

Structural Analysis and Design of Multistoreyed Framed Buildings

By

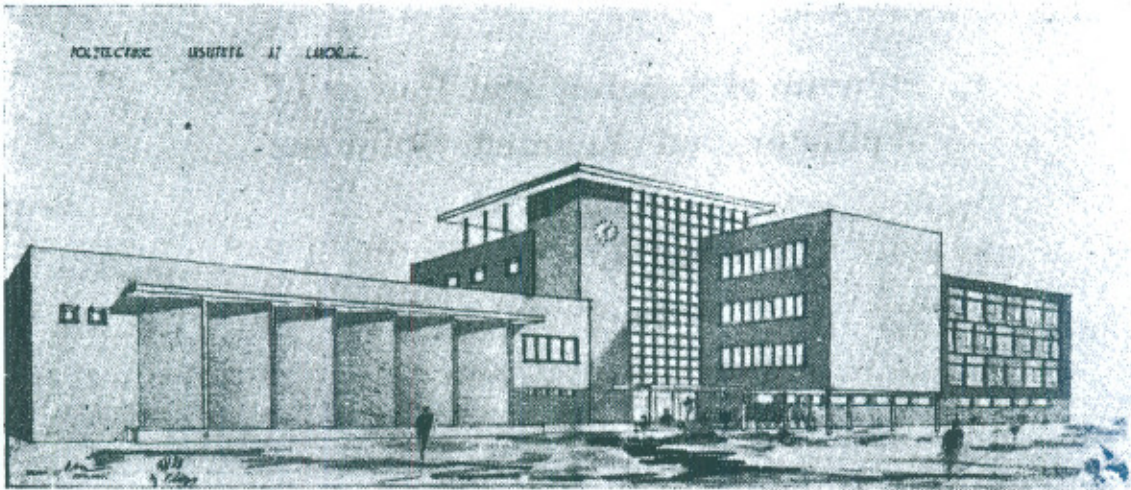
MR. GHIAS-UD-DIN HABIB

I. INTRODUCTION

Continuous frame analysis is a very important design subject for a structural engineer, for in this field he is confronted with the conflicting requirements of achieving sufficient accuracy and at the same time expending a minimum of effort and calculation. For this purpose there are many analytical procedures established and evolved by eminent persons like Castigliano, Clapeyron, Maxwell, Poisson, Hardy Cross, Grinter, Kani and so on. In the presence of these the aim of writing this paper is to lay down in a simple form a procedure for adopting suitable methods, which may be easily understood, and are also comprehensive enough to solve the various problems, that arise in designing complicated framed buildings. Construction of a number of such buildings are in progress under the charge of the author at Lahore, and he has the opportunity of structurally designing a number of these. He has, therefore, ventured to sum up his observations in the form of this paper, so that it may be of some use to those who are interested in the design of multistoreyed buildings. While tackling this problem the paper deals with, in the beginning various properties of a rigid frame and different kinds of loads to which it is subjected. It then gives various methods of stress analysis, finishing in the end with illustrations for stress analysis, which are self explanatory.

The scheme for the construction of Government Institute of Polytechnic at Lahore was approved by the Government of West Pakistan at the estimated cost of Rs. 31 lacs. The site selected by the department for this purpose was the premises occupied by the existing Government Technical Institute at Railway Road, Lahore.* The site being limited, the new building has been planned as five-storeyed structure consistent with architectural, structural and functional requirements, so that it is suitable from the point of view of accommodation, convenience and comfort. The structure is fully rigid and the frame that has been adopted consists of two or three panels having long verandahs with rooms flanking on one side only. The site where the Polytechnic building is being constructed consisted of made-up soil up to 15' depth. Boring was carried out at different places up to 100' depth which revealed that the soil below 15' was satisfactory.

* *Superintending Engineer Construction Circle, Lahore.*



Polytechnic Institute at Lahore

II. RIGID FRAME AND ITS PROPERTIES

(1) Building Bent

A building frame known as building bent consists primarily of girders carrying vertical load and rigidly connected to columns so that all the members of the frame carry bending moment, shear and axial forces. The stress analysis of such structure involves not only its geometry but also its elastic properties like modulus of elasticity, cross sectional area, and moment of inertia. The bending moment in such frames at the ends of a beam cannot be transferred to the next span without subjecting the columns to bending. Thus instead of transmitting the bending moment in full to the beams the moment is transferred partly to the beams meeting at the joint and partly to the columns above and below the beam. The columns, particularly in the case of rigid frames are subjected to considerable bending due to unbalanced loading. In a rigid joint where several members meet the bending moment of one member composing the joint is balanced or resisted by the bending moment in the remaining members, and each member at the plan of juncture with the other members is capable to resist the bending and shear stresses to which it is subjected.

(2) Span of the Frame

In structural analysis a frame is represented by single line diagrams. Actually depths of beams and widths of columns amount to sizable fractions of their respective clear lengths which are considerably smaller than the respective centerline distances. The block of concrete formed at a joint by the members is extremely rigid in both directions contrary to the assumptions, that the member has sufficient flexibility over its entire length. It is however a practice to consider the lengths of the beams to be given by centre-to-centre distances between joints, which though not strictly exact is closer to the truth than the

alternative of using clear lengths, which cause good deal of reduction in the size of a frame. Since the diagram of negative moments is very steep in the neighbourhood of supports, the theoretical moment at the joint is considerably larger than that at the face of the column. For this reason it is economical to use "moments at the faces of supports" for design of beams and girders. The same situation arises for columns. But for these the moment curve is not as steep as in girders so that the difference between the theoretical joint moment and that at the face of the girders is relatively smaller. Also in view of the simultaneously acting axial force, a change in moment affects but slightly the required cross section. For this reason theoretical moments at joints without reduction are used for columns.

(3) Moment of Inertia of the R. C. Frame

The design of flexural members is based on the supposition that the part of the concrete which is in tension is ineffective. It would seem, therefore, that moment of inertia for computing stiffness of a member should be determined in the same manner *i.e.* discounting the concrete in tension. This however would not give a measure of true stiffness of the member, because though the concrete is capable of resisting only a moderate amount of tension, hairline cracks from this cause form only in limited portion of the lengths of the beam and the sum of the widths of all such cracks is extremely small as compared to the span. The actual bending rigidity is, therefore, satisfactorily characterized by the moment of inertia of the full section. The contribution of reinforcement is, however, neglected which compensate to some extent for the neglect of cracks. For T-shaped members allowance is made for the effect of the flange which is assumed to have the same effective width for computing moments of inertia as is considered in stress computations. For continuous T-beams the flanges are also effective in regions of negative moments where they are in tension, as there is little difference between concrete in compression and concrete in tension in regard to its effect on the flexural stiffness of the member. For columns, likewise moment of inertia is computed for the full unreduced concrete section, neglecting the influence of reinforcement.

(4) Conditions of Supports

The frames are supported at various points of other structure, which in the end transmit the loads to the underlying soil. Thus, columns either rest on individual footings or raft foundations or they are supported on piles. In partially framed building columns are placed only in the interior while the outer ends of the girders are supported by brick walls. In all these cases it is important to decide whether the supports are hinged, rigid or lie between these two extremes. It is, however, not possible in frame analysis to account for inter-

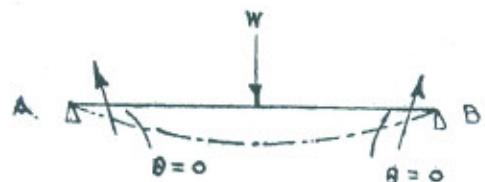
mediate restraints. Moments in a frame are affected to a considerable degree by the choice so made because the stiffness of a fixed end member is four thirds of the stiffness of a hinged member. An assumption of support conditions must, therefore, be made which is both realistic and safe. In the case of buildings resting on the strip footings, soils offer but little resistance to rotation of the footing, and hence hinged joints are assumed. When the foundations rest on a raft as provided in the case of polytechnic building complete fixity of the bottom of the column is assumed. If outer ends of the beams rest on brick walls the joints are treated as hinged.

