PAPER No. 126.

ENERGY OF FLOWING WATER, CRITICAL FLOW AND STANDING WAVES.

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CONTENTS. PARA.

NOTATION.

$A\ A_1\ A_2$ etc.	.. areas.
$B\ B_1\ B_2$ etc	.. width of crest.
C	.. constants.
$D\ D_1\ D_2$ etc.	.. depths. D_c critical depth. D_n neutral depth.
E	.. elevation of weir crest above a datum.
g	.. gravity constant
H	.. a total head acting.
i	.. Mr. Burkitt's intercept.
K **k** & $\mathbf{k}^1$	.. constants : " $\propto$ " a combination of constants.
m	.. mean hydraulic depth.
L	.. a length of channel.
$P\ P_1\ P_2$ etc.	.. total pressures.
Q	.. total discharge in cusecs.
q	.. discharge per foot run.
$s\ s_1\ s_2$ etc.	.. slopes.
T	.. Top or surface width of any channel.
$V\ V_1\ V_2$ etc.	.. velocities. V c critical velocity.
W	.. weight of 1 cubic foot of water.
Z	.. The ratio of actual depth to depth of normal flow.

AN EXAMINATION OF THE "STANDING WAVE."

PART I.

EXPERIMENTS TO DETERMINE THE SHAPE AND POSITION OF A STANDING WAVE.

Preliminary.

1. Every one engaged in hydraulic work is familiar with the phenomenon known in the Punjab Irrigation Branch as the "Standing Wave." The theory is not so familiar, and the attempts made to explain it only emphasized the complexity of the problem.

2. This was brought home to the Author in 1921 by the failure of a semi-modular bifurcation to function as designed. Again, in 1923, out of three syphons designed by the late Mr. Elsden, for passing a Distributary below a drainage in Nokhar Sub-Division of the Upper Chenab Canal, two functioned smoothly with but little loss of head, while the third created much disturbance both at the intake and outfall, resulting in heavy maintenance charges, and considerable loss of head.

3. The same problem was constantly cropping up in the design of Weirs, etc., but the determination to make experimental observations was crystallised by the Author's experience at Suleimanki Head Works in 1926-27.

4. During 1926 the Author received instructions to collect and collate the calculations involved in the design of Suleimanki Weir. Two considerations served to direct attention particularly to the position of the Standing Wave below the Sluices.

i. Superintending Engineer, II British Circle, impressed upon the Executive Engineer, Head Works, the necessity of so regulating as to retain the Standing Wave upon the *pacca* floor.

ii. Examination of the Weir, downstream of the glacis in the cold weather of 1926-27 shewed that considerable action had taken place. Much of the loose stone protection had been washed away and in two places the *pacca* toe wall had been broken up and a number of concrete blocks had vanished. This had occurred in spite of the fact that during the time the Author was in charge of the Division, at no time was the visible Standing Wave permitted to leave the *pacca* floor.

With a higher normal river in 1927 the damage was even more extensive.

5. An examination of the original calculations to determine the position of the Standing Wave was not very helpful. They were in a orm difficult to follow and, unfortunately, an important part of the preminary considerations could not be found.

The Author then turned to the only work connected with the subject ith which he was acquainted, namely, Mr. Burkitt's valuable paper on Broad-Crested Weirs" read before the Punjab Engineering Congress 1919.

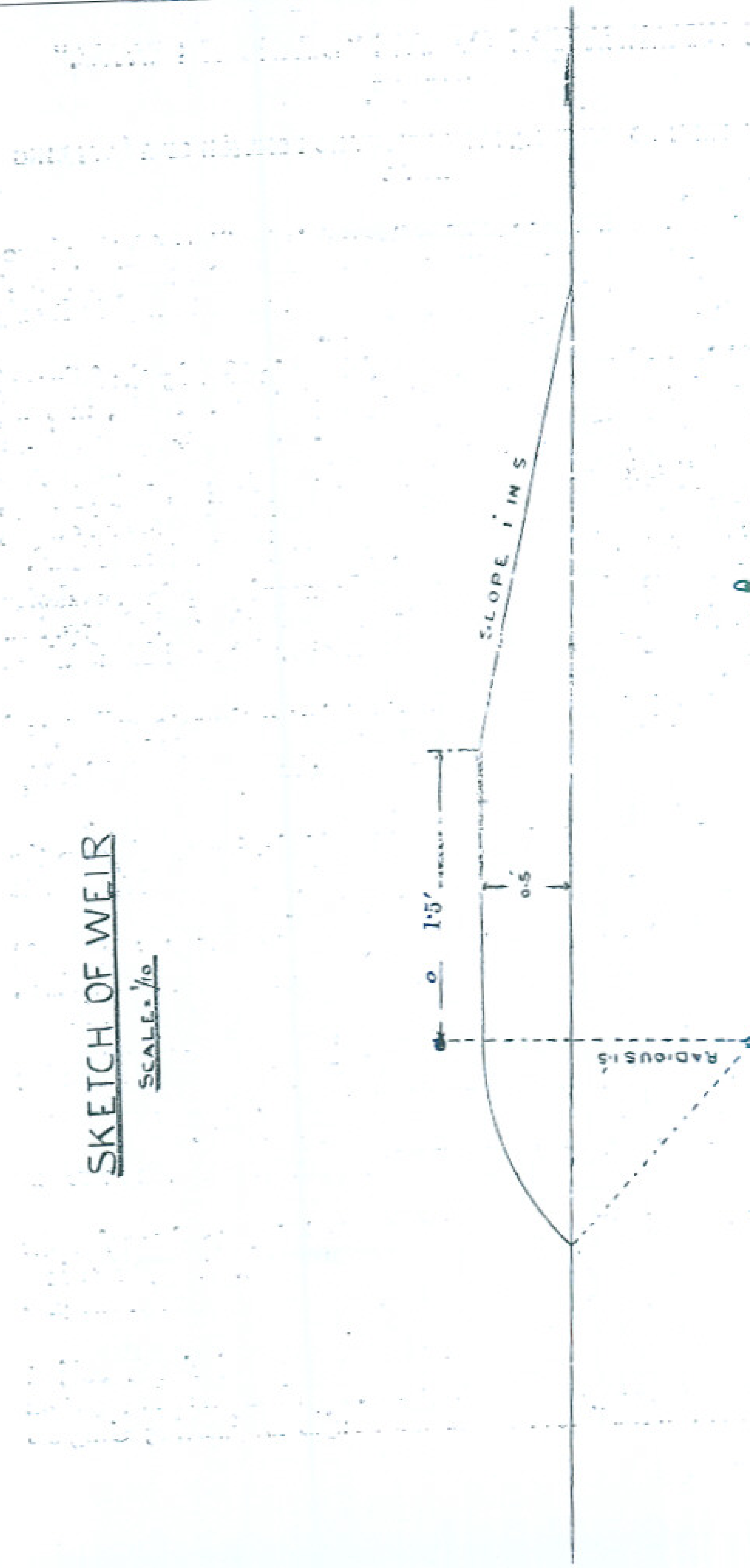


fig 1

In these calculations and formula, Mr. Burkitt made certain assumptions whose justification the present Author failed to appreciate. In one case at least—the assumption that the critical depth occurred on the downstream edge of the Weir—the assumption appeared to be definitely incorrect.

6. It became necessary to find out to what extent Mr. Burkitt's formula were applicable and to see what other work had been done on these lines. The concurrence of the Local Government was obtained to the Author carrying out certain experiments at the City and Guild's College (London) while on leave.

Arrangements were made in May and the work was carried out in June and July of 1927.

7. Most fortunately for the Author, Mr. Robertson, Superintending Engineer (on leave) was actually engaged on work of his own at the College and readily consented to associate himself with the enquiry.

Description of the Apparatus.

8. The experiments were carried out in the new hydraulic laboratory in the Goldsmith's Extension.

The steel trough in which the experiments were made, was 80 feet long, 2 feet wide and 1·5 feet deep, and had been erected accurately horizontal. The model Weir fitted into this trough, was built up of wooden slats screwed on to wooden profiles and was bolted to the bottom of the trough.

The whole was carefully smoothed and the points of contract between the wooden Weir and the steel trough were carefully puttied up to ensure further smoothness (fig. 1.)

9. Water was supplied by an electrically driven centrifugal pump of 9″ delivery and a capacity of 3,500 gallons per minute at 1,500 revolutions. The water was lifted into a reinforced concrete tank 12′-6″×6′-0″×6′-0″.

