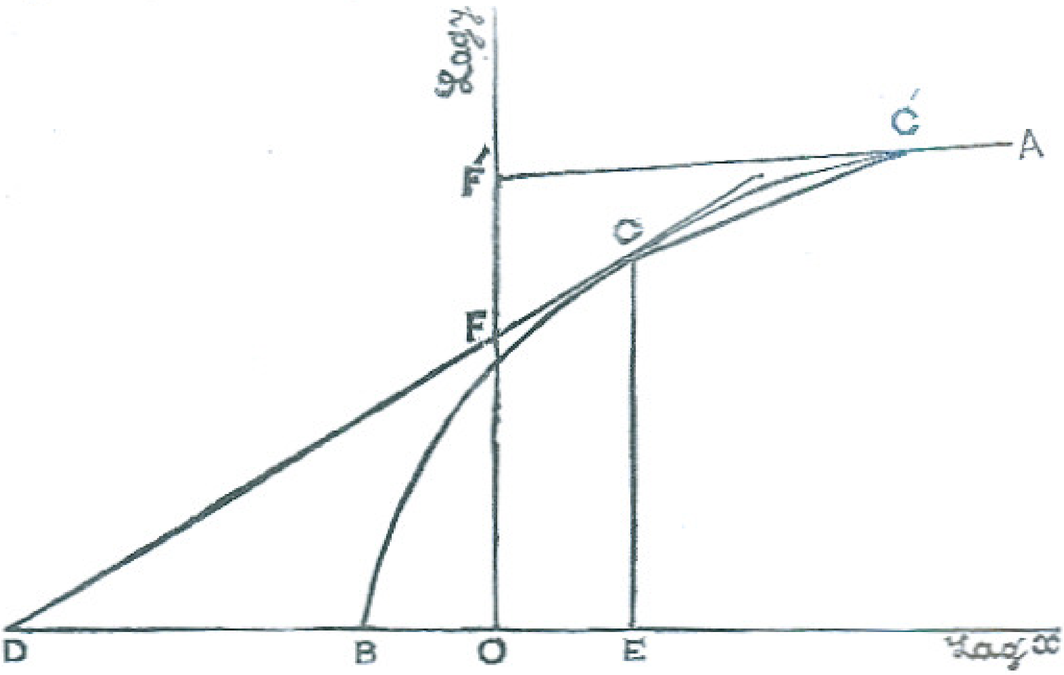
$$\log y = \log a + n \log x \quad (2).$$

By differentiating

$$\frac{d \log y}{d \log x} = n \quad (3).$$

The fact that equation (2), the logarithmic-equivalent to equation (1), is the equation of a straight line enhances its utility by enabling us to examine the comparative rate of variation of the exponent, n.

But in many physical problems it is found that the exponent n does not have a constant value through the observed range of x and y. The exponent n is in other words a variable, and its locus of log y = f log x, is a curve which we may represent by A B in the figure.



Locus of log y=f (log x) illustrating the nature of the index of relative variation.

At any point of the curve C, its minute element, not distinguishable from a straight line, has an inclination $\frac{CE}{DE}$ which is homologous with n in equations (3) & (1), and which we may call n_2.

The value of n_2 varies from point to point of the curve.

The variation at any point C on this curve would then be represented by the tangent line C. D. of which the equation is: $\log y = \overline{F.O} + n_2 \log x$.

$$\text{Whence } y = \log^{-1} \overline{F.O}\, x^{n_2};$$

comparing this with equation (1) it is seen that $\log^{-1} \overline{F.O}$. corresponds to a. Replace it by a_1, and we get

$$y = a_1 x^{n_2}$$

For any other point as C' on the curve the tangent intersects the axis of log y at a point different from F, and this corresponds to a different value of a_1. In other words, if we would express in an equation of type (1) the same relation between the two variables that is expressed by the logarithmic locus in the figure, we must make the coefficient as well as the exponent variable. The values of a_1 and n_2 are evidently functions of the independent variable, x.

To distinguish that this equation is one in which co-efficient and exponent are both variants he uses the symbols

$$y = vx^{i}$$

The exponent i is the instantaneous ratio of the variation of y to the variation of x, and is the first differential coefficient of log y with respect to log x, and is spoken of as the index of relative variation. Much attention is given to it and the discussion of the variation of such values is used as a mode of treating empirically the relations between the various factors of the general problem of traction.

