The Regime Concept and The River Nile

By

GERALD LACEY, C.I.E., M.I.C.E.

SYNOPSIS

The Author, utilising observations made at the Beleida discharge site on the Nile in 1921, *correlates the mean velocity, hydraulic mean depth, gradient, and sediment transported and shows that these observations provide unexpected confirmation of current "empirical" regime equations derived from a study of hydraulic data from sandy bedded canals in India and Pakistan.*

He emphasises that the linear resistance equation derived by Phillips, is applicable not only to sandy bedded rivers but also to those transporting very fine material, such as the White Nile at Malakal. He suggests that regime equations applicable to mobile sandy beds are inapplicable to rivers or canals transporting fine sediment.

He draws attention to the analogy provided by the White Nile at Malakal with the Sind canals (to which it would appear that the Blasius and Phillips equations simultaneously apply) and concludes that the Sind canals present an entirely new regime problem, very different from that of sandy bedded channels, which urgently demands a solution.

Introduction

In the year 1921 an elaborate series of discharge observations was undertaken by the Egyptian Irrigation Department at Beleida, a discharge station on the Nile 38 kilometres upstream of the Roda gauge in Lower Egypt. Not only were observations recorded of the mean velocity, the hydraulic mean depth and the slope, but also, in furtherance of a campaign of "silt research" initiated in 1918, weekly determinations of the "silt" transported were made.

2. The observations were taken continuously during the flood season and thus present an exceedingly valuable opportunity of studying the influence of sediment transport on velocity, for, as Leliavsky, a recognised authority on this subject, has emphasised in his recent publication[1] the diameter of the sediment particles remained sensibly constant. These observations were discussed by A. B. Buckley, O.B.E., A.M.I.C.E. in a paper entitled "The Influence of Silt on the Velocity of Water Flowing in Open Channels" which he presented to "the Institution of Civil Engineers" in 1923[2]: In the writer's view this valuable

paper, together with the subsequent discussion, is a mine of information which has been sadly neglected, and of which the merits have never been fully recognised. One reason may be that in the discussion a number of authorities expressed different and conflicting opinions. This conflict was never resolved. It is the intention of the writer, in the light of our present knowledge of "regime" to attempt a reconciliation.

3. It is necessary first to consider the manner in which the hydraulic data were observed. The discharges were taken by means of Price current-meters from motor launches, each current-meter being returned to the rating tank after 75 hours use. The sediment samples were obtained by lowering sampling bottles of from 1000 to 2000 c.c. to the desired depth. The cross-section was measured only at Beleida. The slopes were computed from the gauge at the discharge site and a second gauge at a point 14.5 kilometres up-stream. As a result of these observations we are presented *inter alia* with the mean velocity in metres/second, V, the hydraulic mean depth in metres, R, and the slope, S, which it is sometimes convenient to express in centimetres per kilometre, *i.e.*, parts per hundred thousand. The sediment load, Z, represents the sediment in grammes per cubic metre of water.

4. We have thus recorded observations of V, R, S, and Z. Of these observations that of R is probably the most accurate. Any error in measuring R has its effect on the computation of the *mean* velocity of the entire cross-section which includes current-meter error. The gross slope as measured must differ from the local slope but this may imply only a difference in the numerical coefficient. The errors in sediment sampling are mitigated by the great range in load from the onset to the peak of the flood period. Probably the values of R and V have the greatest statistical significance. The slope range is very small and for this reason S and Z may be classed together, in a lower class than that of R and V. The reason for this discussion is that in any multiple correlation of the mean velocity V, with Z, R, and S the precise indices assigned to Z, R, and S depend on their statistical significance.

5. A first examination of the hydraulic data by Buckley showed that the coefficient of rugosity of the Nile during the low waterperiod, irrespective of whether the Bazin, the Kutter or Manning equations were employed was markedly higher than in the flood period. It is now generally agreed that these equations, of which the Manning is preferable, can properly be applied only to a rigid boundary channel and not to a channel with a mobile and incoherent bed. The influence of the "silt" or sediment on the rugosity was therefore examined.