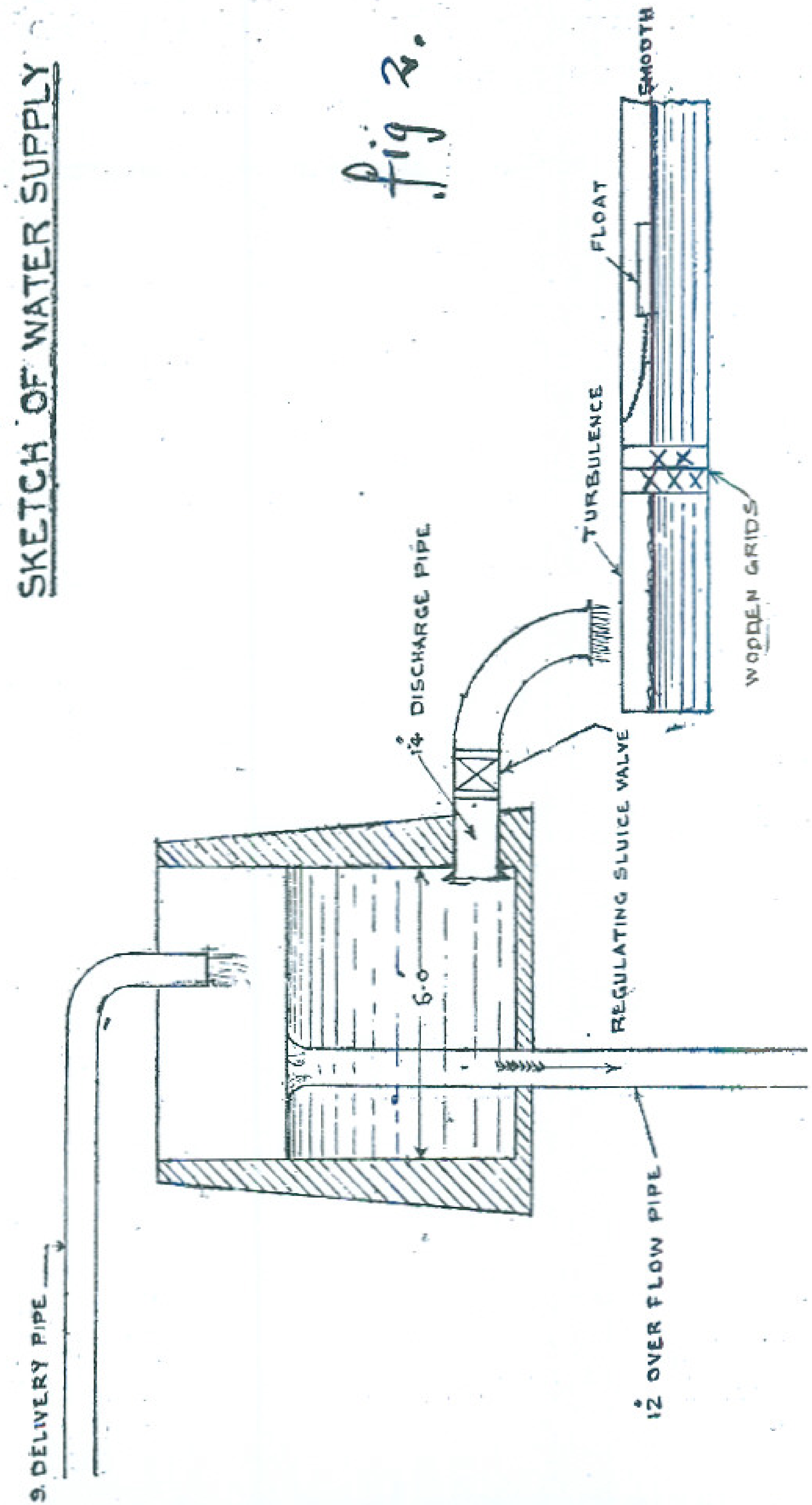
Projecting through the bottom of the tank and 4′-6″ high was a 12″ diam. pipe which returned the surplus water to the Pump Sump (fig. 2).

At the bottom of the tank was a 14″ discharge pipe, leading water into the trough. The discharge was controlled by a large Sluice Valve, discharging vertically into the trough.

The consequent turbulence was checked by a series of wooden grids nd a float anchored to the sides of the trough by a light chain (fig 2.)

The flow in the trough up to the weir was extraordinarily smooth.

It was also constant during any one series of experiments to within 2%.


SKETCH OF WATER SUPPLY
Fig 2.
SMOOTH
FLOAT
TURBULENCE
WOODEN GRIDS
14" DISCHARGE PIPE
REGULATING SLUICE VALVE
6·0
12" OVER FLOW PIPE
9" DELIVERY PIPE

10. After passing over the model weir, the water flowed down the trough for 20 feet where it met the "Karri" regulator which permitted regulation of the depth downstream.

Small wooden Karris 2′ long $\frac{3}{4}''$ wide and $\frac{1}{2}''$ thick fitted with small distance pieces on each end were placed in suitable grooves in the trough. In addition to this vertical Karris could be fitted also to permit of very fine regulation.

11. After passing the regulator, the water flowed down a pipe to a "two way" sluice valve, from which it could be directed into either of two large steel tanks. This discharge over a given period (measured with a stop watch) was passed into one tank. The contents were measured before and after the discharge on a gauge glass fitted with a scale.

The maximum variation of discharge of all the numerous observations was only 0·2%. The tanks were also fitted with discharge valves back to the pump sump.

Diagram I shews the lay-out of the apparatus described above and is not to scale.

12. On each side of the steel trough was a brass rail. These two rails were accurately parallel and level and extended some five feet above the weir and about twenty feet below it.

These two rails carried a girder carriage. Sliding transversely on the girder carriage was the actual instrument for tracing the water profile. This consisted of a cylindrical brass block drilled through the centre to receive a screwed needle. The screw thread engaged with a brass nut of pitch 1/100 foot per revolution (fig. 3.)

The nut was marked off in 1/10 and 1/100 revolution, thus permitting a reading to be carried to the fourth place of decimals. In practice, owing to a slight periodicity of the water surface, it was not found possible to read accurately to nearer than three places of decimals of a foot.

The body of the instrument also carried a vertical scale marked off in feet, 1/10 feet and hundredths.

13. An ordinary piezometer gauge, connected by brass piping to a series of holes drilled *across* the trough, was used for measuring the depth downstream.

The reading so obtained was accepted so long as the Standing Wave was well away from the holes. Where the Standing Wave was near or downstream of the holes, the depth of water below the Standing Wave was measured by two observers independently, as well as with the profile ıeedle.

It is freely admitted that this part of the apparatus shews a weakness. t would have been an improvement for the purpose of these experiıents had there been several such pressure indicators at intervals down ıe trough. This point will be referred to later on.

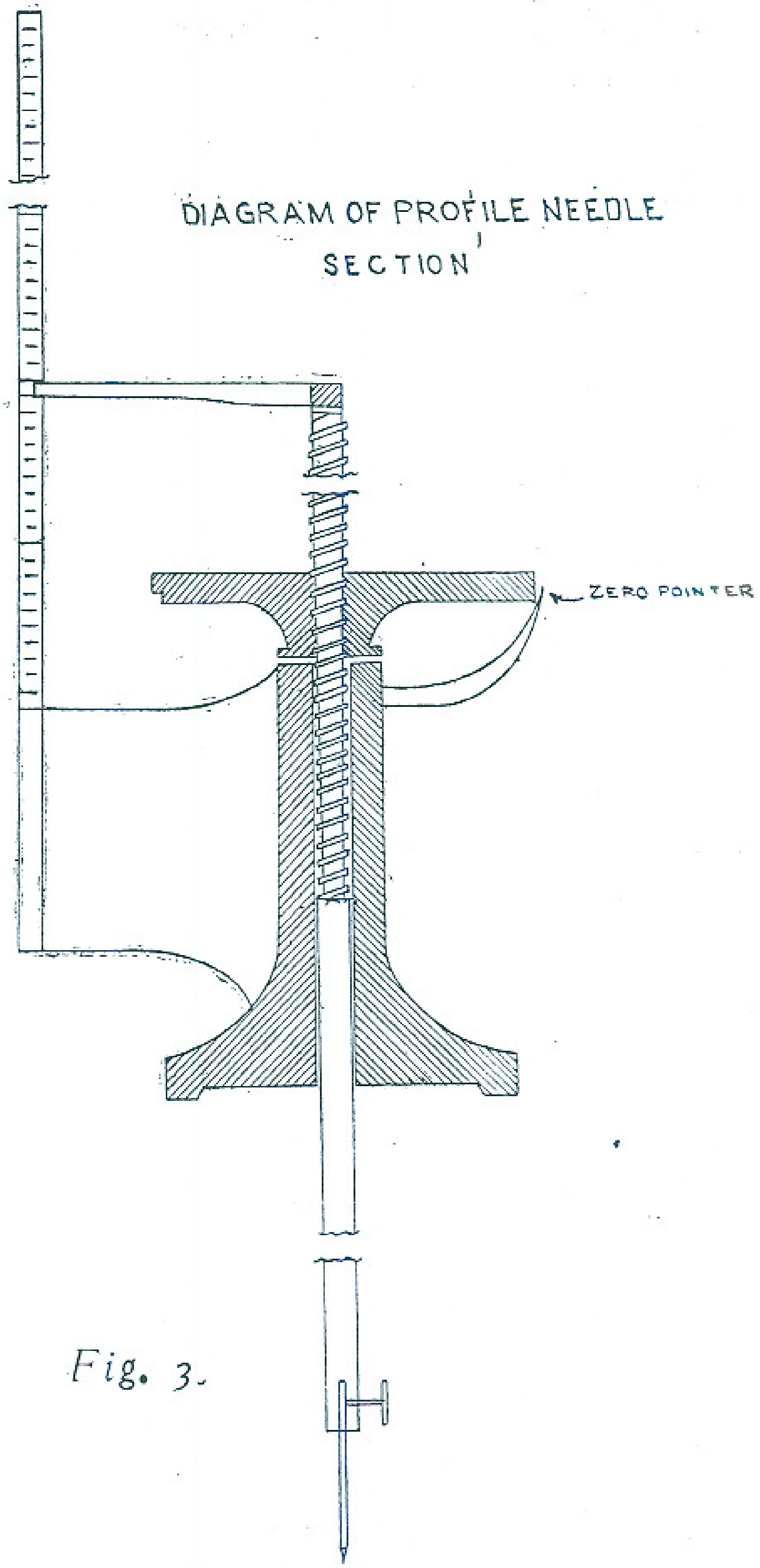


Fig. 3.