Again, recurring to the figure and giving attention to a restricted portion of the curve, for example the part between C and C′ the value of i corresponding to the point C is the inclination of the line C F; the value of i corresponding to the point C′ is the inclination of the line C′ F′. Between the two are a continuous series of other values. The inclination of the chord connecting C and C′ considered as a ratio or exponent, is intermediate between extreme values of i. If the sequence of values follows a definite law, the value given by the chord equals some sort of mean derived from the others, and in any case it is in a sense representative of the group. It may be called a synthetic index of relative variation between the indicated limits. If the co-ordinates of C be log x′ and log y′ and the co-ordinates of C′ be log x″ and log y″, then representing the synthetic index by I.

$$I = \frac{\log y'' - \log y'}{\log x'' - \log x'}$$

As the direction of chord depends on the position of C and C′ upon the curve, so the value of I depends on the limits between which it is computed. As the direction of the chord gives no information concerning the direction of any part of the curve, so the value of I cannot be used to determine any particular value of i. It is used for the comparison of different functions for which the data span approximately the same range of conditions.

In dealing with the relations of capacity to velocity Gilbert employs the synthetic index of relative variation to discuss the problem. This is characterized by the symbol I, the index with respect to mean velocity being written Iv and distinguished as I_{vq}, I_{vs}, I_{vd}, I_{w}, when associated with the special cases, constant discharge, constant slope, constant depth, and constant width, respectively. These distinguishing symbols are used, since to make a definite comparison between capacity and mean velocity, it is necessary to postulate constancy in some accessory condition. If slope be constant, velocity changes as discharge, and if discharge be constant, velocity changes with slope, and if depth be constant, velocity changes with simultaneous changes of slope and discharge.

The computations of the index are then made by the formula

$$I_v = \frac{\log C' - \log C''}{\log v'_m - \log v''_m}$$

in which C′ and C″ are specific capacities and v'_m and v''_m are the corresponding mean velocities. Graphically, I_v is the inclination of a line connecting two points of which the co-ordinates are, for the first, log C′ and log v′m and for the second, log C″ and log v''_m. Where the data serve to place more than two points in the logarithmic plot $C = f(v_m)$ a line is drawn with regard to all.

The following table gives the means of indexes for several grades of debris, the latter being tabulated below. The table represents averages for practically the entire range of the conditions covered by the experiments.

Table of synthetic indexes.

Grade.	Number of separate determinations.	Mean I_{vq}	Mean I_w	Mean I_{vs}	Mean $\frac{I_{vq}}{I_w}$	Mean $\frac{I_w}{I_{vq}}$
A	5	3·62	1·87	2·02	1·93	0·52
B	18	4·35	1·94	4·00	2·24	0·45
C	19	3·57	1·85	2·96	1·93	0·52
D	9	4·50	1·87	2·97	2·41	0·42
E	5	5·17	1·83	3·08	2·83	0·35
F	5	8·73	2·27	4·11	3·84	0·26
G	9	9·56	2·60	3·85	3·68	0·27
H	3	10·01	3·20	7·81	3·12	0·32
	..	5·33	2·05	3·85	2·60	0·38

The following table of grades of sand and gravel gives us the grades of debris used:—

Grade name.	Sieves used in separation (meshes) to 1 inch	D_o mean diameter of particles (foot).	F_1 No. of particles to linear foot.	F_2 No. of particles to cubic foot.	Range of D or F_1	Range of F_1
A ..	50/60	0·00100	1,002	1,910,000,000	1·13	1·44
B ..	40/50	0·00123	872	1,023,000,000	1·17	1·60
C ..	30/40	0·00166	602	417,000,000	1·44	2·99
D ..	20/30	0·00258	388	111,500,000	1·56	3·80
E ..	10/20	0·00561	178	10,770,000	1·95	7·41
F ..	6/8	0·0104	95·9	1,685,000	1·40	2·74
G ..	4/6	0·0162	61·8	451,000	1·43	2·92
H ..	3/4	0·0230	43·4	156,000	1·36	2·51