6. First it should be stated that the term "silt in suspension" requires

qualification. The wash load, the fine silt and clay, with diameters of 0.60 mm and less may well be in permanent suspension, but the balance of the transported load other than particles in the bed layer, consists of sand both coarse and fine in very active "saltation". The values of Z as recorded cannot, therefore, be dismissed as "suspended" and entirely unrelated to the sand on the bed. The task of correlating V, R, S and Z was entrusted to Leliavsky and the very ingenious solution which he obtained, the Beleida fromula, takes the form

$$V=[147+3.92(Z-10)^{.383}]R^{.85}S^{.72} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

7. This formula the writer terms the Leliavsky equation. Those who desire to know how Leliavsky evolved it must make a close study of Appendix D of the Buckley paper. Briefly, the method Leliavsky employed was as follows: First he postulated an equation of the form $V=B\ R^a\ S^b$ in which B is a variable depending on the quantity of silt, Z, which results in the formula $V = f\ (Z)\ R^a\ S^b$

Secondly, during the rise and fall of the flood there must be at least two dates on which the charge was the same. He selected five such pairs of observations; for each such paif obtained an equation of the form

$$\log\ (V_1/V_2)=a\ \log\ (R_1/R_2)+b\ \log\ (S_1/S_2)$$

This is an equation in the first degree for a and b giving a straight line on plotting and thus any two pairs on plotting intersect. All such pairs would ideally intersect at one point. In practice the first approximation is a circle drawn tangentially to all the lines. Leliavsky thus obtained an approximate solution and assigned a value of 0.85 to 'a' and of 0.72 to 'b'.

8. Leliavsky relied on five pairs, *i.e.*, ten observations in all to determine the powers of R and S, and as these are the only observations given in the paper they are tabulated below :

TABLE 1.—LELIAVSKY BASIC BELEIDA DATA, 1921

S. No.	V	R	S**	Z	Date	Stage
1.	0.650	4.300	6.85	400	Aug. 10-11	Rising
2.	1.000	6.300	8.20	400	Nov. 15	Falling
3.	0.750	4.750	7.25	600	Aug. 11-12	Rising
4.	1.200	7.270	8.14	600	Nov. 2	Falling
5.	0.915	5.350	7.85	800	Aug. 13-14	Rising
6.	1.350	8.050	8.44	800	Oct. 22-23	Falling
7.	0.979	5.720	7.85	1000	Aug. 15	Rising
8.	1.369	8.475	8.04	1000	Oct. 7-15-16	Falling
9.	1.060	6.060	7.92	1200	Aug. 17	Rising
10.	1.368	7.920	8.06	1200	Sep. 27	Falling

Note.—Z ppm; S** slope in cms per Km. V metres per second; R metres.

9. The function B which Leliavsky introduced is so contrived as to give a constant value when the value of z falls below 10 but it is debatable whether this is essential. It might be urged alternatively that there was a "critical charge" below which the bed was inactive and thus derive a simpler equation of the form

$$V = \text{const } Z^c \ R^a \ S^b \ldots\ldots (2)$$

10. Mr. Buckley, when concluding his paper, and paying tribute to the remarkable agreement between the observed velocities and those computed from the Beleida (Leliavsky) equation remarked that "it appears to be the consistency of the slurry at the bottom of the channel which influences the friction and not the amount of silt actually in suspension". But, as stated by the writer, the values of Z as recorded cannot be dismissed as "suspended" and entirely unrelated to the sand on the bed.

11. Kanthack in the discussion on the Buckley paper commented that very little was known about the quantity and behaviour of "rolling sand and silt in a canal or river" and continued "it was obvious, however, that in so far as the coefficient of roughness was concerned, the character of that material must be, in most cases, the predominating factor, and not the quantity of silt in suspension." It is however clear that the quantities of fine and coarse sand in saltation cannot be regarded as suspended "silt".

12. Dr. Unwin remarked that it was somewhat unreasonable to demand of empirical equations, such as that of Kutter and Manning, designed to apply to "streams with permanent beds", that they should be applicable to "Egyptian streams with sandy beds". Of the Beleida equation he commented that although it seems to fit some of the cases quoted with great accuracy it did not seem consistent with the Buckley theory of the action of silt in the water. Professor Dixon stated that the only recent experiments that he knew of which were worth recording—those of G. K. Gilbert—"instead of showing that silt in running water tended to increase the velocity showed really that there was an opposite tendency." Mr. Lindley quoted observations which tended to show that "the carriage of silt used up more of the energy due to the fall of the channel." These diverse opinions suggest that, in the light of more recent knowledge, elucidation is possibly best sought by means of a re-examination of the Nile data.