14. A scale clamped to the side of the trough determined the posi- 1 of the needle with reference to an arbitrary zero.

ıe Experiments.

15. The object of the observations was to obtain as accurately as possible, the actual profile taken up by water flowing over the weir.

To this end the pump was started up, the valve adjusted and matters allowed to attain a steady condition.

After calibrating the needle, the mean reading for the trough floor was recorded. The extreme variation was ·003 feet. The extreme variation in the surface of the weir was between ·001 and ·002.

16. Observations of the position of the needle and the height of the water were then recorded.

Owing to the slight pulsations in the surface upstream, already mentioned, extreme care had to be taken to get a mean reading. The pulsation never exceeded ·001 foot each side of the observation recorded and more generally the total variation was less than ·001 foot.

Observations were carried on until the standing wave was approached. Thereafter the froth from the wave made observation difficult. Extreme care was taken to divert the froth to get further readings. As soon as the pulsation was marked, or when the froth definitely prevented it, the observations noted above had to be discontinued.

17. At this point an attempt was made to record a physical condition.

If the tips of the fingers were lightly drawn along the jet, the surface could be felt to extend a definite distance into the wave. At one point, the jet was felt to have taken a distinct upward turn : the water no longer brushed the tips of the fingers but impinged thereon.

The point was recorded to $\frac{1}{100}$ foot only in height but it is freely admitted that the observation is not an accurate one. It was taken to be the beginning of the standing wave proper.

18. This test was carried further. There was found to be another point, less clearly defined than the former, but still definite where the water impinged on the finger tips giving a sensation of rising oily bubbles.

The point was similarly recorded and from its general position and appearance taken to be the position of maximum wave effect.

19. One last point was also recorded both in regard to position and height, *viz.*, the point at which water ceased to boil and flowed more or less smoothly away.

Lastly, the depth d/s was read off on the gauge, and recorded.

20. The above observations were recorded for each experiment in five distinct series.

A series consists of a number of experiments for the same disch carried out at varying depths d/s. For ease of reference, Table I gi the series No., Experiment No., discharge and depth downstream, actually observed.

Several discharges were observed in the course of each series.

21. At the conclusion of the observations, these were plotted to the following scale :—

Water surface vertically $12\frac{1}{2}''$ to 1 foot.

Distance horizontally $2\frac{1}{2}''$ to 1 foot.

These curves are shewn as diagrams Nos. II to VI.

It is necessary at this point to refer to Table I. Please note that the discharges for series III and V respectively are 2·32 and 2·34 cusecs. Although plotted separately, these two series are taken as one for purposes of calculation, and the mean discharge taken, *viz.* 2·33 cusecs. The error is only 0·85%.

Results of the Experiments.

22. A casual examination of diagrams Nos. II to VI shews that they are too definitely characteristic below the jump to admit of the physical points recorded as per paras. 17, 18, 19 to be quite meaningless.

Attention is invited to the following general points :—

(*a*) The general adherence to the original water surface, as the S/W is forced up the glacis is remarkably close.

This was to be expected as a result of previous investigations.

(*b*) Immediately prior to the jump there is a definite departure from the original water surface. This was unexpected as it does not appear to have been recorded in any of the investigations which the present Author has examined. As a consequence the precise position at which the section should be taken upstream of the jump for purposes of calculation, appears very doubtful.

(*c*) When the S/W is off the glacis and far down the channel, the trough of the wave is large and round. Also the wave itself has a general slope and does not approach an erect position.

Whereas, as the wave climbs up the glacis the trough becomes narrow and the surface of the wave more nearly approaches the vertical.

(*d*) Conversely, the *length* of the wave in proportion to its height appears to increase as the wave climbs up the glacis. This might appear opposed to the observations of the Engineers of the Miami Project but will be referred to again later.

(*e*) The only cases obtained in these experiments where the wave rises higher than the actual d/s water surface, have been when the wave has been well down the channel. This again, fails to conform to the published views of the Miami Project Engineers.

(*f*) In no case does the critical depth exist at the downstream edge of the crest of the weir.

This took the Author by surprise. Along with others who have gone more deeply into the subject than had the Author, he assumed that the water surface would obey the well established theoretical law that the critical depth can only occur on the downstream edge of the crest.

This fails to support Mr. Burkitt's assumption on this head (*vide* his 1919 Congress paper) and is more in conformity with Mr. Crump's implied views in his Technical Paper No. 26. *viz.*, that " the throat must be long enough to *include* the critical point."

(*g*) The " Position " of the standing wave is clearly a matter of grave doubt. In every case there is a considerable space in which the standing wave may be said to be beginning, taking place or finishing.

The matter is even more doubtful than the diagrams shew, because the profiles beyond the standing wave although consistent among themselves could not be traced so accurately (*vide* para 17 *et seq.*).

In Part III of this paper the Author presents his method, believed to be original, of tracing the water profile over masonry works, in a channel. This process is an essential step towards the solution of the standing wave problem.

Prior to presenting this, the fundamental principles are discussed in Part II and no claim to originality can lie. Part II may be omitted by any one familiar with the subject.

PART II.

THEORY OF THE STANDING WAVE AND ALLIED PHENOMINA.

1. The Author has found it impossible to separate the theory of the standing wave from other considerations of the flow of water. With a view to presenting a logical whole, a beginning is made with a brief exposition of the theory of critical flow.

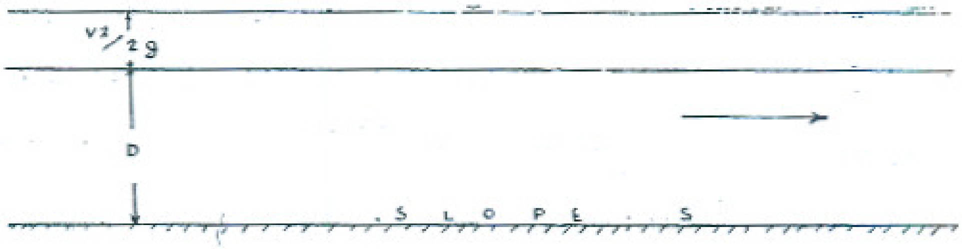


fig 4

Let Q be a fixed discharge flowing in a *Uniform* channel of any shape, of cross sectional area A and at a mean velocity V.

Then Q=A V.

Also the mean depth D will be constant, the surface slope will be constant and will be that required to overcome friction, and will be parallel to the bed.

The flow will be " Stable." That is to say, the velocity and depth will remain unchanged until either the bed slope or the shape of the channel alters.

3. The total energy of unit mass of the water at any point consists of :—

i. Potential energy, i.e., height above a fixed datum.

ii. Depth (or pressure) energy, i.e., the mean depth D over the bed.

iii. Kinetic (or velocity) energy $V^2/2g$.

The " total energy " line may be plotted by setting off the velocity energy above the surface line. The slope of the " total energy " line is equal to the friction and for stable flow is parallel to the bed and the surface.

Definition.

4. The " energy of flow " is the sum of the depth and velocity energies.

5. If the slope of the bed or the shape of the channel alters, there will be a corresponding change in the mean velocity and in the water surface.

Provided the friction remains unaltered the "total energy" line will remain unchanged.

For example, let a small smooth obstruction be placed in the above channel. (In the diagram the obstruction is shewn in the bed. It may be a constriction of the sides or a combination without altering the truth of the following proposition).

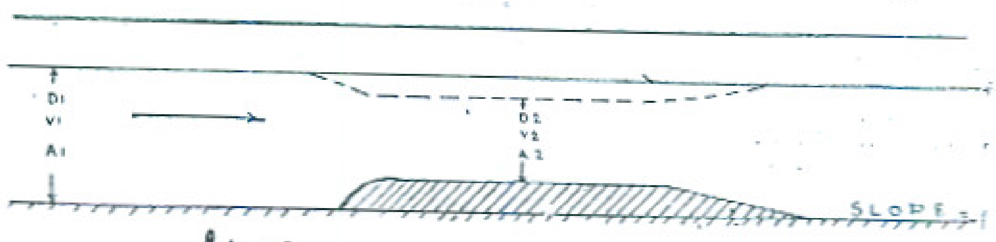


fig 5

Q the discharge is constant.

$Q = A\,V$

$D = f\,(A)$

Assume various mean depths of water in this channel. Deduce A: Calculate V

D is the depth energy

$\frac{V^2}{2g}$ is the velocity energy.