(5) Convention of Sign and Direction of Bending of Members at a Rigid Joint



Certain conventions with regard to signs have to be followed if errors are to be avoided. A moment in the clockwise direction is considered +ve and a moment in the anti-clockwise direction is considered -ve. If due to a moment at the support, a member AB to the right of the support is bent upward, the moment is (anti-clockwise) and -ve. If AC is bent up, the moment is (clockwise) and +ve. If AB is bent downwards, the moment is +ve and if AC is bent downwards, the moment is -ve. In case of a column AB fixed at the bottom A, if the lower portion of the column *i.e.* AB is bent to the left the moment is -ve and if to the right, the moment is +ve. Similarly for the top of the column AC a bend to the left is due to a +ve moment and bend to the right is due to a -ve moment. If the direction of bending of one member out of two forming an angle is known, the direction of bending and, therefore, the sign of the moment of the other member is easily obtained. The joint being considered rigid, the angle between the two members cannot vary and the moments in the two members must balance.

(6) Concept of Fixed End Moments and Shear



When a load is applied on a beam it deflects and end rotates through a certain angle. If ends of beams are restrained by moments and change in angle is zero, the ends are considered fixed and the restraining moments are called fixed end moments. These fixed moments are computed as the product

of co-efficient, loading and span length. The co-efficients are independent of adjacent beams or other members in the frame. The fixed end moments make up a major part of the actual moment under beams. The object of frame analysis is to determine the moments which when added to the fixed end moments gives the actual final moment.

Shear at the end of a beam, that is part of a frame, is determined as the sum of the shear in the beam considered simply supported, and a correction due to the difference between end moments produced by the frame action. The correction is usually small compared with the simple beam shear especially in interior spans. In the case of beams with fixed ends it is necessary to evaluate the bending moment before shear forces can be determined. This is the converse of the procedure that is followed when the beams are simply supported. The shearing forces at the ends of the beams which is due to fixed end moment is given by $\frac{M_A - M_B}{L}$ at A, $\frac{M_B - M_A}{L}$ at B where M_A and M_B are the numerical values of the moments at the ends of the beams. These formulae are followed exactly with respect to the signs. The shear force is greater than the simple support reaction at the fixed end which has the greater numerical value of fixed end moment.

(7) Concept of Stiffness and Carry Over Factor

As the ends in buildings are not fixed, the fixed end moments must, therefore, be modified to suit whatsoever rotation take place at the joints. This modifying factor is denoted by 'K' and equals $\frac{4EI}{L}$ which is referred to as the absolute value. A relative value of $K = \frac{I}{L}$ is preferred when E is constant throughout the frame. The stiffness K at a point 'B' of a beam equals the moment at B required to give 'B' a unit rotation. Applying a moment M_{BA} at B will induce at A a moment $M_{AB} = \frac{1}{2}M_{BA}$. The factor of 1/2 is called the carry over factor. The stiffness is a function of cross-sectional dimensions and is, therefore, not initially known and must be estimated.

III. LOADS COMING ON THE FRAMED STRUCTURE

(1) Vertical Load

For the purpose of calculating load to be carried by columns, walls and footings in buildings of more than two stories in height, the superimposed loading on the roof and other floors is obtained on the basis of loads specified for such buildings as per standard specifications. For stories below the top one, a

reduction of the superimposed loading is permissible such as for story below top-most it is 10% and then for the next it is 20% and so on subject to the maximum reduction of 50%. Over horizontal members of a frame considerations are limited to combination of dead loads on all spans with full live loads on two adjacent spans for negative support moments and with full live loads on alternative span for positive span moment. Columns the vertical members are required to resist the axial forces from loads on all floors plus the maximum bending due to loads on a single adjacent span of the floor under consideration. Roughly 90% of any bending moment is due directly to the loads on the two adjacent spans where the moment is required which means one can safely approximate as far as more vertically distant live loads are concerned.

(2) Wind Load

Wind forces induce stresses in all buildings to some degree, depending on the type of building. In case, however, the overall dimensions of the building are such that its height is not more than twice the effective width and the frame work is stiffened by walls and floors, calculations for wind effect are not required except with regard to foundation and roof. In R. C. framed structure it is an essential part of a wind pressure problem to ascertain the pressure acting on each individual bent of a building because all concrete members are integrally and rigidly connected with adjacent members, and all bents extending in a given direction cooperate in resisting the wind acting in that direction. The axis due to wind on a structure depends on the velocity of the wind and the shape and size of the exposed members. The external wind pressure (P) is considered to be the numerical sum of the positive pressure on the windward vertical face and negative pressure and suction on the leeward vertical face, the positive pressure and suction being both equal to $P/2$. For the purpose of design the wind pressure is taken on gross area of the vertical projection of the structure. On open framed structure the area used in computing wind pressure is taken as one and half times that net area of the framing members on the side exposed to the wind. The outer rows of columns and their foundations are designed to resist the total wind load on the projected area unless the roof is capable of transmitting half the load in which case the row of columns on the other side are designed to carry other half of the load. This half is assumed to be shared equally by all subsequent lines of columns. When the outer columns are designed to resist the whole wind load, the remaining columns resist no wind load. Wind pressure on a vertical surface is adopted in accordance with the values given in the following table. The normal permissible stresses caused by the wind in the members are exceeded by 25 per cent in cases where such increase is due solely to the stresses induced by wind. This higher working stress is allowed in view of the transient nature of the load and also because

the structure is of a sufficiently elastic nature to allow it to absorb such loads without permanent effect. The increase is not permissible on foundations because the soil is not capable of acting in the same elastic manner as the frame itself.

H	10	20	30	40	50	60	70	80	90	100	125	150
P	12	15	18	20	22	23	25	26	28	29	30	34

H=Average height in feet of the exposed surface above the mean retarding surface.

P=Horizontal effect of wind in lbs per sq. ft.

(3) Seismic Load

The ability of a structure to resist seismic load depends largely on its ability to absorb energy, and it is, therefore, of utmost importance to ensure that it can do so. The type of the soil on which the building stands is considered to be one of the most influential factors in the production of the seismic force. The effect of the ground motion is to feed energy into the structure. If the structure is built on a solid expanse of rock, it is fair to assume that the whole of the energy input is taken up through kinetic energy of motion of the mass, the strain energy of deformations of the structural members and final damping. If however the structure is built on a soft soil strains are produced in the foundation and energy is dissipated by internal friction in the soil which becomes softer and the probability of damage increases. The irresistible, wave-like motions of the ground under earthquake causes vibration both in the horizontal and vertical directions. The vertical movement is generally very small, and because the structure is designed to resist vertical load, damage due to it is negligible. For the computation of earthquake force for design purpose the selection of acceleration is made with reference to the intensity of shocks which had occurred in the locality. The inertia reactions due to earthquake are computed as fixed proportion of the dead and live load and for convenience are applied at the different floor levels. The buildings are designed to stand minimum horizontal acceleration of 1/10 of the acceleration due to gravity or a force equal to 1/10th of the weight of the structure. In such cases the working stress for combined vertical and horizontal forces is increased by 33% as the structure is subjected to seismic pressure in a very infrequent manner. The steps in a rational computation of earthquake stress in a framed building are firstly the estimation of the bending stress in the vertical column in each storey produced by the horizontal seismic forces, and secondly it is

the estimation of the additional bending stress imposed on the horizontal beams of the floor system connected to the columns.

(4) Temperature Load

The assessment of moments for sideway due to temperature variations in a frame with multipoints of freedom involves calculations as for transverse loading. Due to temperature difference of the roof and floors sway takes place in the framed structure. The variation of temperature develops no moments in columns as there is no relative movement of vertical members of the frame, which are all of same height. The horizontal members at the top which are exposed to atmospheric conditions are affected by the variation of temperature, whereas the raft slab being underground practically gets no temperature effect. There is, therefore, a relative displacement between these two horizontal members. This gives rise to moments in columns which on account of rigidity of frame are transferred to other members of the frame. Variation of temperature in the top frame is much higher than the protected inner frames and so a variation of 60°F for the topmost storey and 40°F. for the lowest storey is generally assumed.

With the above temperature variation, expansion of each length of horizontal frame is calculated. To begin with the frame is assumed to be rigid at one end and free to expand towards the other. With these values of deflections, the moments induced at all the joints are calculated. For this assumption the frame remains unbalanced and sideway correction becomes necessary. To do this correction, each individual storey is given certain deflection keeping the joints of the other storey rigid and thus framing simultaneous equations of moments for these unknown values of deflection. When these unknown are determined, correction factors are known by which the figure of original moments in the frame due to actual temperature variation are corrected and the final results tabulated.