13. The most striking criticism of the Beleida (Leliavsky) equation, despite the fact that it fitted the rising and falling stages very well, was made by Dr. Hurst. All such formulas he stressed were empirical and many could be found to fit given sets of observations. The Beleida equation fitted the facts at the Beleida and Khannaq discharge stations on the Nile ; the same

result, however, could be obtained by not using the silt content. His colleague, Dr. Phillips, had found that the very simple formula for the Nile at Khannaq

$$V = 2180\ R\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

fitted the facts as well as the more complicated one.

14. This formula, which is a linear resistance equation, is of great importance. The writer referred to it in the reply to the discussion on his paper "Stable Channels in Alluvium". The Beleida formula, as is natural when an additional variable is introduced, is a better fit than the Phillips equation; *both* equations however are substantially correct and the two can be reconciled. Dr. Hurst quoted for the Beleida site the alternative Phillips version

$$V = 2080\ R\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and finally for the White Nile in the Sudan, at Malakal, where there is very little silt, showed that a good fit could be obtained, from the formula

$$V = 4600\ R\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Dr. Hurst commented that the fact that the Phillips equation fitted a non-silty river as well as silty rivers robbed the Beleida formula of significance. He also considered that the large variations in the observed quantities V, R, and S, and the errors involved in measuring S made any other than a statistical investigation by recognised methods unacceptable.

15. Unfortunately no such statistical examination of the Beleida observations by recognised methods has, to the writer's knowledge, been published, and the tables of all the hydraulic elements submitted with the Buckley paper, including 192 observations at Beleida, were not printed in the proceedings of the Institution of Civil Engineers. For the present, therefore, the writer employs the observations in Table 1. Before attempting an analysis of this data one can only reflect on the progress that might have been made had the merits of the Phillips equation and the Leliavsky equation been recognised.

16. The following analysis by the writer is made on the assumption that the diameter of the sediment remains constant as stated by Leliavsky. We can rest assured that year in and year out there is no change, but witnin any flood cycle one would expect a small difference in the particles exposed on the bed during the rising and falling flood periods. The regime concept ideally postulates constant discharge, constant diameter of the bed particle and constant load: thus Q, the discharge, 'm' the diameter of the bed particle in millimeters, and Z the load in p.p.m. are regarded as *independent* variables. In the Nile the wash load of true "silt" from Ethiopia might possibly be regarded as an independent variable, but the fine and coarse sand transported is a dependent variable, picked up from the bed and thrown down during the annual flood cycle. An analogy of a kind is provided by the Punjab canal systems. Regime in canals is regarded as an annual overall stability, and hence the problem is

over simplified. During the cold weather when the clear water admitted picks up the canal sediment the value of Z is then a dependent variable, and a function of the discharge, as on the Nile.

17. The Phillips equation will first be examined and the relevant hydraulic data are given in the following table :

TABLE 2
BELEIDA DATA
PHILLIPS EQUATION

S. No.	Z	V	R	S**	RS**	V/RS	Remarks
1*	400	0.650	4.300	6.85	29.455	2206.8*	rising
2	400	1.000	6.300	8.20	51.660	1935.7	falling
3*	600	0.750	4.750	7.25	34.438	2177.8*	rising
4	600	1.200	7.270	8.14	59.178	2027.8	falling
5*	800	0.915	5.350	7.85	41.998	2178.7*	rising
6	800	1.350	8.050	8.44	67.942	1987.0	falling
7*	1000	0.979	5.720	7.85	44.902	2180.3*	rising
8	1000	1.369	8.475	8.04	68.139	2009.1	falling
9*	1200	1.060	6.060	7.92	47.995	2208.6*	rising
10	1200	1.368	7.920	8.06	63.835	2143.0	falling av:2105.5