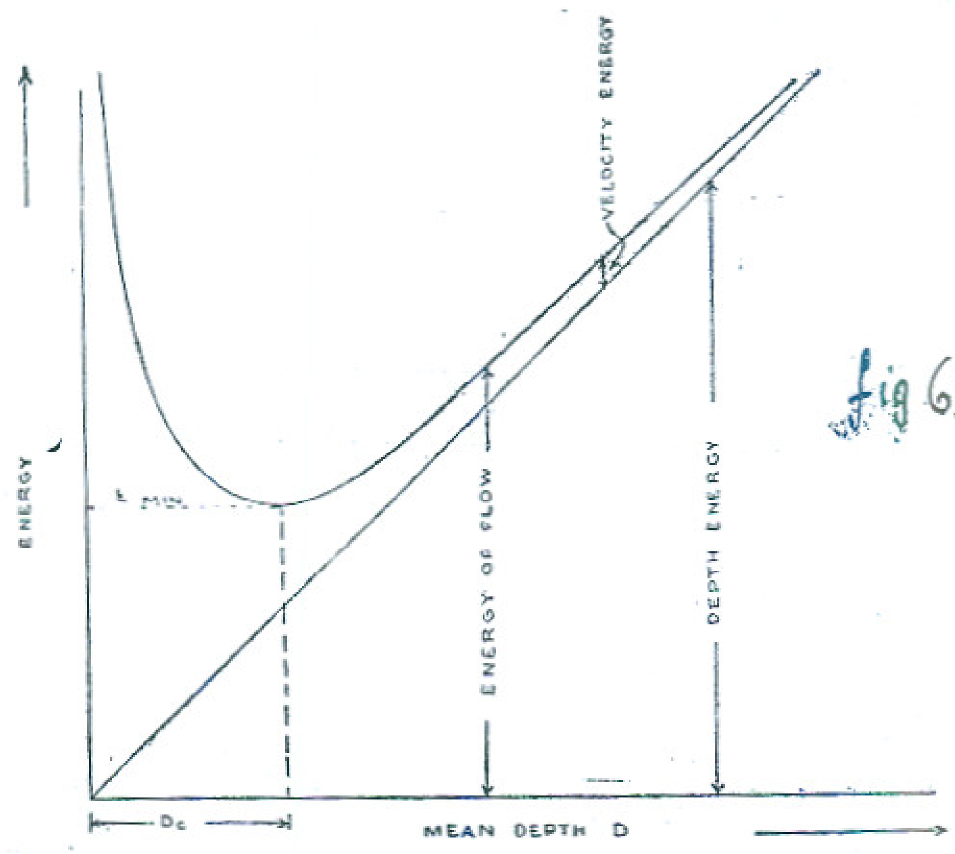


fig 6.

Sum of these two is energy of flow. Plot, on axes, D horizontal and energy vertical. The result will be two curves as shewn.

Clearly, plotting depth energy against mean depth gives a straight line passing through the origin. Also plotting the velocity energy on top of (or in addition to) the depth energy must give a distorted parabola :—

i. Asymptotic to the energy line approaching infinity as the depth approaches zero.

ii. Asymptotic to the depth energy line as the depth approaches infinity when the velocity would be zero.

7. This curve gives the "energy of flow" for any depth.

It has a minimum point. This minimum point is the "critical point." It gives the critical depth D_c, the critical velocity V_c, and the minimum energy of flow.

It must be noted that the critical point depends only on the shape of the channel and the discharge passing, and that for every value of the energy of flow, above the minimum, there are two values of D and corresponding values of V.

8. With the assistance of this diagram it is possible to plot an "ideal" surface profile for any known change in mean depth or elevation of bed. The actual will differ from the ideal somewhat, owing to the following considerations :—

i. Changes in stream line cannot be abrupt on approaching a change of section.

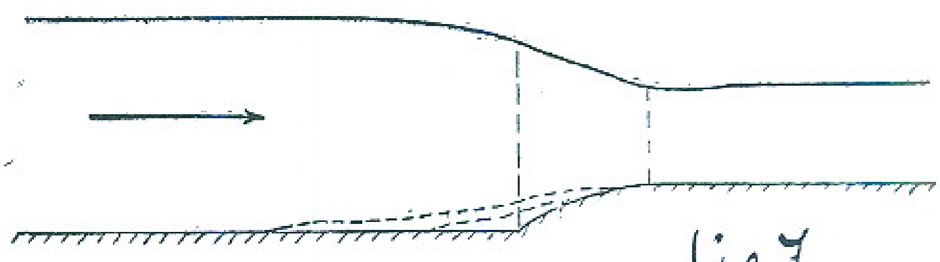


fig 7.

Hence changes of surface slope will not be abrupt as would be indicated by the surface profile plotted from the diagram.

ii. A change of velocity and/or a change of depth postulates a change in frictional losses. The change in frictiona losses has been neglected in the figure 5 for para. 6.

iii. Momentum—to be considered later.

9. Further consideration of the curves in fig. 6, para. 6, brings t light another important limit. The obstruction in any channel (or chang of slope) may be increased until critical depth and velocity are reache

The water is now passing with minimum energy of flow. This energy of flow is measured by $D+\frac{V^2}{2g}$ Any further constriction requires the same amount of $D+\frac{V^2}{2g}$ to pass the same discharge and will result in " heading up ".

10. What happens to the water after passing the critical section is also of importance. There are three possibilities.

Consider first the case in which the water after passing the critical section remains at critical depth and velocity. (This state of flow is unstable and results in pulsation).

The velocity has materially increased hence friction increases therefore the slope of the bed, the surface and the total energy line all increase, as indicated in fig. 8. (The obstruction is shewn in the bed for sake of simplicity only).

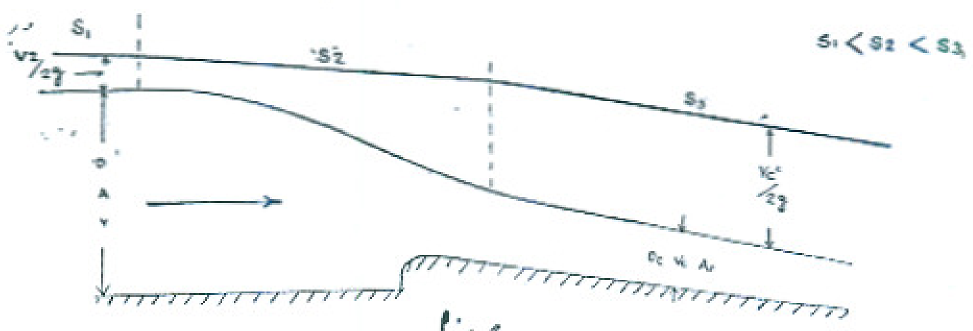


fig 8.

11. The second case is equally summarily dismissed. The water after passing the critical section resumes normal flow. If such occur, the limit of obstruction was reached (*vide* para. 9). Provided the removal of the obstruction is sufficiently gradual to avoid undue frictional losses, this recovery postulates parallelism of flow of the stream lines and this is the only *case* in which such parallelism occurs within the obstruction or " Work " (fig. 9).

Such parallelism is violently unstable and causes great fluctuations which may be calculated with difficulty as can be shewn.

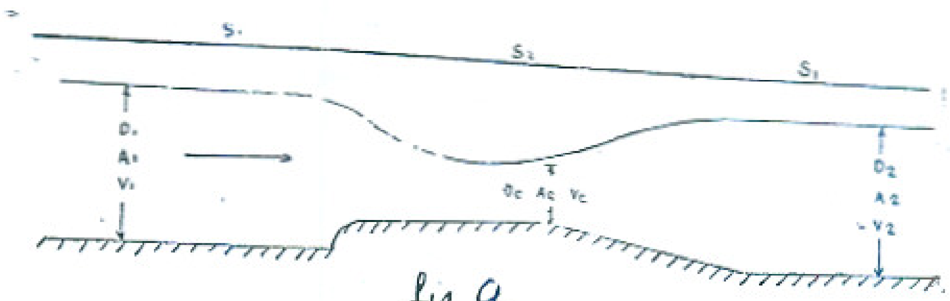


fig 9.

12. The third case is clearly that in which the water after passing the obstruction increases in velocity beyond the critical velocity.

Reference to the curves in fig. 6. indicates that any further increase in velocity postulates a decrease in depth and an increase in energy of flow. Put another way, since $\frac{V^2}{2g}$ increases the surface must fall. But $D+\frac{V^2}{2g}$ also increases so there must be a fall in the bed also, but not so much as in the surface, because D decreases.

Owing to the increase in velocity, friction also increases therefore the slope of the total energy line also increases.

So that for stable flow at a depth below the critical (and corresponding velocity above) the state of affairs is indicated by fig. 10.