IV. STRESS ANALYSIS OF R.C. FRAMED STRUCTURE

The main object of stress analysis of a structure is to determine with reasonable accuracy bending moments, shears, and axial forces produced in various members of external loading. Columns of a frame are analysed to resist the axial forces for loads on all floors plus the maximum bending due to loads on the adjacent span of the floor. It is generally conceded that moments cannot be determined in columns with the same degree of accuracy as in beams. A beam moment is obtained as the sum of fixed end moment, and an additional term of a correction derived by analysis. But a column moment simply equals the correction obtained by analysis, and is, therefore, as a rule far more sensitive

to changes in assumptions and much more susceptible to faulty analysis.

There are several methods of designing an indeterminate structure. Some are analytical and the others are experimental. The conventional analytical method of analysing structure is computed by the theory of elasticity based on Hook's Law. Under working load this theory as applied yields results which sufficiently coincides with reality. In the 19th century the fundamental theories of "Slope deflection", "Three moments", "Least work" etc., were developed to solve indeterminate structures. These methods, however, are not of any practical advantage since their application results in a system of many algebraic equations, solution of which represents extremely laborious task. During the early part of the 20th century a simpler method of stress analysis *viz.* the method of successive approximation was evolved. In 1932 Prof. Hardy Cross improved upon this method by advancing his method of "Moment Distribution". During 1942 Dr. C. V. Kouch effected considerable simplifications through a method of approximation namely the distribution by "Deformation method". Besides these developments a method which is of great practical utility was evolved by Dr. Kani of Germany. This is an iteration method different from moment distribution and presents a solution of stress analysis of complicated multi-storeyed structure under all kinds of loadings with comparative ease. Recently a modified method of moment distribution has been presented by Mr. Lloyd C. P. Yam in which sideway is automatically allowed for without employing a temporary support device. After gauging advantages and disadvantages of all these methods it has been found to be very expedient to employ Hardy's method of moment distribution for loading without sideway and then use "factor method" for sway due to unsymmetrical loading or unsymmetrical disposition of the frame. For transverse seismic or wind loading stress analysis is conveniently carried out by Dr. Kani's method.

Amongst experimental methods which are used where analytical methods become very complicated and cannot be applied are (a) Photo-elastic method in which polarised light is passed through loaded models made of transparent material, and then the direction and intensity of stresses are observed. (b) Begg's deformer, in which elastic models are deformed with the help of plugs and gauges and the resulting deformation is accurately measured through micrometer microscope. (c) Elastic strain gauges, working of which is based on the principle that the electrical resistance of a wire changes when it is strained. A small current is passed through the strained gauges which are very small pieces of wire attached at various points to the surface of the structure. The subsequent strain in the structure is increased by the variation in the current, considerable amplification being necessary to observe photographically recorded variation.

ANALYTICAL METHODS

(1) Moment Distribution by Hardy Cross

The classical methods of tackling indeterminate structure are methods of only stress analysis as distinct from design of its various members and for this reason they have not proved of any practical value to a designer. This drawback has been overcome by the method of "Moment Distribution" which consists series of cycles each converging to the precise final result. The series are terminated whenever one reaches the degree of precision required by the problem at hand. The method is based on the fact that whenever a stiff joint in a structure absorbs an applied moment with rotational movement the moment is taken up or resisted by the various members of the joint in proportion to their respective stiffnesses. Also when a member is fixed at one end, and a moment is applied at the other (free end) in such a way that the free end while rotating retains its original position the moment induced at the fixed end is half of the applied moment. In a rigid frame deflection of axes and rotation of joints govern the distribution of moments, shears and thrusts. It is, therefore, important throughout the computations to visualize deflections and rotations in relation to the corresponding moments and shears to understand physical significance of the analysis.

The ease of computation and rapidity of application in analysing structures by moment distribution with non-translatory joints for simple frames are beyond dispute, but when problem arises where sidesway occurs, a prohibitive increase of computational labour results. By the method of "Moment Distribution" simple frames of not more than two stories under transverse loading can be dealt with easily. Under such loading when the frame is fixed at the bottom, slight bending of the whole structure in the vertical plain takes place. The result is that the joints in the frame do not remain fixed in position but move slightly in the horizontal direction. This movement results in increased moments in the members.

In order to obtain correct moments of the joints subject to sidesway the method adopted is first to obtain the moments in the frame assuming that no sidesway occurs, which is easily accomplished by the method of moment distribution. If a transverse or unsymmetrical loading gives horizontal thrust, that apparently does not satisfy the statical requirements, *i.e.* the algebraic sum of all the forces must be zero. The building bent tends to move sideways setting moments at the corners, and it therefore becomes obvious that an external force must be added in line with the frame if horizontal displacement is to be prevented. The magnitude of this force is the difference between the reaction due to the imposed transverse load and the shear due to the induced moments

in columns. In order to ascertain the additional moments in columns caused in the members of the frame due to the absence of such a counteracting force, a set of end moments are assumed in the members of the frame and the shear due to these moments is worked out, and the assumed moments in the frame are corrected in the proportion of these two shears. These are then the additional moments in the members due to sidesway, and are added to the moments determined with no sidesway, to obtain the total moments. The method, however, becomes unworkable in the case of multi-storeyed multibay frames which renders it desirable to look for other methods of analysis which are simple in application and give correct results.

(2) Factor Method of Stress Analysis

In practice it is found time and labour saving to make to certain extent an approximate analysis of any structure. As it is, many of the materials used in ordinary construction do not behave exactly according to Hook's Law. Moreover the value of modulus of elasticity varies, and certain phenomena of plasticity, creep, shrinkage etc. are neglected. It may, therefore, be good enough to adopt some approximate solution for analysing stresses due to unsymmetrical loading or due to unsymmetrical disposition of a framed structure. There are a number of approximate methods of analysing a building frame acted upon by sidesways. The two such methods are known as "Portal Method" and "Factor Method". Out of those two "factor method" is more accurate while portal method is easier in application but in actual practice latter method is preferred.

The factor method depends on certain assumptions regarding the elastic action of the structure. For each girder and each column value of $K = \frac{I}{L}$ and for each joint "Girder Factors" $g = \frac{\sum Kc}{\sum K}$ are computed. Value of 'g' thus obtained is written at the near end of each girder meeting at the joint. For each joint also columns factor $c = (1-g)$ is calculated. Value of 'c' thus obtained are written at the near end of column meeting at the joint. For the fixed column bases of the first storey 'c' is taken as 1. Then to the number at each end, half of the number at the other end of the member is added. The sum thus obtained is multiplied by the 'K' value for the member in which the sum occurs. For columns this product is called moment factor 'C' and for girders moment factor 'G'. The column moment factors 'C' are actually the approximate relative values for columns end moments for the storey in which they occur. The sum of the column end moments in a given storey are as shown by statics the total horizontal shear on that storey multiplied by the storey height. Hence the column moment factors 'C' is converted into columns end moments by direct proportion for each storey. Similarly the girder moment factors

'G' are actually approximate relative values for girder end moments at each joint, which are further corrected by direct proportion for each storey. The sum of the girder end moments at each joint is equal by statics to the sum of the column end moments at that joint.

(3) Kani's Iteration Method

As stated before moment distribution is not considered as last word in framed analysis, so that any new developments may appear superfluous. Moment distribution as apparent has its limitations. No matter how simple it may appear it becomes cumbersome when used for frames involving multiple sideway. Unfortunately the most common type of frame, the multi-storeyed structure, falls in this category, Dr. Kani's of Germany has evolved a method of structural analysis which mostly overcomes these shortcomings and gives an easy solution of complex multi-storeyed buildings under both transverse and unsymmetrical vertical loading. It is like the method of "Moment Distribution" a method of successive approximation but its procedure is quite different. There is no particular restriction as to the sequence in which the joints are chosen. Any arithmetical mistake made during the solution is automatically rectified. The formulae used in finding the final moments are :

$$(1) M_{ik} = \bar{M}_{ik} + 2M'_{ik} + M'_{ki} + M''_{ik} \quad \left\{ \begin{array}{l} \text{This is a simplified method of} \\ \text{writing down the slope deflection} \\ \text{equations.} \end{array} \right.$$