18. The resulting equation is

$$V = 2105.5\ R\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

It must be borne in mind that the constant in Eq. 4 was obtained from the entire range of data and, therefore, the result from only ten observations is satisfactory. It might also be pointed out that this particular relationship applies equally well to ft/sec units. The observations during a rising flood are marked with an asterisk. The good fit during the rising flood-values of V/RS should be noted. The equation calls to mind the early regime canal equation

$$V = \text{const. } RS/f$$

but the great value of these Nile observations lies in the fact that they confirm the Inglis-Lacey relationship[3, 4].

$$V = \text{const. } RS/m^{\frac{1}{2}}$$

which implies that when the diameter of the bed particles are constant the Phillips equation is quite independent of the sediment transported. If we consider the methods of pairing two out of the three variables V, R, and S such as

$$V^2/R;\ SV;\ R^{\frac{1}{2}}S$$

all three such pairs are functions both of sediment dimeter and charge but the immediate effect of employing all three variables, V, R, and S is to eliminate either charge or the particle size according to the grouping.

19. The data in Table 2 are plotted in Fig. 1(*a*). Attention is drawn to the "shift" from left to right owing to the difference between the regime of a rising flood from a falling flood. It should be noted here that the rise was much more rapid than the fall. It will be noted that the rising flood results which are above the line show a remarkably good correlation.

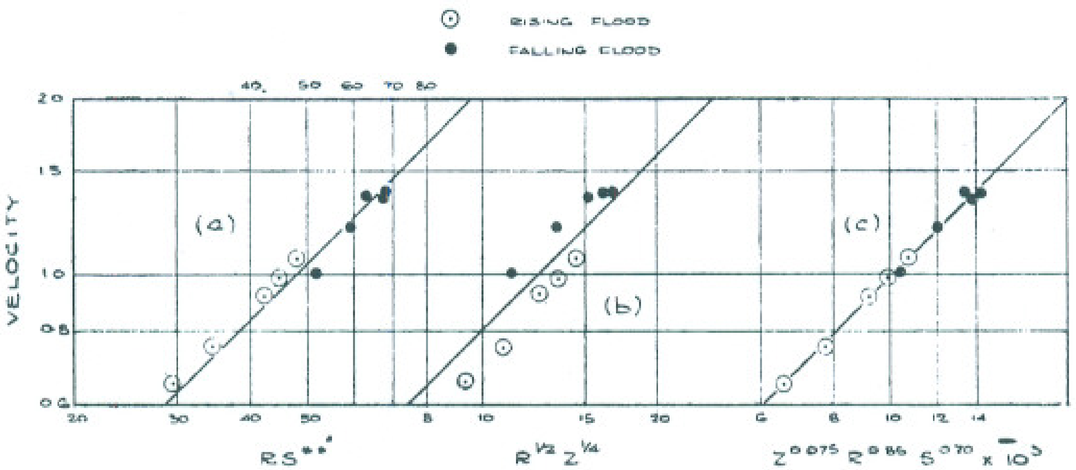

Fig. 1.

20. The Phillips equation was obtained by a correlation of V. R. and S. A correlation of V, R and Z was not attempted. In terms of the Inglis-Lacey equations, this when the diameter of the sediment particles was constant should take the form

$$V = \text{const. } R^{1/2} \; Z^{1/4}.$$

This relationship has been tested and the relevant hydraulic data are given in the following table :

TABLE 3
BELEIDA DATA
REGIME EQUATION

S. No.	Z	$Z^{.25}$	V	R	$R^{.5}$	$Z^{.25}R^{.5}$	K	
1*	400	4.472	0.650	4.300	2.0736	9.273	0.0701*	0.0796
2	400	4.472	1.000	6.300	2.5100	11.225	0.0891	
3*	600	4.949	0.750	4.750	2.1795	10.786	0.0695*	0.0797
4	600	4.949	1.200	7.270	2.6963	13.344	0.0899	
5*	800	5.318	0.915	5.350	2.3130	12.301	0.0744*	0.0819
6	800	5.318	1.350	8.050	2.8372	15.088	0.0895	
7*	1000	5.624	0.979	5.720	2.3917	13.451	0.0788*	0.0782
8	1000	5.624	1.369	8.475	2.9112	16.373	0.0836	
9	1200	5.886	1.060	6.060	2.4617	14.490	0.0732*	0.0779
10	1200	5.886	1.368	7.920	2.8142	16.564	0.0826	
							av: 0.0795	

21. For each pair of observations, rising and falling the mean value of K the constant has been shown. The derived equation is

$$V = 0.0795\ R^{.5}\ Z^{.25} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

It will be seen that the means of the pairs show a quite good correlation, The data have been shown in Fig. 1 (*b*). The "shift" is greater than in Fig. 1 (*a*). The values of K for the falling flood are :

Z	K
1200	0.0826
1000	0'0836
800	0.0895*
600	0.0899*
400	0.0891*

Note the remarkable correlation for the three values of K, marked with an asterisk and well below the peak of the flood.