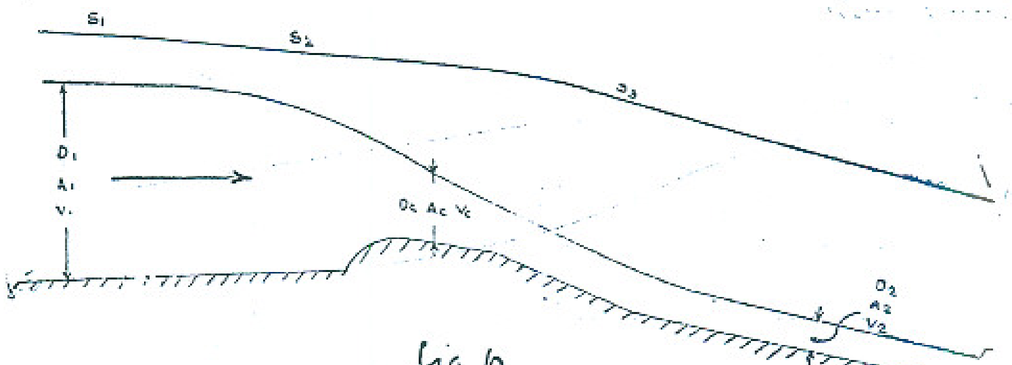


fig 10

13. So far only the transition from velocity below critical to velocity above critical has been discussed. It is necessary now to look at the converse before dealing with the Standing Wave itself.

Consider a discharge passing at a velocity above critical, at one point in a channel and the same discharge passing another point in the channel at a velocity less than critical. What happens in the interim? Can a smooth change be effected?

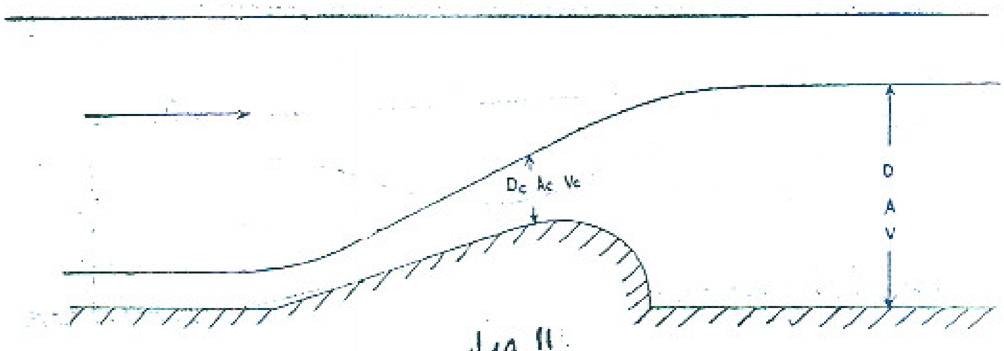


fig 11.

14. Clearly the converse of the conditions in para. 12 holds. In order to effect a smooth transition, there must be a smooth increase in depth and decrease in velocity, *i.e.*, a smooth change in the energy of flow.

This entails no disturbance of the total energy line, whose slope is fixed by considerations of friction in the channel alone. (As usual the change in friction over the obstruction is neglected).

The profile of the bed of the channel may be determined therefore by setting off $D + \frac{V^2}{2g}$ from the total energy line until the critical depth and velocity is reached. Thereafter this becomes a special case of the general one treated in para. 11.

15. If the rise in the bed profile required by the arguments advanced in para 14 does not exist, then the transition is effected naturally in the phenomenon known as the standing wave.

Before going on to treat of this in detail it is necessary to consider a factor not so far mentioned, save in para. 8 iii.

16. All moving bodies have momentum, the product of mass and velocity.

The rate of change of momentum is proportional to the force acting.

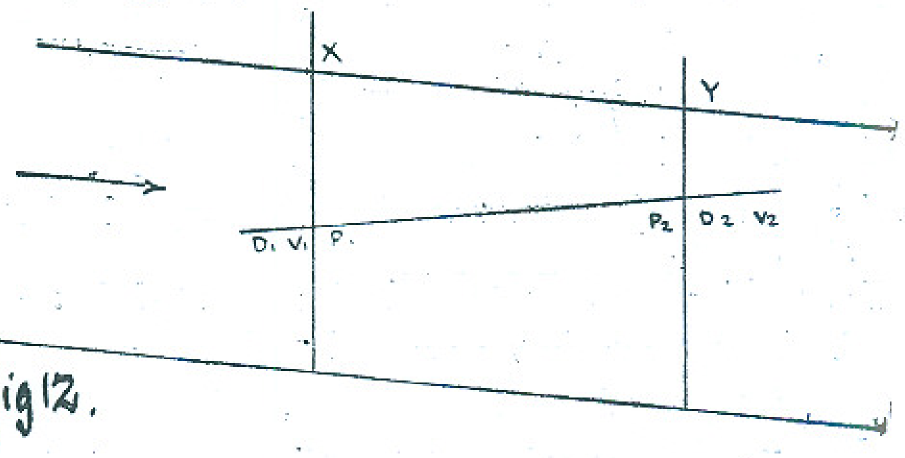


fig 12.

Consider a uniform channel of any shape. Water is flowing therein upstream of the section X at a velocity higher than the slope justifies. It tends to assume stable flow downstream of the section Y. Let D_1, V_1, P_1 be the mean depth, mean velocity and total pressure over the section at X & D_2 V_2 P_2 be the same at section Y.

The slope of the total energy line exactly overcomes friction (as usual the *change* of friction is neglected). Therefore the change of momentum per second equals the force producing it.

$$\frac{W}{g}Q\,V_1-\frac{W}{g}Q\,V_2=P_2-P_1$$

Transposing :—

$$P_1+\frac{W}{g}Q\,V_1=P_2+\frac{W}{g}Q\,V_2$$

That is to say, provided no other force is acting, the sum of the total pressure over any section plus the momentum at that section remains constant.

17. But examination of the form of this expression shews that it is a function of D the depth. If D varies, V changes, also A. And if A varies P the total pressure must change.

The following conclusion may safely be drawn and will be readily understood. Any change of depth (and all that such change connotes) is caused by pressure on the boundary of the channel, and the resultant pressure is the sum of all small items of pressure parallel to the stream line.

18. It was shewn in para. 16 that the quantity given by the expression $P+\frac{W}{g}Q\,V$ remains constant provided no outside force comes into play.

It was shewn in para. 17 that the value of this expression alters with a change of depth and that this change of depth connotes the play of some external force. If the new value of $P+\frac{W}{g}QV$ be worked out for the new depth, the difference in values of the expression is the exact measure of the force brought into play.

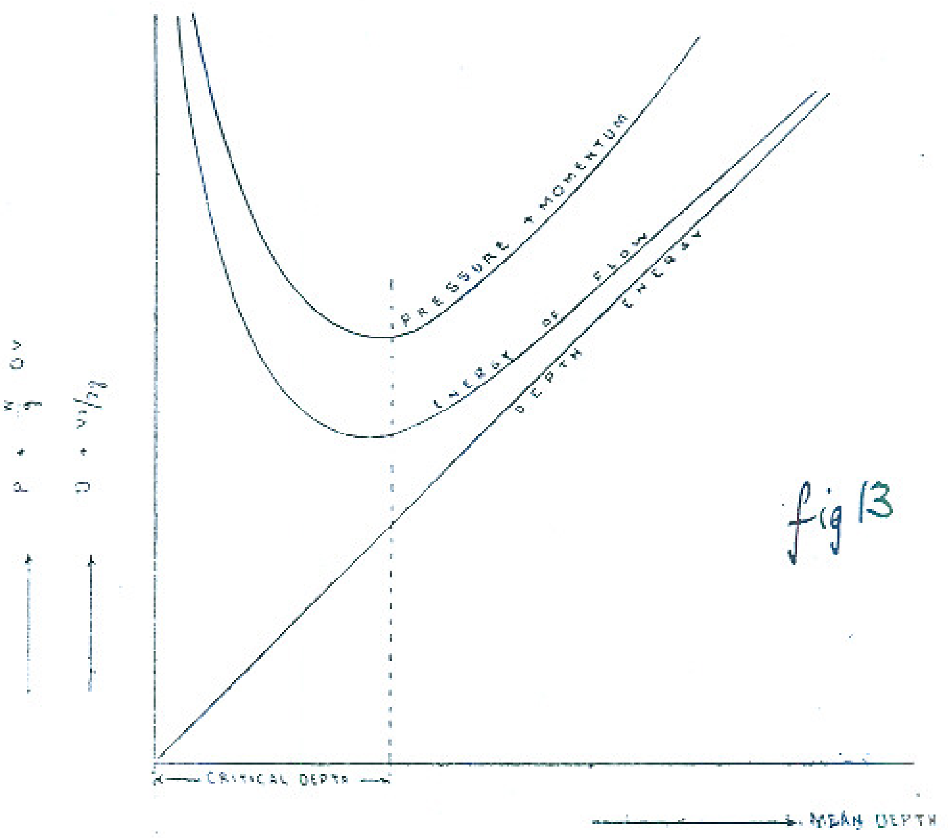
19. Assume various values for D and calculate the value of the expression $P+\frac{W}{g}Q\,V$. Plot on axes, D horizontal and "pressure plus momentum" vertical as was done for the expression for "energy of flow" in para. 6.