$$(2) \left\{ \begin{array}{l} \Sigma M'_{ik} = -\frac{1}{2}(\bar{M}_i + \Sigma M'_{ki}) \\ M'_{ik} = U_{ik}(\bar{M}_i + \Sigma(M'_{ik} + M''_{ik})). \end{array} \right. \quad \left\{ \begin{array}{l} \text{This equation gives the basis of the} \\ \text{method. The sum of the fixed end} \\ \text{moments, the far end moments and} \\ \text{the displacement moments is multi-} \\ \text{plied by the corresponding distribu-} \\ \text{tion factor } U_{ik}. \end{array} \right.$$

$$(3) U_{ik} = -\frac{1}{2} \frac{K_{ik}}{\Sigma K_{ik}} \quad \left\{ \begin{array}{l} \text{The distribtion factor for } M'_{ik} \text{ is} \\ \text{found by dividing } (-\frac{1}{2}) \text{ in propor-} \\ \text{tion to the stiffnesses. } K_{ik} \text{ is the stiff-} \\ \text{ness of member ik.} \end{array} \right.$$

$$(4) \Sigma M''_{ik} = -\frac{3}{2} \Sigma (M'_{ik} + M'_{ki}) \quad \text{Sum of displacement moments.}$$

$$(5) V_{ik} = -\frac{3}{2} \times \frac{K_{ik}}{\Sigma K_{ik}} \quad \left\{ \begin{array}{l} V_{ik} \text{ is the displacement factor for} \\ \text{the columns and is obtained by} \\ \text{distributing } -\frac{3}{2} \text{ in the proportion to} \\ \text{the column stiffness.} \end{array} \right.$$

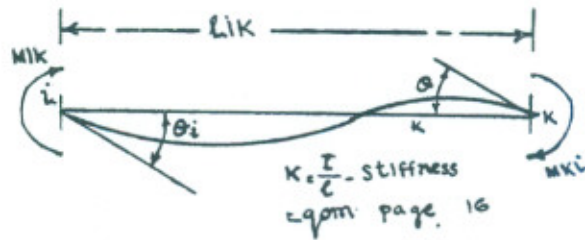
where M_{ik} = fixed end moment at i,

$2M'_{ik}$ = Moment caused at i by rotation of end i which may be termed as near end moment.

M'_{ki} = Moment caused at i by

rotation of the far end K which may be termed as far as end moment.

M''_{ik} = Displacement moments.



Starting with any joint all values of M'_{ki} i.e. all the far end moments are assumed zero and approximate value of M'_{ik} at that joint is obtained by using equation (2). This sum M'_{ik} is distributed at any joint over the members meeting there in proportion to the stiffness of the members. Proceeding to the next adjacent joint, the value of M'_{ik} at this joint is again calculated and distributed. The process is continued from joint to joint. For the next cycle, the first approximation of the far end moments is included in the sum of the unbalanced moments, and distributed giving a better approximation for M'_{ik} .

The process is repeated till no further change in the distributed moments occur which is generally after 3 or 4 cycles. When the loading is unsymmetrical and there is sidesway the equations 4 and 5 are applied. These equations show that to obtain the near end moment, the sum of fixed end moments, the far end moments and the displaced moments is multiplied by the corresponding distribution factor. After obtaining the first approximation for M'_{ik} for all columns at one floor, assuming M''_{ik} (displacement moment) to be zero a first approximation for M''_{ik} is found by using the equation (4). The value for M''_{ik} is then included in the equation (2) to give a 2nd approximation for M'_{ik} . The two operations are repeated till no further changes in the value of M'_{ik} and M''_{ik} occur. The calculations are then stopped and the final end-moment is found by using equation (1).

V. DESIGN OF FRAMED STRUCTURE

(i) Preliminary Design of the Structure

In a statistically determinate structure, moments and forces are calculated before the sizes of the various members are known and so the direct design of the structure is possible. But in the case of a rigid structure it is not so

because bending moment is not only dependent upon position of loads and modulus of elasticity but also upon length and the moments of inertia of the various members of the frame. It is, therefore, evident that for indeterminate structures certain assumptions are made which make the analysis and design of the structure a process of "guess and check". Thus to attain the objective a preliminary estimate of the approximate sizes of members are made using simple beam reactions as shears, and fixed-end moments as the final end moments of a particular span. Alternatively the approximate moments and shears coefficients may be used for preliminary design. An accurate moment analysis is then carried out to check the assumed sizes for making adjustments and changes as may be required, and also to obtain the complete information on moments and force necessary for designing details of the various members. The moments and shears thus obtained are evidently not accurate if the final sizes of members are not the same as those used for computing 'I' and 'K' which necessitates to repeat the process involving lengthy calculations. This shows the practical importance of making the best possible preliminary design before beginning an elaborate analysis.

In building frames, the sizes of girders are usually governed by the negative moments and the shears at the supports, where their effective section is either rectangular or they are shaped as T-beams. Moments in columns are generally smaller than in girders so that their sizes are primarily governed by the axial loads they carry. To determine these sizes preliminary sections are computed which are required if axial loads alone were present, and then to increase them slightly to provide for the additional influence of moments. Moments are larger in exterior than in interior columns since in the latter dead load moments from adjacent spans largely balance. In addition the influence of moments as compared to that of axial loads, is large in upper floor than in lower floor columns, since the moments proper are usually of about the same magnitude, while the loads are the largest on the lower floor columns.

(ii) Detailed Design of the Framed Structure

Design of a framed structure is carried out in order to provide for equilibrium of each beam, sidesway equilibrium of each storey, and rotational equilibrium of each joint. In order to achieve these ends, moments and other forces acting on the frame are computed and design of the members is carried out on the basis of elastic theory. Design of R. C. structural members using this theory is well known, and needs no description except that under modern trend of design, measures are devised to work out most economical sizes for various members of the frame like columns, beams and slabs. When unlimited section of a column is permitted it is cheaper to use the minimum percentage of steel permissible which is 0.8% of the core area of the column with a normal

concrete strength of 2500 lb/sq. in. When the size of the column is restricted a stronger concrete may be used and the percentage of steel used may be anything but not exceeding the maximum limit of 8%. Generally in ordinary building construction Tee and Ell beams are used and strength of concrete in compression is made adequate, as in practice it is uneconomical to use compressive steel. In slabs also all compressive stresses are carried by the concrete. Two-way reinforced slabs have a much greater percentage of steel than one way slabs but such slabs are always economical provided the conditions permit their use. This is so because in one way slab there is 20% distribution steel which does not carry any load but is usefully utilized in the two way designed slab.

In the case of framed structure special attention is always given to its foundation. Exploration of site is carried out to a considerable depth and the interpretation of the bearing capacity is carried out very carefully depending on soil properties, ground water conditions, and foundation characteristics, the allowable bearing pressure varies also with the sensitivity of the structure to deformation, and in the case of framed structure this is usually less than the safe bearing capacity of the soil. While the foundations of flexible structures can frequently be designed on the basis of independent footings since movements are either independent of the load or have little effect on the stresses in the structure, rigid structure requires estimation of the probable total, as well as differential movements due to the foundation loads. Foundations for such structure are, therefore, laid as a raft or group of piles to ensure that there is no differential settlements, and if there is some it is not much effective.

Tall multi-storeyed buildings are usually provided with a structural frame consisting of beams and columns capable to resist lateral force of earthquake. All portions of the building are firmly tied together, and is stiffly braced so that it tends to move as a unit. The approximate coincidence of the central mass, and centre of rigidity is desirable. If this is not possible consideration of due rotation is taken into account. Foundations are laid rigidly interconnected so as to be able to transmit lateral seismic forces with the least deformation. Concrete floors act as horizontal diaphragms which distribute the lateral seismic forces to the frames. An independent section of a building is restricted to maximum lengths of 80' with a proportion of length to breadth as 3. Where "crumple" sections are provided to deal with lack of symmetry in building, a complete separation of the parts are made except on a level below the beams of the footings. Where the "crumple" joint is owing to temperature stresses, the foundation beams and footings are continued. Stairways when built so that they act as diagonal braces between the connected floors are liable to be damaged by earthquake movement. To avoid such damage stairways connecting structural elements are very carefully designed so that inter-

connection of adjacent floors by means of stairs are provided with sliding joints to eliminate the bracing effect or the construction is made strong enough to transmit shear between adjacent floors.