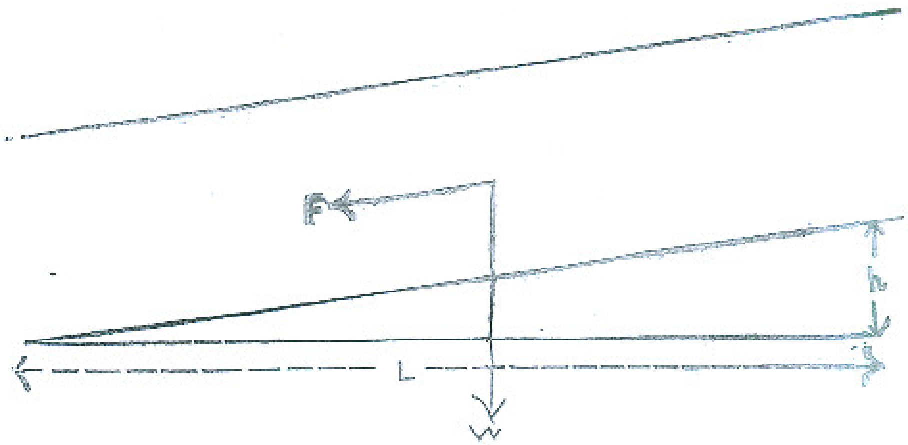
The comparison of I_{vq} with I_w affords an estimate of the relative variation of mean velocity and slope. The rate of variation of capacity with mean velocity being I_v, the ratio of variation of mean velocity with capacity is I/I_{vq} and the rate of variation of capacity with slope being I_w the rate of variation of mean velocity with slope is $\frac{I}{I_{vq}} \times I_w = \frac{I_w}{I_{vq}}$. By means of this or its inverse ratio $\frac{I_{vq}}{I_w}$ we are able, as already stated, to examine the results of other experimenters and to compare them with these experiments.

The principles of steady, uniform flow, in channels (as ordinarily defined).

1. In any channel where there is uniform flow, the two factors involved are the frictional resistance and the accelerating force; these two must be equal or acceleration would occur.

2. The frictional resistance is assumed to be proportional to the square of the velocity, the area of the channel in contact with water, its roughness and the density of the liquid.

The general formula of flow is then obtained as follows:



F=accelerating force parallel to the channel gradient.
f=friction resisting force.
s=area of surface in contact with water for a length L.
V=velocity of flow in feet per second.
A=area of cross section in square feet.
L=length of channel considered in feet.
p=wetted perimeter in feet.
r=hydraulic radius $=\frac{A}{p}$ in feet.
h=fall in length L in feet.
S=gradient of canal $=\frac{h}{L}$
d=density of liquid.
W=weight of water in length L in pounds.
C_a & C_b = constants.

Then:—

$$\frac{F}{W}=\frac{h}{L} \quad \text{or } F=\frac{Wh}{L}$$

and $f=C_b\,s\,d\,V^2$

but $s=pL$;

therefore $f = C_b\,p\,L\,d\,V^2$.

$F=f \quad \therefore \frac{Wh}{L}=C_b\,p\,L\,d\,V^2$

from which:

$$V^2=\frac{Wh}{C_b\,p\,L^2 d} \quad \text{but } W=A\,Ld$$

therefore

$$V^2=\frac{A\,L\,d\,h}{c_b\,p\,L^2\,d}=\frac{1}{C_b}\cdot\frac{h}{L}\cdot\frac{A}{p}=\frac{1}{C_b}\,S\,R.$$

$$\text{or } V=\sqrt{\frac{1}{C_b}}\;\sqrt{S.\,R.}=C_a\sqrt{S.\,R.} \quad \text{(Chezy's formula).}$$

This might be written in a general form

$$V^n = C_c S.\,R.$$

as the exponent 2 for V is merely an assumed value.

DISCUSSION.

THE AUTHOR introduced his paper and said that parts might be found elliptical and needing further explanation. For example, he had omitted the whole question of shape and size of channels. Again he might be blamed for slurring over the comparison of Chezy's equation $V^n = C. S. R.$ with his own equation $V^n = C''d$. In this case, as C. R. was dependent only on the size and shape of the channel, it might be expressed as a definite quantity for a given depth.