The resulting curve has the following characteristics :—

i. As D gets smaller, (and therefore A) P becomes negligible but $\frac{W}{g}Q\,V$ becomes very large, and the curve is asymptotic to the vertical axis. Since D and V are in inverse relation the "Pressure and momentum" curve approaches the energy of flow curve—and may even cross it.

ii. As D becomes large, V becomes small, but P increases rapidly since P is a function of D^2. This leg of the curve must diverge from the "Depth Energy" line. But the corresponding leg of "the velocity energy" curve is asymptotic to the Depth Energy" line, so it must follow that as the Depth increases, the two legs of the "Pressure plus momentum" and "Energy of flow" curves diverge.

iii. The "pressure plus momentum" curve has a minimum point which coincides with the critical depth, and (like the "energy of flow" curve) there are two depths corresponding to each value of the expression. A reference to fig. 13 will make the above points clear. The three curves have been combined into one diagram for ease of reference and comparison, suitable scales having been chosen.



20. Before discussing the application of this "pressure plus momentum" curve to the Standing Wave, certain illustrations of "Smooth" change will make the phenomenon easier to understand.

Consider water flowing in a uniform horizontal channel of any shape at a velocity higher than $V_{c.}$ The velocity tends to decrease. The force acting is friction, *i.e.*, slope of the total energy line.

At any two sections, the depth has changed. The difference in value of the expression $P + \frac{W}{g} Q V$ read off the curve in fig. 13 is a measure of the force acting (*i.e.*, the friction.)

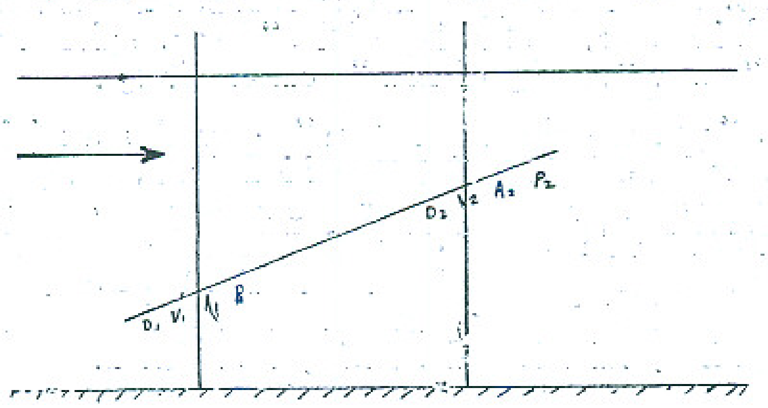


fig 14

21. Again : consider the case of an obstruction in a uniform channel of any shape in which the flow is stable.

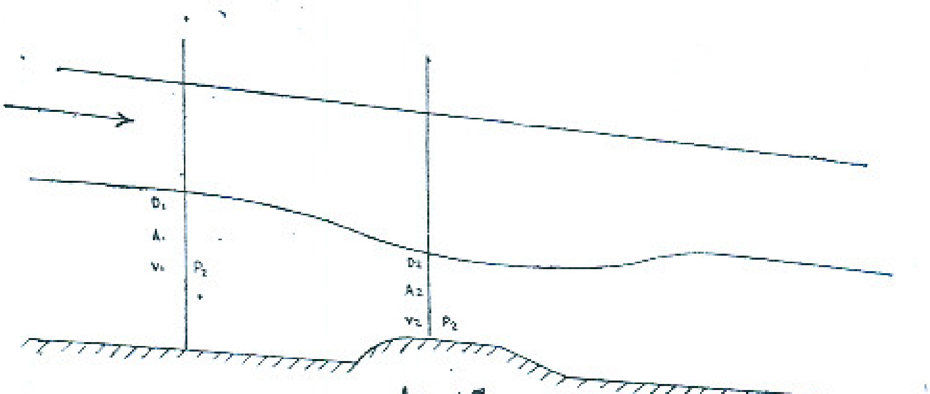


fig 15

Here, the change in value of " pressure plus momentum " is a measure of the pressure exerted by the obstruction in a direction parallel to the stream line.

22. In parenthesis the above considerations make clear at last why divergence on the downstream side of a work must always be more gentle than on the upstream side.

i. Friction can only act one way *i.e.*, retarding the flow.

ii. The pressure of a masonry work in an upstream direction is practically unlimited. Its useful effect is only limited by the extent to which turbulance must be avoided.

iii. The pressure of a masonry work in a downstream direction is limited to the extent to which the friction can compel adherence of the stream lines.

It is not proposed to pursue this further : it is outside the scope of this paper.

23. Returning now to para 15 the case to be considered may be restated thus.

A discharge Q is flowing down a uniform channel of any shape whose slope is such that the stable velocity is below the critical.

Suppose that, for some reason, at one point the velocity is above the critical. The case briefly touched upon in para. 20 is presented.

In order to secure the smooth transition from the high velocity to the low, a rise in the bed level is essential *vide* para. 14. But it is postulated in para. 15 that this modification of channel section does not exist.

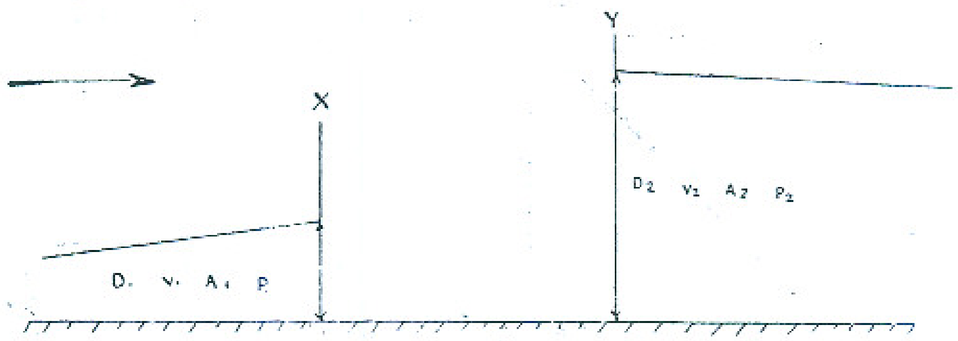


fig 16.

If no obstruction exists to supply the necessary pressure to ensure a change of value of the expression $P + \frac{W}{g} Q V$, it follows that the value of this expression *must* be the same at section Y as it was at section X (save for the very trifling effect of friction over the distance between X & Y.)

In other words, the change of conditions involved in passing from velocity above critical at X to velocity below critical at Y must be sudden. And a standing wave is formed.

24. Since the value of $P + \frac{W}{g} Q V$ is the same in both cases, a reference to fig. 17 shews that the conditions precedent to any standing wave are obtained by drawing a horizontal line to cut the two legs of the pressure plus momentum curve. If any one piece of information is known (*i.e.*, a velocity or a depth—or if one of these can be calculated) the precise relations between conditions above and below the standing wave, are known with certainty.

25. Nor is this all. Let ZZ′ be a horizontal line drawn as directed in para. 24 (fig. 17).

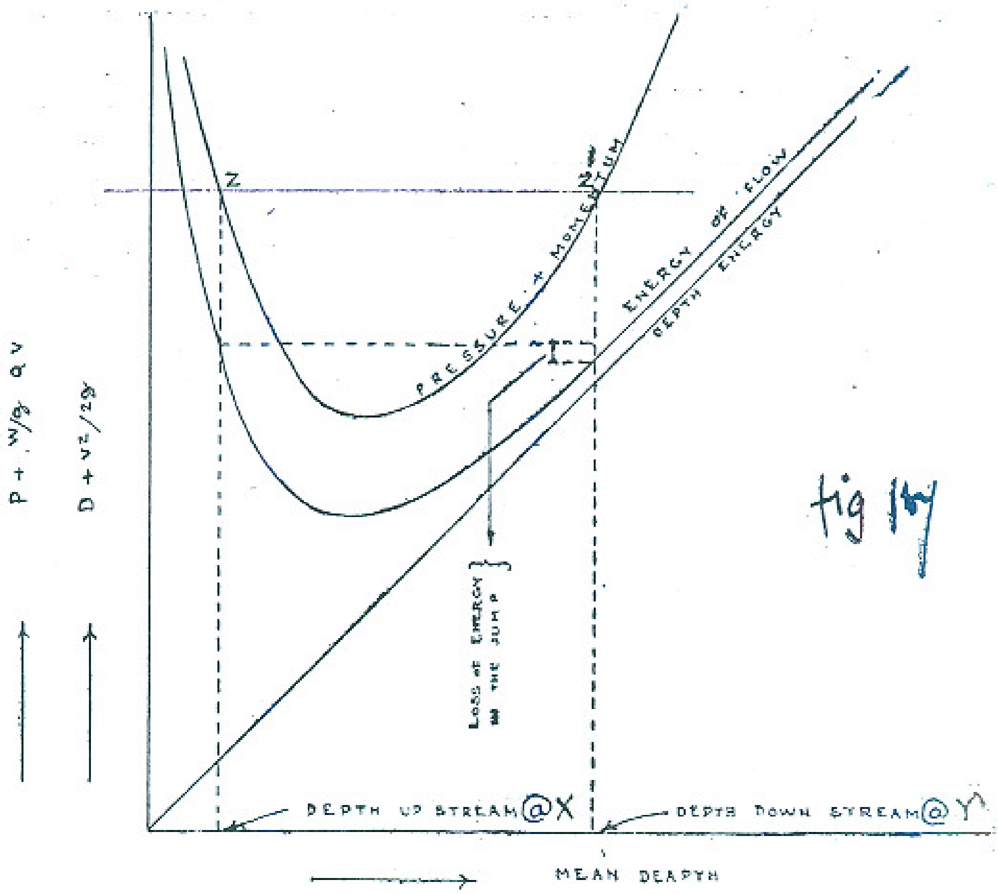
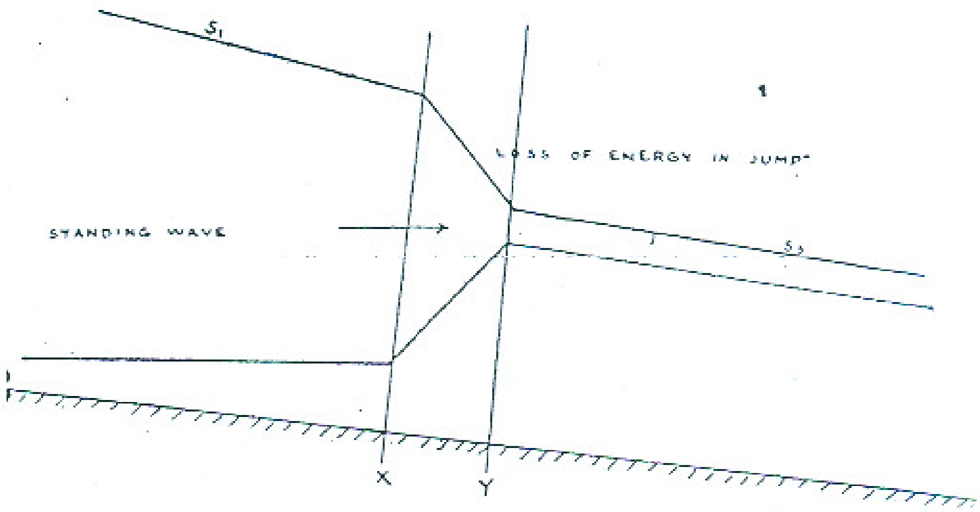