In framed structure depth of girders has to be kept very low to avoid the increase in floor height. To achieve this end prestressed concrete is sometime employed for the design of beams. In such cases, the structure below ground level is entirely built of ordinary reinforced concrete and is designed as a base for the superstructure frame which generally consists of single span prestressed beams at each floor level cast in situ or precast between R. C. columns. The beams are cored for a system of post tensioning the cables. Full continuity between the columns and beams is provided for, as the cables are so prestressed that no rotation of the beams relative to the columns occur during prestressing. The sizes of the cables vary at each floor level according to the moment distribution throughout the frame.

(iii) Illustration of Stress Analysis and Design of Framed Structure

The design of Lahore Polytechnic was carried out on the basis of the procedure laid down in this paper. To illustrate the various methods in detail an unsymmetrical frame of one wing of the building consisting four storeys, with basement, has been selected. The storey height is 12 feet. From the preliminary calculations of the design, sections of girders and columns were computed. The value of stiffness factor, distribution factor, displacement factor and fixed end moment at each of loaded members are worked out as per table given.

The stress analysis without sidesway has been carried out by the method of "Hardy Cross Moment Distribution". In this method the unbalanced moment at each joint is calculated and is multiplied by the distribution factor of each member. The value thus obtained is written under F. E. M. with opposite sign as that of unbalanced moment which process is called balancing. Distribution factor is obtained by dividing stiffness of the member by the sum of stiffness for all members at a joint. Moments equal to half the balancing moments are induced at the adjacent fixed joint with the same sign. This operation is known as carry over. The moment carried over represents an unbalanced state which must be eliminated by a balancing operation. Balancing and distribution are repeated as many times as necessary in order to reduce the residual moment to a negligible amount. The moments appearing at the ends of each member is added algebraically to obtain the final moments.

The frame under investigation is subjected to unsymmetrical loading and moments are, therefore, developed due to sidesway. These moments have been balanced by "factor method" as per tables given. For

each joint "girder factor" 'g' and column factor "(1-g)" are computed and written at the near ends of the members *i.e.* girders and columns. To each of the above figures is added half of the number at the other end. Then sum of above figures at each end of member is multiplied by the 'K' value of that member. The relative value thus obtained for columns are equated to the total horizontal force of that storey multiplied by storey height and by statics of equilibrium of each joint moments are obtained.

Dr. Kani's method has been employed in order to analyse the frame subject to seismic load as per tables given. In this method, fixed end moments of each member is written at the end of each member, and unbalanced moments are entered within the circle at each joint. These unbalanced moments are multiplied by the corresponding distribution factors to give a first approximation, and these values are written against the relevant member under the F.E.M. values. The displacement is calculated by adding algebraically the column moments at both ends of each column, and multiplying it with corresponding displacement factors for each column. Far-end moments of all members meeting at a joint are added algebraically to the unbalanced moments of the joint along with the displacement moments. The process is repeated from joint to joint till two consecutive approximation do not show any appreciable change. The calculation is then stopped. The net final moments are found by algebraically adding F.E.M. the value of last approximation in the above process and the sum of near and far end moments.

From the moment distribution method it transpires that maximum support bending moment on a ground floor beam is 67.1 T. ft. against 66.0 T.ft. adopted in preliminary design. As regards columns moments, it transpires that there is no sidesway due to unbalanced moments as worked out on page 33. After balancing the moments by "Factor Method" the maximum moment to which the beam is subjected is $70-26$ T. ft. The additional B.M. to which beam is subjected is added to the moments obtained from moment distribution method after correcting for internal sway, so that total moment on beam is equal to $-70.26+14.32=55.94$ T.ft. for which the beam has been designed. The elastic shear to which beam is subjected due to moments obtained by moment distribution after correcting for internal sway is $\frac{70.26-33.71}{26} = \frac{36.35}{26}$ = 1.4 tons. The additional shear due to transverse loading is $\frac{16.40-26.35}{26}$ = $\frac{-9.95}{26} = -0.38$ T.ft. The total shear is, therefore, $1.4-0.38=1.02$ tons. The

mid-span B. M. in the beam $\frac{2646 \times 26^2}{8 \times 2240} - \frac{70.26 + 33.71}{2} = 99.5 - 52.00 = 47.5$ T.ft.

which is less than 63.86 calculated above, so it is not considered in design. Similarly the maximum B.M. in a column worked out by moment distribution method and after correcting it for widesway by factor method is -23.33 T. ft. The additional B.M. in this column due to seismic load worked out by Kani's method is -8.33 . Hence total moment in the column is equal to $-23.33 - 8.33 = -31.66$ T. ft for which column has been designed.

VI. ECONOMICS OF PLANNING AND DESIGNING FRAMED STRUCTURE

The technique of building construction has made very rapid progress in the recent past. With larger and higher structures coming into existence, emphasis has shifted in this country from the simple load bearing construction to the framed reinforced concrete structure, because of the consequent economy in space and cost, and structural rigidity. The design of a framed structure broadly falls under two classifications *i.e.* (i) partially framed structure (ii) fully framed structure. The adoption of fully load bearing walls becomes difficult beyond a height of five storeys because of the difficulty of accommodating footings. The thicker walls also increase the cost and reduce considerably the floor space available. From three to five storeys partially framed type is the most economical because beyond this again difficulty of laying the foundation of the load-bearing walls is encountered. For building with five or more storeys a fully framed construction is always more advantageous. The problem of evolving an economic design for building under a set of conditions may be classified as (a) to decide whether a structure should be of a simple load bearing type or partially framed, or a fully framed reinforced type. (b) If a fully framed construction is decided what should be the framing plan of the building. Assuming that a fully framed R. C. construction has been decided upon, it should be borne in mind that the distribution of load on the beams and columns of a building is affected by the relative sizes of the panels and by the arrangement of columns. It therefore follows, that at least for one arrangement of the columns the frame will work out to be the most economical. The type of economical frame generally consists of a well defined corridor with rooms flanking it on either side. To evolve a suitable framing plan to be consistent with architectural and functional requirements endeavour must be made to make it suitable from the point of view of accommodation, convenience and comfort.

Cost of frame work in one typical bay of Lahore Polytechnic has been worked out in detail assuming that the building is not subjected to seismic load for the sake of comparison. The frame adopted has panels of 34 ft. span at

8'-6" centre to centre with a floor height of 12 feet. The plinth area rate for a five storeyed framed building works out to be Rs. 25.50 per sft against Rs. 26.00 per sft for load bearing walls. From this comparison of cost it may also be inferred that the plinth area rate of fully load bearing type of building increases with the number of storeys, and it becomes uneconomical for three or more storeys when compared with the partially or fully framed type. In spite of the many studies that have been made there still remains a wide diversity of professional opinion as to whether or not a rigid framed type of construction is likely to show any economy of material as compared to an alternative type of simple structure. However the question of economic advantage may be summed up conservatively by saying that other things being equal, there is no good reason to accept that an indeterminate framework will show much economy in material, that in many cases it will be at a clear disadvantage in this regard, but that in other cases especially favourable to it, it will be more economical than any other type practically feasible.

Since in any statically indeterminate structure a member cannot change lengths nor a support shift its relative position without setting up stresses throughout the structure it is clear that the effects of inaccuracies in the lengths of members due to changes in temperature and settlement of foundations require very careful consideration. There can be no doubt that its sensitiveness to such effects constitutes a valid general criticism against rigid framed construction, but it must not be forgotten, that the importance of these effects varies widely with different conditions.