Thus $C. R. = C_{(1)} d^m$, where m was something less than unity, being nearly equal to unity in the case of wide shallow streams and diminishing as these became narrower and deeper.

$\therefore V^n = C_{(2)} d^m S$ and where S was constant, as for streams in regime.

$V^n = C_{(3)} d^m$, which was of the same form as the author's equation when $m=1$, *i. e.*, when channels became so wide that the effect of banks was negligible.

In the Author's equation, the effect of banks as indicated by remarks on page 55, was taken into account by the exponent.

For purposes of comparison it was not necessary to go any further, since it was only the author's object to ascertain how far Chezy was right in assuming 2 to be the value of n, the exponent of V.

Again, in comparing Gilbert's results, a small error in notation should first be corrected. On page 65 of Appendix, I_w was not associated with velocity, but was written for capacity and slope, width being constant.

It was to be noted that the mean of Gilbert's synthetic indices $\frac{Ivg.}{Iw.} = 2{\cdot}6$ (see page 66). This meant that slope varied as $2{\cdot}6$ power of the velocity, *i. e.*,

$S \propto V^{2{\cdot}6}$, or, $C. S. = V^{2{\cdot}6}$, which was of the same form as Chezy's equation, when R, the hydraulic gradient was absorbed into the exponent.

These omissions were due to the fact that the exponent rather than the co-efficient was dealt with. Although in the equation $y = vx^i$, expressing the relation between the variables of flow, Gilbert showed that the exponent and the co-efficient were both varying, the author was still justified in dealing only with the exponent, since the slope measured the ratio of change and gave the law, while the co-efficient merely gave the condition. The danger was that the condition might be made more of than the law, as in discharge equations, where modifications were applied to the co-efficient while the exponent was fixed. By so doing, however, the law was destroyed and the equation became purely empirical and irrational. To get the purely rational equation, the Author had enunciated a law which he called the fundamental law (*vide* page 51).

Here f. d.=c.[1] v^2. The units of this would be :—

$$\frac{\text{feet}}{\text{seconds}^2} \times \text{feet} = \frac{\text{feet}^2}{\text{seconds}^2}$$

which was rational.

In other accepted equations, a discrepancy would be found. This was due to the introduction of "potential," an outside influence created by eddying impulses at the prominencies of the rough boundary when there was position. It might be objected that to obtain the fundamental equation, it was incorrect to write M. a. s.=O. If the condition of flow was considered, it would be seen that it became uniform under a succession of impulses which opposed acceleration and velocity was dependent on the momentum of and rate at which these impulses recurred. The quantity M. a. s. was therefore the measure of real work by acceleration between the interval of two successive impulses over an infinitesimal space S, which in the limit became zero. The result showed that the exponent of v_1, where impulses opposed only gravity was always 2. When more than gravity had to be considered as potential, the exponent was greater than 2.

The idea that the modified velocity absorbed and carried on potential was new perhaps, but the Author could think of no better way of expressing it. In other words, two equal velocities, one over a smooth boundary and the other over a rough, were quite different as one had an exponent 2, while the other had an exponent greater than 2, and the greater the exponent the more the eddying energy and silt capable of being transported.

He apologised that his conclusions were incomplete and pointed out that this type of investigation clearly belonged to a Research Department and he was content if he had put forward some new ideas worthy of consideration.

He drew attention to the final practical application of some of the principles put forward, to the design of canal and distributary heads. In this connection, he described the application of his theory to the groyne.

Mr. W. P. Thompson prefaced his remarks with an appreciation of the Author's efforts to bring to the notice of the Congress an analysis of the conditions of flow in water. To appreciate the circumstances, it was necessary to bear in mind the diagram drawn at page 50. A body of water in uniform motion in a straight line under the action of gravity was a phenomenon of some rarity and some importance and the wonder was that it did not attract closer study by a greater number of enthusiasts. Two forces act on the body in uniform motion, one measureable, the gravitational force, and a resistance, sufficiently indefinite. This resistance, the Author stated, was not friction but lateral restraint, all three of which appeared to be the same thing and between resistance, friction and lateral restraint there was sufficient material for an Athanasian exposition of the hydraulic faith. As the result of the conflict between gravitational force and resistance, we had the eddies, a visible manifestation of the effects of resistance ; these eddies had a certain property in that they helped in the suspension of silt ; but having done this work, it was difficult to accept the Author's argument that they were in consequence, of less resisting power to the force of gravity (from which they derived their existence) and that in consequence of this diminished power the velocity of the body of water tended to increase. Arguing in this direction, we come to perpetual motion.