fig 14

$S_3 < S_1$



If the final depth downstream be known then point Z′ is known and Z is found by drawing the horizontal line Z′ Z. Drop verticals from Z and Z′. These verticals cut the "energy of flow" curve. The values of the energy of flow are therefore determined for section at X and Y respectively. The difference between these two values is the loss of energy in the Standing Wave.

26. In all that has been said above the sole assumption has been that the channel was *uniform* in section, but of any shape. The "obstruction" remained undefined also. As the experiments to be examined have a special characteristic, it becomes necessary to describe briefly the special effect of this characteristic on the general theories sketched above.

Therefore in what follows, the conditions are :—

i. The discharge remains constant as before

ii. The depth downstream can be varied at will either by varying the slope or "heading up"

iii. The channel remains *Uniform* in section but of any shape.

iv. The "obstruction" is obtained by varying the bed line, **i.e.,** although the section remains constant in shape it changes in height.

v. This change in height is uniform in separate stages of the obstruction *vide* fig. 18.

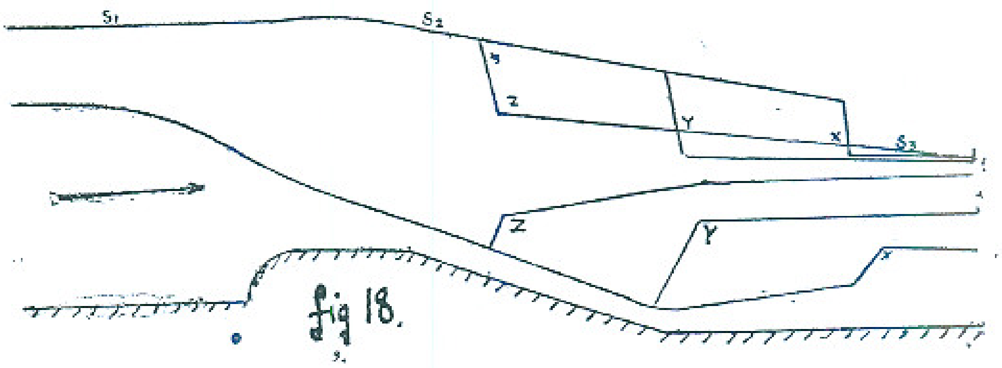


fig 18.

27. The long section sketched above indicates a regular Weir, but the actual shape of the cross section does not matter for the moment.

Above the Weir, the depth is great, the velocity small. While passing over the Weir the critical section is passed and the velocity increases. The velocity is a maximum at the toe of the weir. Thereafter the slope of the channel is such that the stable velocity is slightly below the critical velocity.

The film will decrease in velocity (the necessary pressure being supplied by the friction) until the velocity is slightly above the critical, at the point on the pressure plus momentum curve, horizontally opposite the point (on the other leg) defined by the conditions downstream. The jump thereupon takes place. (Case X of Fig. 18).

28. Consider now the effect of decreasing the slope or of heading up, downstream. The depth downstream of the jump increases. The point on the leg of the pressure plus momentum curve moves up towards the right. The corresponding point horizontally opposite must move up towards the left, requiring a smaller depth and a greater velocity in the stream upstream of the jump.

This is satisfied by the Standing Wave moving slowly forward as the depth downstream increases. This also postulates a corresponding increase in the destruction of energy, in the standing wave.

Finally a point is reached where the standing wave takes place directly over the toe of the glacis (Case Y of fig. 18).

Definition.

For future use, this depth is now defined as the "neutral depth."

29. The standing wave has been getting bigger because the stream above the standing wave has been decreasing in depth and increasing in velocity. From now on, as the depth downstream is continually increased the standing wave climbs up the glacis. In so doing the stream above the standing wave is successively deeper and slower. The standing wave is therefore *less* in height and the destruction of energy in the Standing wave is less. (Case Z fig. 18).

At first sight this appears paradoxical but is simply explained by applying the "ideal" profile derived from the total energy line.

30. Consider the case of a discharge passing down a channel at velocity less than critical, and work backwards. (fig. 19).

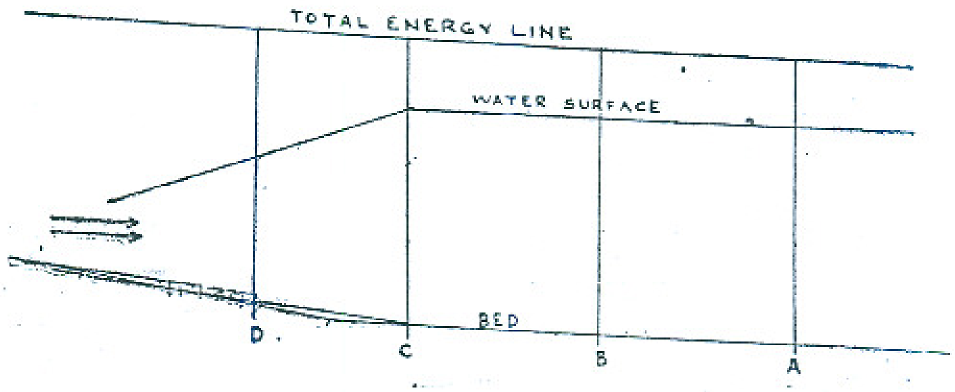


fig 19.

The slope of the total energy line represents the friction. At section, A the depth and velocity are known. In working backwards, section B is reached. The slope of the bed is parallel to the total energy line therefore depth and velocity remain unaltered. At section C is the toe of the glacis: Conditions still do not appear to have changed. But at section D, the bed is at a greater elevation than before. Hence the velocity must have increased to pass the same discharge therefore the surface must be depressed.

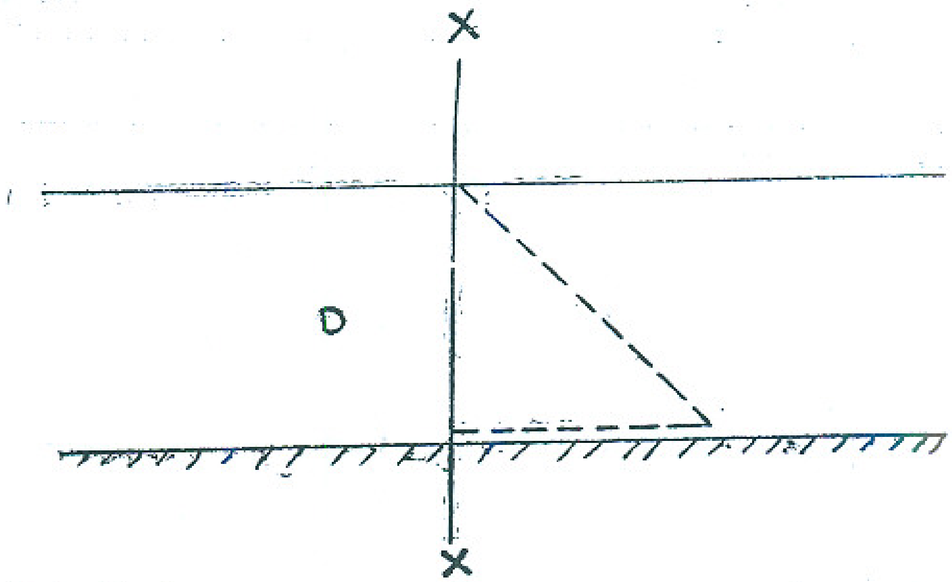
(Obviously at section C the water will not adhere to the "ideal" profile for reasons already given).