22. It will be observed that Equations 6 and 7 provide alternative means of computing the mean velocity V. As both correlations are good an improved correlation can immediately be obtained by taking the geometrical mean of the two expressions. This would give us

$$V = \text{const. } Z^{1/8}\ R^{3/4}\ S^{1/2}$$

which brings to mind the form of one of the earliest regime equations with the very necessary addition of the sediment charge, Z. A study of Figs. 1 (*a*) and 1 (*b*) however shows that this does not necessarily give the best fitting multiple correlation and it was this which Leliavsky sought. Thus Fig. 1 (*a*) is a better correlation than Fig. 1 (*b*) which should be given a lower weightage which is obtained by taking the entire equation to a lower power. Whatever weightage is adopted the resulting equation will always be in the form

$$V = \text{const. } Zn\ R^{(1-2n)}\ S^{(1-4n)} \quad \ldots\ldots\ldots\ldots (8)$$

23. Now Leliavsky obtained for the Nile at Beleida a value of the power of R of 0.85. This would correspond to a value of n of 0.075 and finally

$$V = \text{const. } Z^{.075}\ R^{.85} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

Note that by adopting a power of S of 0.70 instead of 0.72 the difference, owing to the range of slope being so small, involves a maximum departure, plus or minus, of only 0.20 per cent. This equation has accordingly been examined and the computations are given in the following table :

TABLE 4

BELEIDA

$$V = K\,Z^{.075}\,R^{.85}S^{.70};\; K = V\,S\text{-}^{.70}Z^{.075}\,R^{.85}$$

S. No.		$Z^{.075}$	V	R	$R^{.85}$	S-1	$S^{.70}$	K
1	400	1.567	0.650	4.300	3.455	14.599	822.29	98.72
2	400	1.567	1.000	6.300	4.780	12.195	724.98	96.79
3	600	1.616	0.750	4.750	3.760	13.793	790.24	97.54
4	600	1.616	1.200	7.270	5.399	12.285	728.72	100.23
5	800	1.651	0.915	5.350	4.160	12.739	747.47	99.58
6	800	1.651	1.350	8.050	5.887	11.848	710.48	98.68
7	1000	1.679	0.979	5.720	4.403	12.739	747.47	98.99
8	1000	1.679	1.369	8.475	6.150	12.438	735.06	97.45
9	1200	1.702	1.060	6.060	4.625	12.626	742.82	100.03
10	1200	1.702	1.368	7.920	5.807	12.407	733.78	101.56
								av: 98.96

24. The resulting equation is

$$V = 98.96\,Z^{.075}\,R^{.85}\,S^{.72} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

and should be compared with the original Leliavsky Eq. (1). It is simpler in form and had Leliavsky introduced Z as Z^c in the same manner that he postulated $R^a\ S^b$ he would have arrived at a comparable result. It will be seen from Fig. 1 (*c*) that the difference between the rising and falling stages has been eliminated and the correlation is superior to that of Figs. 1 (*a*) and 1 (*b*). One reason that might be advanced is that there was in fact a slight difference in the grade of bed material exposed during the rising and falling stages of the flood. There are two Inglis-Lacey equations which may have some bearing on this point. The equation

$$V = \text{const.}\ RS/m^{1/2}$$

in which the charge is implicit has already been quoted, but there is yet another

$$V = \text{const.}\ Z^{1/6}\ R^{2/3}\ S^{1/3}$$

which contains the charge, Z, the precise grade of sand being implicit.