SARDAR BAHADUR PRABH SINGH drew attention to the last paragraph, but one on page 48 wherein it is stated that water is frictionless, and also to the last five lines of the same page wherein it is laid down that impressions were communicated in water from the boundary throughout the mass. These two statements appeared to be contrary.

He had personally had experience with a groyne intake as described on page 60 and had found it had not the desired effect in preventing undue silt deposit and he had to apply other remedies and do away with the groyne.

MR. NICHOLSON noticed the Author had used a silt index at the bottom of page 56. He asked whether this was the same as the Wood's Silt Index?

He considered that the solution of the problems involved in the paper were possible only by actual experiments and that mathematical solutions were of little value.

He would like to ask the Author one question, *viz.*, whether a channel carrying a greater amount of silt would carry a greater amount of water. There were varied opinions on this. For example, some time ago on the Sirhind Canal they had thought that less water was passing during periods when silt was heaviest. On the other hand tests made in Egypt disclosed that channels carried more water when the silt was heaviest.

THE PRESIDENT said he remembered that the Author while in charge of the Bhatinda Division of the Sirhind Canal had constructed a number of groynes as described in the paper. He would be glad to know how they had worked.

MR. B. H. WILSDON said Mr. Claxton had attempted a very interesting but difficult problem. In discussing it, he could not usefully follow the Author's line of reasoning as too many exceptions to questions of principle as well as treatment would have to be made. The treatment of "potential" and the distinction drawn between turbulence produced by "potential" and "lateral restraint" respectively were cases in point.

In Mr. Claxton's formula, $f = \frac{cv^n}{d}$, f was defined as a force. If the exponent of V=2, the equation was dimensionally homogenous with C a perfect number. If n was allowed to vary, this was no longer true, so that, while the formula might be used to represent empirically, variations found by experiments, it could have no physical basis. In Wood's formula for the Silt Index, $I = V/0{\cdot}4d$, the index had the dimensions [time^{-1}]. This would correspond with an angular velocity such as was expressed in the usual notation for the vorticity of a fluid.

$$\zeta = \frac{\delta^2 \Psi}{\delta x^2} + \frac{\delta^2 \psi}{\delta y^2}$$

The formula was therefore rational, if it was supposed the silt charge was proportional to the vorticity. But although it was generally agreed that the silt charge depended on the vorticity, physical proof was needed that it was proportional. If, as seemed more reasonable, the energy of the vortex was equated with the energy of the silt charge, the formula should contain the square of the vorticity. The precise function of the linear dimensions of the stream then still remained to be determined.

Progress in the decision of such problems could be made along two lines. On the one hand, the collection of empirical evidence as to the relation of silt charge to velocity and the dimensions of the canal was essential. On the other hand, it was necessary to proceed along more theoretical lines. By working exclusively with empirical formulæ such as had been dealt with in the paper, the influence of physical factors which could not be measured with a foot rule might be lost sight of entirely and it was precisely these factors, such as viscosity and surface tension which controlled conditions of turbulent flow, and in particular the critical conditions. Then, it was necessary to consider to what extent conditions applicable to one channel held for another with an altered scale : thus, how far was it justifiable to assume that the same laws held good for an 8,000 cusec channel as for a water course. The question could only be investigated by reducing experiments to conditions of dynamic similarity.