All such ideal profiles can be traced with ease, from the total energy line by the aid of the "Energy of flow" curve in figs. 6, 13, 17.

31. It will be readily understood now that the statement in para. 29. is true. It follows that part of the slope, observed in practice to follow a standing wave, is due to the continued depression of the glacis below water.

32. The limiting position of the standing wave is theoretically the point at which the continued depression of the profile results in a depth (on the "obstruction") equal to the critical depth. As soon as this occurs the standing wave *cannot* form, but complete recovery takes place. This is a special case of the general one discussed in para. 14.

33. Another special characteristic of the conditions of these particular experiments is that the channel is rectangular at any cross section.



If the depth at any cross section X X is D, the total pressure P

$= \frac{1}{2} D\ W \times D \times B = D^2 \frac{WB}{2}$

Where B is the width of channel at the section.

For rectangular sections, therefore, the expression

$P + \frac{W}{g} Q\ V$ (vide para. 16) may be written.

$D^2\ \frac{WB}{2} + \frac{W}{g} Q\ V$ and is constant.

Therefore $D^2 . \frac{B}{2} + \frac{Q}{g} . V.$ is constant.

34. Yet another special characteristic of these experiments is that the width B remains constant, approaching and over and issuing from the weir.

For any one series, therefore, one combined diagram (*vide* fig. 17 may be used to determine conditions at any point in the channel or on the weir.

If the approach to or the departure from the Weir was *not* parallel (*i.e.*, if B did not remain constant) separate diagrams must be drawn for each of the sections selected for treatment. The same argument holds for a variable batter.

The logical result must be noticed also, *viz.*, that the critical depth varies with the shape of the section and in the case of rectangular sections, with B, provided the discharge Q remains constant.

35. A further deduction is that standard diagrams for *rectangular* sections may be drawn on a basis of discharge per foot run. If such diagrams be drawn for 1, 2 3 cusecs etc., up to our working limit, they will hold for and can be applied to any rectangular sections over a work of any type whether the water-way is converging, constant or diverging.

The use of such standard diagrams in designing the approach to water-way of and divergence from works would be considerable : *e.g.*, syphons of all types : so called "flumed" bridges : falls : measuring devices like broad crested Weirs, etc., etc.

36. Before concluding this part, a few useful expressions must be derived.

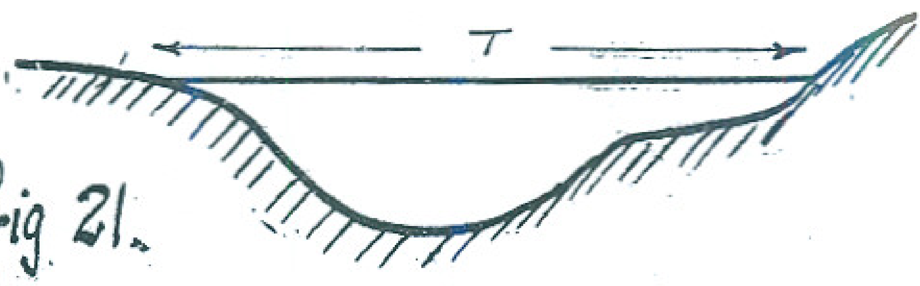


fig 21.

Let T be the surface width of any regular channel (**i.e.,** free from casual obstruction) of depth D.

$$\text{then } \frac{dA}{dD} = T \text{ and } A = f(D)$$

Energy of flow of unit mass

$$= D + \frac{V^2}{2g}$$

$$\text{but } Q = AV$$

$$\therefore H = D + \frac{Q^2}{A^2} \times \frac{1}{2g}$$

$$= D + \frac{1}{[f(D)]^2} + \frac{Q^2}{2g}$$

Differentiate with respect to D

$$\frac{dH}{dD} = 1 - \frac{f'\,D}{[\,f\,(D)\,]^3} \times \frac{Q^2}{2g}$$

$$= 1 - \frac{T}{A^3} \times \frac{Q^2}{g}$$

Equate to zero to find minimum.

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

---

37. Substitute A V_c for Q

then $\frac{A}{2T} = \frac{V_c^2}{2g}$ =velocity head.

at critical depth.

For a rectangular channel

$A = B\,D_c$

& $T = B$

$\therefore \frac{Q^2}{g} = B^2 \quad D_c^3$

or

$$\frac{V_c^2}{2g} = \tfrac{1}{2} \; D_c$$

i.e., at the critical depth, the velocity head equals half the depth

or

$$V_c = \sqrt{g \; D_c}$$

Again in a ~~rectangular~~ channel ~~if the velocity head equals~~ half the depth, the total energy of flow

$$= D_c + \frac{V_c^2}{2g}$$

$$= D_c + \tfrac{1}{2} D_c$$

$$= \frac{3}{2} D_c$$

or the critical depth $= \frac{2}{3}$ H, total head acting.

---

38. If D_1 V_1 and D_2 V_2 be the depth and velocity above and below a standing wave, the general formula connecting these, is easily obtained by equating the change in momentum per second to the total pressure acting. Then :—

$$D_2^2 + D_2 \ D_1 = \frac{2 \ Q}{g} V_1$$

$$\text{or } D_2 = - \frac{D_1}{2} + \sqrt{\frac{2 \ V_1^2 \ D_1}{g} + \frac{D_1^2}{4}}$$

Substitute $\frac{Q}{D^1}$ for V_1 in the former equation,

$$D_1 \ D_2 \ \frac{(D_1 + D_2)}{2} = \frac{Q^2}{g} = D_c^3$$

39. There is nothing original in the foregoing, save possibly in the arrangement, and some of the deductions. The formula can be found in any text book.

Volume III of the report on the Miami Conservancy Project contains particularly clear exposition of the theory of flow. The Author is indebted to an article by Mr. J. Hinds in Engineering News Record, dated 25th November 1920, for the ingenious " pressure plus momentus " curve and its application to the Standing Wave.

PART III.

Determination of the Water Profile over a Weir.

1. After much cogitation, the Author decided to deal with this part of the problem in a separate part as the solution it offers is subject to criticism to an extent that the theory expounded in Part II is not.

2. It is necessary first to prove that the minimum energy of flow of the discharge occurs at the downstream edge of the Weir.

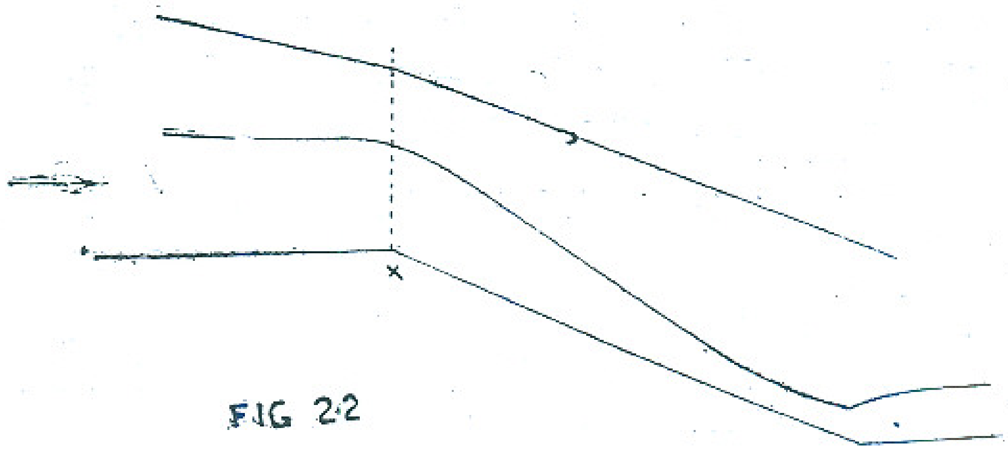


FIG 22

(*a*). Upstream of the section X, the total energy line, must *approach* the profile of the weir, because the profile of the weir is horizontal and the total energy line must steadily descend owing to the consumption of energy by friction.

Therefore at all sections upstream of X, the energy of flow must be greater than at X.

(*b*). Once past X, the water increases in velocity. (If the glacis were long enough, it would eventually reach stable flow, where the slope of the total energy line would be parallel to the glacis).

But until stable flow is reached, the actual velocity of the water is less than that of stable flow.

Therefore the consumption of energy due to friction, is less than that represented by the slope of the glacis.

Therefore the total energy line must successively diverge from the profile of the glacis.

So at all sections downstream of X, the energy of flow must be greater than at X.

(*c*). From the above considerations it follows that the minimum must occur at X.

But in passing from a depth greater than the critical to a depth lower than the critical the energy of flow must pass through the minimum.