If we were to assign each equation the same weight by taking the geometrical mean we would obtain the equation

$$V = \text{const.}\ Z^{.0833}\ R^{.833}\ S^{.667}$$

and the writer has thus obtained the equation

$$V = 70.48\ Z^{.0833}\ R^{.833}\ S^{.667} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

which gives a good correlation. As, however, Leliavsky reviewed *all* the observations full weight must be given to the powers which he assigned. This analysis goes to show that the Leliavsky equation, with its excellent correlation during rising and falling stages, actually took into account slight changes during the flood of the bed material exposed.

25. It is to be concluded that the observations on the Nile go far to confirm the conclusions of research workers in India and Pakistan. They also encourage workers to hope that "regime" equations, often dismissed as "empirical" with all that such a term implies, may yet have a firm physical basis, as yet only partly elucidated. It should be recognised that the majority of observations of regime channels in India and Pakistan show much greater "scatter". For this the reason is not far to seek. All the Beleida observations were at one site with only a small change in the diameter of the bed particle, a sand of the same characteristics throughout, with the same coherence or lack of choerence, and the same size distribution. The velocities ranged from 2.0 to 4.5 feet per second and the depths from 14 to 28 feet. To obtain comparable results from canals entire canal systems would require to be observed and, the observations having been made, each channel would have its own characteristics, there would be no constancy in the grade of bed material, nor in its size distribution. The Nile floods present a unique opportunity for observations which would be invaluable to those concerned with the design of canals with *mobile sandy beds.*

26. When the bed material is very fine, with diameters of the bed material of 0.10 mm approximately, or less, a very different problem arises. In certain of the Sind Canals these conditions obtain. In a recent valuable paper by Sethna[5] he has dealt with this problem. He found from his observations that in many cases the Phillips equation gave an excellent correlation, that the beds could be treated as smooth, and that there was very little correlation with the precise size of the material, provided it was small. These are probably the conditions at Malakal, on the White Nile in the Sudan. Here there is no sand, and the velocities are very low. Thus during the flood of 1921 the maximum velocities at the peak of the flood were of the order of 0.55 metres/sec., or 1.80 ft./sec. and at low stage fell to 0.35 metres/sec. or 1.15 ft./sec.; it will be readily agreed that at this site the Nile is a very sluggish river. At low stages the depth of water would be of the order of two metres, or say 6 feet and if we were to apply the somewhat outmoded equation for the silt factor to the low stage we would have

$$f = 0.75\ V^2\ R = 0.165$$

and for the high stage with a value of R of about 15 ft. and a velocity of 1.80 ft./sec. we would obtain a value f of 0.162. These figures which are at best of only the right order suggest that the bed material was certainly less than 0.10 mm in diameter.

27. It is clear that it is incorrect to compare this river at Malakal with the sandy bedded Nile at Beleida. Nevertheless the Phillips equation was found

to apply which would suggest that this *liner resistance equation* is of great significance and with a wide range of application. The writer has given his views in the discussion on Sethna's paper[6]. Briefly he found that to many of the Sind canals the Phillips equation applied, and, in conformity with Sethna's view of a smooth bed, he therefore examined the possibility that the Blasius equation for smooth turbulence—applicable to channels in which the bed material sheds no eddies-might also apply. The writer found that this relationship held good. The sequel of this investigation is this that many of the Sind channels form a class in themselves and are *not* amenable to the usual regime principles of design. Effectively this implies that before designing new channels in Sind, or remodelling old ones, we must re-examine the entire problem of the regime of channels with fine silt and smooth beds.

28. It must doubtless have occurred to many confronted with the title of this paper that it could have little reference to the design of canals in West Pakistan; the paper when studied may persuade them that the Nile, when explored by the research worker, has much valuable information to yield. The main Nile at Beleida in Lower Egypt provides confirmation that in respect of the regime of channels with sandy mobile beds the existing regime equations—which take account of the load transported are not very seriously in error. The White Nile at Malakal, on the scanty information available, appears to present a close analogy to the Sind canals transporting very fine materials. Neither the Sind canals nor the White Nile at Malakal conform with regime equations applicable to sandy beds, but, irrespective of whether the beds are of loose sand or very fine material, the Phllips linear equation applies and links the two entirely different systems of regime equations together. The Sind canals present an entirely new regime problem which urgently demands a solution, and therefore priority in research.

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