It was a simple matter to shew that the energy of a stream must be determined by an equation of the form

$$E = pV^2 l^3 \phi \, [\, gl/V^2], \, [\, n/pVl \,], \, [\, \gamma/p \, V^2 l \,]$$

Here n = viscosity, γ=surface tension, and p = density and all the terms expressed in brackets after the function sign ϕ were dimensionless. The first of these terms which involved g could only be to the power of unity, so expressing the energy in terms of unit volume

$$E/l^3 = E^1 = pg\, l \, \phi \, [\, n/p \, Vl \,], \, [\, \gamma/p \, V^2 l \,]$$

If then, it was desired to compare the conditions of two channels without knowing the form of the functions on the right of the above equation, it was necessary to do so in such a way that the terms $n/p\,Vl$ and $\gamma/p\,V^2l$ were independent of the scale of the experiment. Thus indicating by the subscript the dimension of an experimental or model stream, it was necessary that

$$n/p\,Vl = n_0/p_0\,V_0 l_0 \,:\, \gamma/p\,V^2 l = \gamma_0/p_0 V_0^2 l_0$$

Only under these conditions would the flow be similar.

In the laboratory, by varying separately n and γ, it should be possibl to arrive at the true form of these functions.

Experimenting on the large scale where it was not possible to vary independently n and γ,

$$E^1 = E_0 {}^1l/l_0 \cdot \phi\, l'_0 [\, n/p\, Vl \,]\, l'_0, + [\, \gamma/p\, V^2 l \,]\, l'_0 = CE_0 {}^1l/l_0$$

Here C was a variable co-efficient which depended on velocity and dimensions as well as the physical constants of the liquid.

Until the form of this function was known, it was not likely that a physical theory for the equilibrium of silt in a flowing stream could be constructed. It was therefore on these points that investigation had been started in the research laboratory.

Lieut.-Col. B. C. Battye said he would confine his remarks to two headings, first, theoretical and second, practical.

He drew attention to the series of monograms on the resistance to flow of fluids in pipes by Mr. Evan Parry published by the New Zealand Public Works Department, which were worthy of study. Mr. Parry gave the results of a paper by Professor Lees published in the proceedings of the Royal Society, Vol. 91 (1914) which he summarised. In this paper, Professor Lees pointed out that resistance to flow varied as a function of the viscosity.

The viscosity of silty water differed from that of clear water and varied with the amount of silt content. It was thus obvious that any discussion on the flow of silty water should take viscosity into consideration.

Lees gave the following formula connecting resistance with flow :—

$$\frac{R}{Pv^2} = a \left(\frac{V}{vd}\right)^x + b, \text{ where,}$$

R = resistance per unit of surface.

P = density of the fluid.

v = mean velocity of flow.

V = kinematic viscosity of the fluid, *i. e*, the physical viscosity divided by the density.

d = diameter of the pipe.

x, a and b were experimental co-efficients.

In the paper under discussion, there was no mention of viscosity and it seemed that this ought to have been taken into consideration. It was true that as d increased, the practical effect of viscosity fell, so that when dealing with large channels, viscosity tended to become a matter of minor importance, but all the same it was a factor.

The resistance to flow at boundaries, was obviously due, as pointed out by the Author, to turbulence, but of molecular dimensions. The energy absorbed in this turbulence was obviously affected by viscosity and it was the kinematic viscosity, *i. e.*, the viscosity per unit of density with which we had to deal.

In this connection, Parry remarked "Above the critical value of the velocity, the resistance is apparently partly viscous and partly an inertia effect, and the relation between the elements a complex one ; so much so that the probability is that it cannot be exactly expressed by any formulæ ; neither is a formula an absolute necessity, though an approximate formula, if obtainable, is undoubtedly a convenience." "In this work, the resistance curve is regarded somewhat in the same manner as a stress strain or magnetic force and magnetization curve is regarded, that is to say, it is a complicated function obtained by experiment and expressed by means of a diagram which shows the relation between two chosen functions." The speaker thought it was almost an impossible task to make any adequate mathematical analysis of turbulences of this magnitude and the b.st that could be hoped to be done was to obtain empirical formulæ and curves based on carefully regulated experiments. The inclusion of kinematic viscosity in the formulæ was obviously a step forward in the right direction.

In regard to the practical application of these questions he drew attention to the fact that at the present time, the successful hydro-electric operation of low head projects involving large quantities of dirty water was almost an unsolved problem. Most schemes of this nature in successful operation, begged the question by adopting large scale seasonal storage ; but where no storage, or at most diurnal storage only was available, the problem was still unsolved ; hence the difficulty of operating some of the schemes of this nature which had recently been brought to the attention of the public.