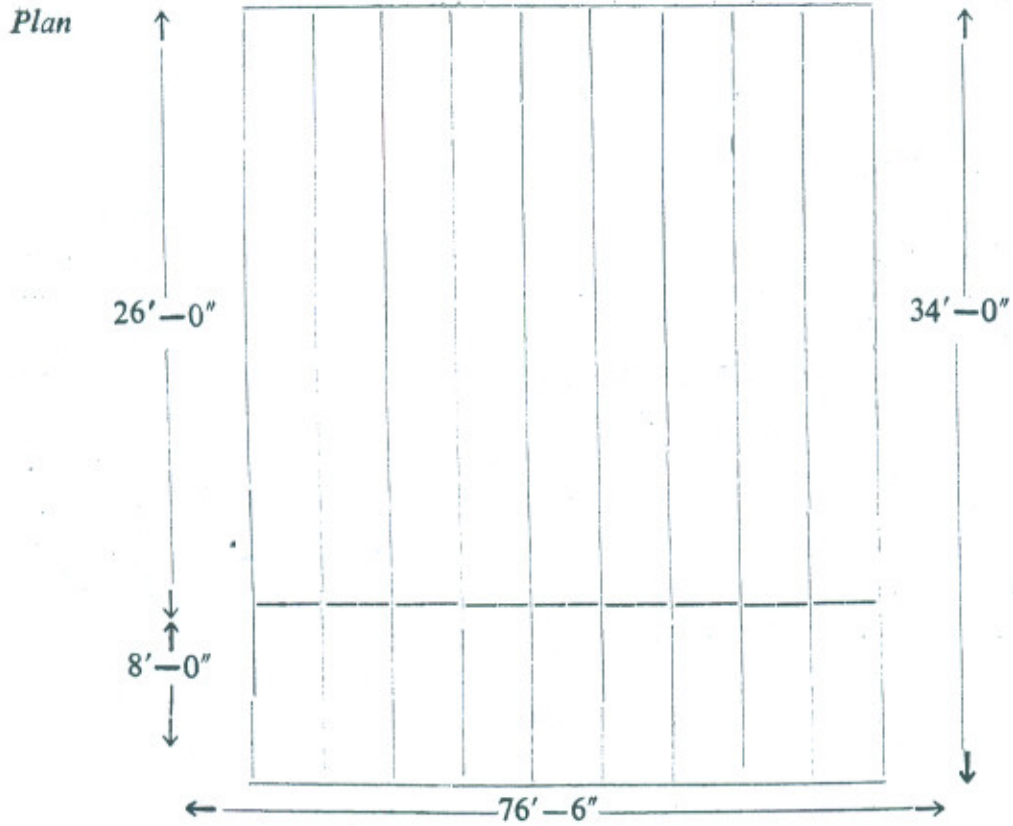
Of the inherent disadvantages of statically indeterminate construction, it is possible that none has made more weight in influencing professional opinion than the fact that the analysis of the stress is a very much more difficult and time-consuming task than in the case for a simple structure; yet when considered rigidly on its merits this objection in general has little to support it. The amount of time and expense involved in making the stress calculations for any structure of considerable magnitude is an exceedingly small item in the entire engineering of the structure. An expert computer will hardly require more than a few weeks to complete it. Moreover the frame analysis may be cut short and carried out floor-wise by the method of moment distribution neglecting vertical carry over moments which are generally small. Also the size of the structural members may be easily fixed without many trials so that maximum allowed stress is produced in them, which is very essential for ensuring a uniform standard of economy.

VII

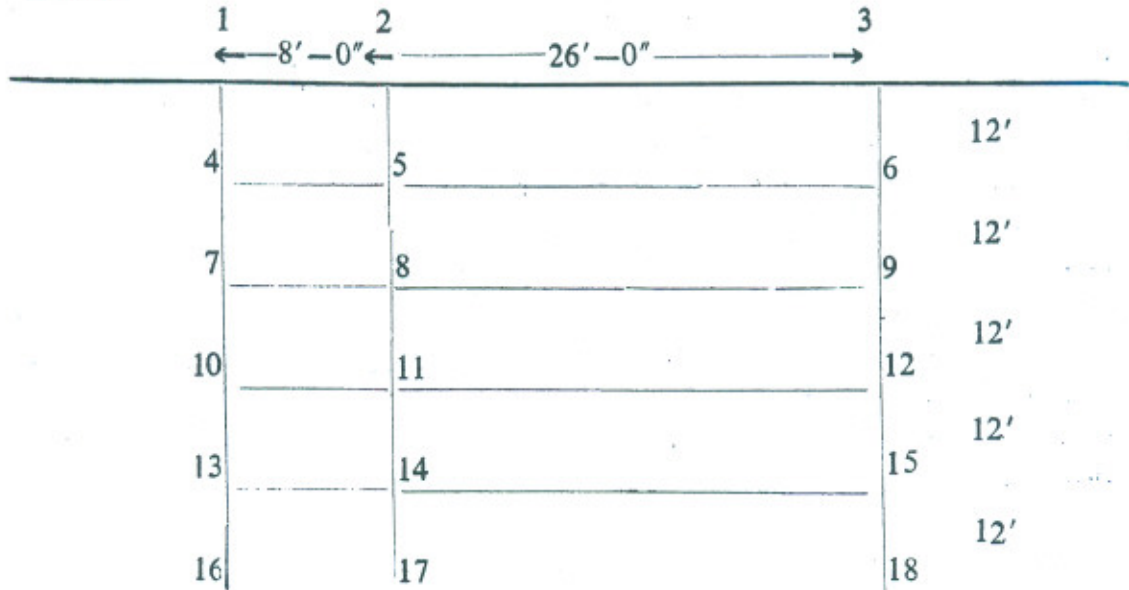
DETAILED CALCULATIONS OF STRESS ANALYSIS

Frame Under Consideration

← 9 Spans of 8'-6" each →



Section



1. PRELIMINARY DESIGN

(a) Assume sizes of various members.

(i) Girders:—18'' × 28'' Basement, ground floor.

14'' × 28'' 1st, 2nd and 3rd floor.

(ii) Columns:—18'' × 18'' Basement, ground floor.

14'' × 14'' 1st, 2nd and 3rd floor.

(iii) Slab:—4''

(b) Loading:

(i) Roof Slab:—Teli = 20 lbs./□'

Earth = 33 lbs./□'

Slab = 48 lbs./□'

Live = 30 lbs./□'

Total ... 130 lbs./□'

(ii) Inter floor Slab

Topping = 18 lbs./□'

Lean cone = 24 lbs./□'

Slab = 48 lbs./□'

Live = 80 lbs./□'

Total .. 170 lbs./□'

(iii) Beams.

Roof beam = shorter span = $130 \times 8.5 + 14 \times 24 = 1436$ lbs./ft.Longer span = $130 \times 8.5 + 14 \times 24$

= 1436 lbs./ft.

Interfloor beam = shorter span = $170 \times 8.5 + 14 \times 24$

= 1776 lbs./ft.

Longer span = $1776 + 870 = 2646$ lbs./ft.

(including the weight of 9'' wall)

Loading on all interfloor beams are similar.

(iv) Columns.

Intermediate column.—Area of loading = $8.5(13+4) = 145$ sq. ft.

(a) Dead Loads :

(i) 3rd floor to roof, $100 \times 145 = 14500$ lbs.

Column = $14 \times 14 \times 11.67 = 2290$ lbs.

Wall (external) = $1480 \times 7.33 = 10820$ lbs.

Wall (partition) = $870 \times 13 = 11310$ lbs.

Beam (main) = $\frac{14 \times 24 \times 144 \times 17}{144} = 5700$ lbs.

Beam (secondary) = $\frac{14 \times 12 \times 144 \times 7.33}{144} = \frac{1230}{45850}$ lbs.

(ii) 2nd floor to 3rd floor.

Floor = $90 \times 145 = 13100$

Col. Wall Beam = $\frac{31340}{44440}$ lbs. — 90280 lbs.

(iii) 1st floor to 2nd floor same as (ii) 134720 lbs.

(iv) Ground floor to 1st floor same as (ii) 179160 lbs.

(v) Basement same as (ii) 223600 lbs.

(b) Live Load :

(i) 3rd floor to roof = $30 \times 145 = 4350$ lbs.

(ii) 2nd floor to 3rd floor = $80 \times 145 + 4350 = 15950$ lbs.

(iii) 1st floor to 2nd floor = $0.9(4350 + 2 \times 11600) = 24800$ lbs.

(iv) Ground floor to 1st floor = $0.8(4350 + 3 \times 11600) = 31300$ lbs.

(v) Basement = $0.7(4350 + 4 \times 11600) = 35500$ lbs.

Total load = $223600 + 35500 = 259100$ lbs.

(c) F. E. Moments.

Roof Beam. Shorter span = $\frac{1436 \times 8^2}{12 \times 2240} = 3.4$ Tft.

$$\text{Longer span} = \frac{1436}{2240} \times \frac{26^2}{12} = 36.2 \text{ Tft.}$$

Inter floor beams

$$\text{Shorter span} = \frac{1776}{2240} \times \frac{82}{12} = 4.5 \text{ Tft.}$$

$$\text{Longer span} = \frac{2646}{2240} \times \frac{26^2}{12} = 66.0 \text{ Tft.}$$

(a) Checking of assumed sizes

(i) F. E. M. of Inter floor (ground floor) beam = 66 Tft.

Breadth of beam = column size = 18"

$$d = \frac{\sqrt{66 \times 2240 \times 12}}{18 \times 126} = \sqrt{780} = 28''$$

(ii) Column load = 259100 lbs.

$$\text{Area of conc. required} = \frac{259100}{680} = 280 \text{ sq. inch.}$$

Take 18'' × 18'' column = 324 sq. inch.

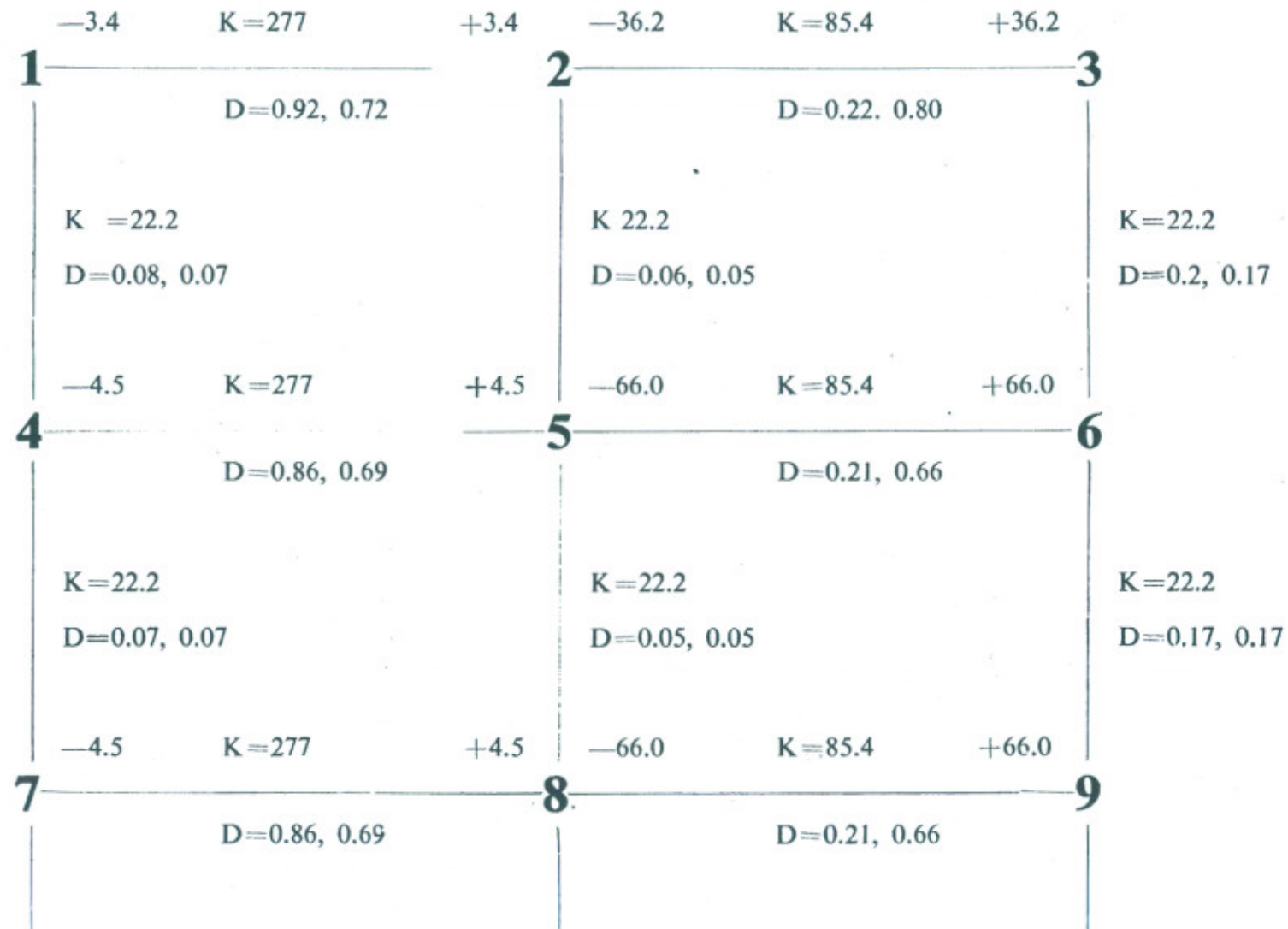
(iii) Slab load = 170 lbs./sq.

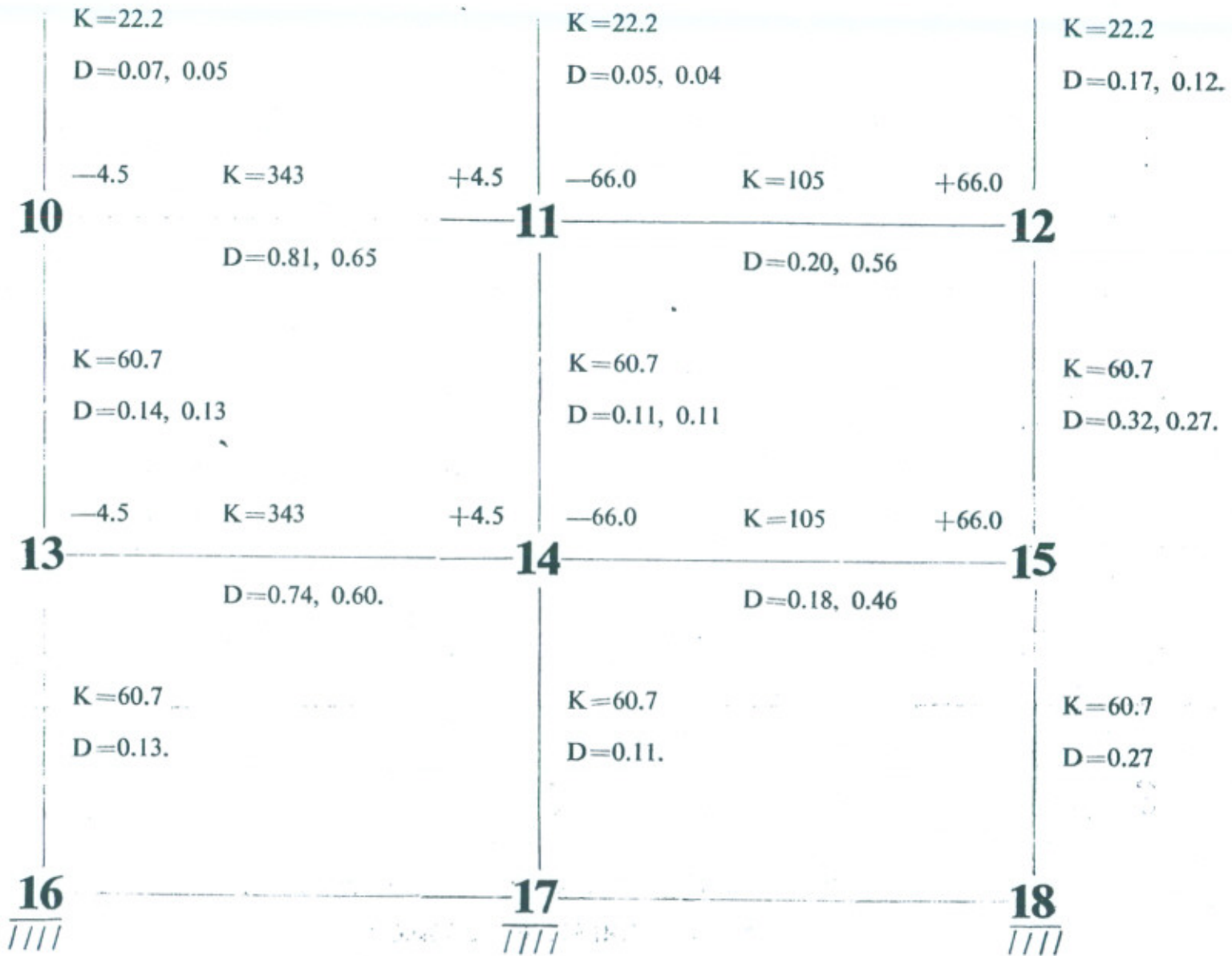
$$\text{B.M.} = \frac{170 \times 8.5}{10} = 1230 \text{ lbs. ft.}$$

$$d = \sqrt{\frac{1230}{126}} = \sqrt{98} = 3.2''$$

D = 4''