In dealing with silty water for power purposes, there were difficulties which Irrigation Engineers did not meet with, as there were fluctuations in velocity due to variation in flow (which of course was due to the varying demand on the plant) as well as due to the unavoidable variations in the dimensions of the cross sections of the water channel.

For example, in the forebay immediately above the power station, the cross section had to be considerably greater than that of the main channel, and when this reduction in velocity was aggravated by changes in demand, it was almost impossible to avoid deposits of silt just where they were least wanted.

Up to the present the only successful remedy had been to fall back on decantation which could obviously only be carried out on a comparatively small scale, and even then it presented many problems, since it was necessary to arrange the water channels, which at one time had to reduce the velocity and drop silt and sand, and at other times had to increase the velocity sufficiently to scour out the deposit without at the same time interrupting the supply.

In mountain streams, the deposits to be handled varied from stones to pebbles, gravel, coarse sand, fine sand and silt. It was found in practice that it was impossible to carry out the decantation of all these various types in one and the same chamber and three different types of decantation chambers had to be evolved, one for small stones and pebbles, a second for gravel and coarse sand and a third for fine sand and silt, and it was the problems involved in the best design both for depositing and then scouring out the deposit, in all three kinds, that were still so much under discussion.

MR. LIVINGSTONE-LEARMONTH asked whether the method of observing discharges through pipes by noting the periodicity of pulsation had been perfected, and if so, would be glad of accurate information thereon.

THE AUTHOR replying to the discussion said that he had not attempted to enunciate a theory of perpetual motion as suggested by Mr. Thompson and thought that acceleration and resistance were appropriate terms to express the forces across any section of a stream. In regard to the points raised by Mr. Prabh Singh, the difficulty about sliding and pressure would be removed if it was remembered that motion was not destroyed but simply restrained. The type of groyne referred to by Mr. Prabh Singh did not fulfil the Author's idea of a suitable groyne completely.

Replying to Mr. Nicholson, the Author's silt index was quite different to the Wood's Silt Index. He agreed that experiments by the Research Department would be most welcome. In the Author's opinion, velocity tended to increase with density of silt, but there were modifying conditions. A great deal depended on the specific gravity of the silt. At Sidhnai the silt which was mainly finely divided clay, increased the velocity in the canal very decidedly. In reply to Mr. Foy, the Author had constructed two groynes in the Bhatinda Division, which had worked very well in his time. It appeared that one had since been dismantled and the old troubles still continued.

In reply to Mr. Wilsdon, turbulence had been taken into account by the Author, both for smooth and for rough boundaries and he did not consider he had under-estimated its effect. It was necessary to separate eddies of restraint from eddies of potential, to obtain the fundamental law. This was a guide through many complex conditions which, till better understood had to be expressed empirically, but in doing so, one was apt to introduce arbitrary co-efficients which destroyed the underlying law. Viscosity had been taken into account, but did not destroy the fundamental equation. That is what lowered the value of the exponent to something below 2, while potential raised it above that value and it was in this way that the fundamental equation served as a guide.

With regard to Lieut.-Col. Battye's remarks, the Author had not seen Parry's monograms and could not say how far the presence of silt modified viscosity. Accepting glass as a smooth boundary, it would be interesting to experiment on viscosity with and without grades of silt.

The Author offered his improved design of groyne as a partial remedy against trouble on hydro-electric installations. By the narrowed section of entry, some head would be lost, but a useful balance might be found at which loss of head was compensated by easier operation.

In reply to Mr. Livingstone-Learmonth, the Author had no accurate information concerning the method of observing discharges through pipes by the period of pulsation, but at Niagara he understood it had been applied quite successfully.

Subsequently Communicated.

MR. W. G. QUINTON wrote that Mr. Claxton's paper was most interesting, in that it approached the subject of silt in an altogether broader way that had hitherto been done by most observers and thinkers. The Chezy hypothesis was the fundamental one up to now for uniform flow, but in that the hypothesis was mathematically rational, the value of the Chezy formula was out of proportion to its true worth, when practical silt problems confronted us, because the whole friction idea seemed unsound, and would never lead anywhere, for the reason that friction postulated heat formation.

As, in practice, this could not be measured, the hypothesis lead nowhere and resulted in the fickle atmosphere of variable constants.

The whole problem of uniform flow might well be considered analogous to that of the fourth dimension. It was impossible to foresee how the elements of velocity, slope, depth and bed width would be affected by a given, or a variable silt intake, at the head of a channel. All that could be said was that a given intake would require a certain Vo and channel dimensions.

If the silt intake was increased or decreased, it was impossible to say that a new velocity would make the channel flow to the same slope and dimensions.

Slope could, however, be fixed fairly well by control points in a channel, *e. g.*, regulators and falls, but even here there was no constancy where "karries" were in continuous use.

Mr. Claxton took f (acceleration) $= \frac{C V^n}{d}$, *i. e.*, $\frac{\text{force}}{\text{mass}} = \frac{C V^n}{d}$

But the acceleration force was simply

$$g. \sin i, \therefore \frac{C V^n}{d} = \frac{g. \sin i}{\text{mass}}.$$

Now, if the slope was fixed, as it could be fixed by control points,

$\frac{C V^n}{d} = K. g.$, where K was a constant or sine of the fixed slope angle,

i. e., $V^n = \frac{K. g. d}{C}$

$V = K_1 d^{\frac{1}{n}}$ which was of the same form as Kennedy's law. Thus Kennedy's law was rational, and K_1 was the balancing factor for rationality, *viz.*—

$$V\left(\frac{\text{feet}}{\text{seconds}}\right) = \text{feet}^{\frac{1}{n}} \times \frac{(\text{feet})^{\frac{n-1}{n}}}{\text{seconds}}$$

This seemed to show that K_1 was a measure of the volume of silt passing per second.

But $Q = Kd^{5/3}$ and $K = Bd + \frac{d^2}{2}$, for trapezoidal channels.

Now volume predicated (feet)^3

$$Bd + \frac{d^2}{2} \text{ was therefore } \propto = \left(\sqrt[3]{\frac{\text{volume of silt}}{\text{per second}}}\right)^{\frac{n-1}{n}}$$

$$Bd + \frac{d^2}{2} = Z\left(\sqrt[3]{S}\right)^{\frac{n-1}{n}},$$

Where S = volume of silt passing per second

S, n, and d could be determined by experiment and thus

$$B = \frac{Z\left(\sqrt[3]{S}\right)^{\frac{n-1}{n}} - \frac{d^2}{2}}{d}, \text{ solved}$$

The above indicated lines for further experiment with silt volume but it was necessary to sound a warning note that a true comparison could only be effected by observing a number of channels of the same slope and discharge, but with a different power of silt intake, by virtue of varying regulator design. It appeared hopeless to experiment in a single channel because the regime changes would be so marked and there would be grave doubts as to any fixity of regime at all, at any period during the experiment.

The factor Z too would probably indicate variations for different qualities of silt, hence the first step seemed to be to obtain data for a number of similar channels offtaking the same quality, but varying quantities of silt.

The ideas expressed above, as thought out from the lead given by Mr. Claxton opened out a very wide field for research, because having obtained data of varying quantities for a given quality of silt, next it was necessary to pass on to varying qualities as well as quantities. Once the quality and quantity factors of silt carried were properly examined, then it would seem that the bedwidth and depth could be co-related finally. The solution of that problem was not in sight yet.

The Author in reply wrote that he agreed that K_1 the balancing factor of Kennedy's law in Mr. Quinton's conception was a measure of the volume of silt passing. In the paper the Author had laid down that the amount of silt in suspension was also a measure of the energy of eddies which was the same thing since his equation was based on the condition that

acceleration $\propto$ resistance of eddies,

i. e., acceleration $\propto$ eddying energy.

Mr. Quinton's proposals to further work out this conception involved a truly formidable task. There was perhaps a nearer solution and that was, to apply Gilbert's curve for varying grades of silt to Kennedy's equation, making the exponent a varying quantity along a similar curve.

This might be done approximately from data ready to hand and afterwards the curve could be perfected by means of other experiments carried out on the lines of Gilbert's experiments suitably modified.