VALUES OF F.E.M 'K' AND 'D' VALUES OF DIFFERENT MEMBERS OF THE FRAMES





MOMENTS DISTRIBUTION
(For max. m Support mt. of Beams—all Spans loaded)

1	0.92	0.72	2	0.22	0.80	3
0.08			0.06			0.20
+0.3	— 3.4	+ 3.4	+ 2.0	—36.2	+36.2	— 7.2
+0.2	+ 3.1	+23.6	+ 1.6	+ 7.2	—29.0	— 5.6
—1.0	+11.8	+ 1.6	+ 0.7	—14.5	+ 3.6	+ 0.4
—0.8	—11.0	+ 8.1	+ 0.4	+ 2.5	+ 1.6	+ 0.2
—0.3	+ 4.0	— 5.5	+ 0.3	+ 0.8	+ 1.2	+ 0.3
—0.1	— 2.9	+ 3.1	+ 0.2	+ 0.9	— 1.1	— 0.2
—	+ 1.6	— 1.4	—	— 0.5	+ 0.4	—
—1.7	+ 3.2	+32.9	+ 5.2	—39.8	+12.9	—12.1
—1.9	—	—	+ 5.9	—	—	—14.4
—0.1	—	—	+ 0.1	—	—	+ 0.1
—0.3	—	—	+ 0.4	—	—	— 0.4
—0.5	—	—	+ 0.3	—	—	+ 0.2
—1.5	—	—	+ 0.9	—	—	+ 0.5
+0.2	—	—	+ 1.0	—	—	— 3.6
+0.3	—	—	+ 3.2	—	—	—11.2

0.07 4 0.07	0.86	0.69	0.05 5 0.05	0.21	0.66	0.17 6 0.17
+0.3	4.5	+ 4.5	+ 3.1	-66.0	+66.0	-11.2
+0.2	+ 3.9	+42.4	+ 1.6	+12.8	-43.6	- 5.6
-1.5	+21.2	+ 2.0	+ 0.8	-21.8	+ 6.4	+ 0.5
-0.7	-18.6	+11.9	+ 0.4	+ 3.6	+ 1.8	+ 0.3
-0.3	+ 5.9	- 9.3	+ 0.4	+ 0.9	+ 1.8	- 0.4
-0.1	- 4.1	+ 5.3	+ 0.2	+ 1.6	- 1.5	- 0.2
	+ 2.7	- 2.0		- 0.7	+ 0.8	
-2.1	+ 6.5	+54.8	+ 6.5	-69.6	+31.7	-16.6
-2.2			+ 6.6			-16.6
-0.1			+ 0.2			- 0.2
-0.3			+ 0.4			- 0.5
-0.8			+ 0.4			+ 0.3
-1.5			+ 0.8			+ 0.6
+0.2			+ 1.6			- 5.6
+0.3			+ 3.2			-11.2

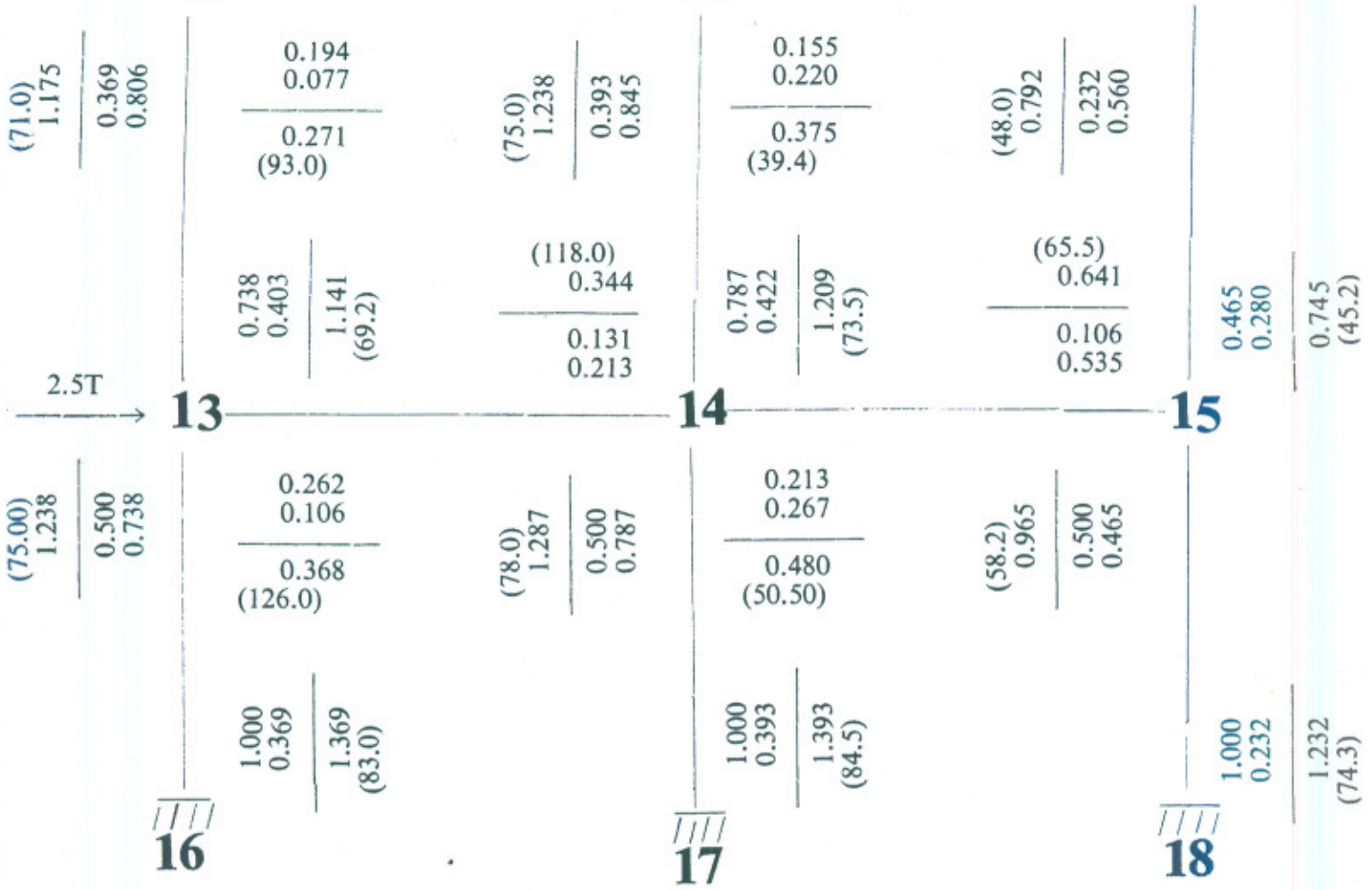
MOMENTS DISTRIBUTION—Contd.

0.07 7 0.07	0.86	0.69	0.05 8 0.05	0.21	0.66	0.17 9 0.17
+0.3	- 4.5	+ 4.5	+ 3.1	-66.0	+66.0	-11.2
+0.1	+ 3.9	+42.4	+ 1.3	+12.8	-43.6	- 4.0
-1.5	+21.2	+ 2.0	+ 0.8	-21.8	+ 6.4	+ 0.5
-0.5	-18.5	+11.7	+ 0.2	+ 3.6	+ 2.1	+ 0.5
-0.3	+ 5.8	- 9.2	+ 0.4	+ 1.0	+ 1.8	- 0.4
-0.1	- 3.9	+ 5.2	+ 0.1	+ 1.6	- 1.7	- 0.1
	+ 2.6	- 1.9		- 0.8	+ 0.8	
-2.0	+ 6.6	+54.7	+ 5.9	-69.6	+31.8	-14.7
-1.6			+ 5.5			-12.8
-0.1			+ 0.2			- 0.2
-0.1			+ 0.3			- 0.2
-0.8			+ 0.4			+ 0.2
-1.0			+ 0.5			+ 0.9
+0.2			+ 1.6			- 5.6
+0.2			+ 2.5			- 7.9

0.05 10 0.14	0.81	0.65	0.04 11 0.11	0.20	0.56	0.12 12 0.32
+0.6	- 4.5	+ 4.5	+ 6.7	-66.0	+66.0	-21.1
+0.3	+ 3.7	+40.0	+ 3.4	+12.3	-37.0	- 8.9
-2.0	+20.0	+ 1.9	+ 1.2	-18.5	+ 6.2	+ 2.7
-1.2	-16.6	+ 7.4	+ 0.6	+ 2.3	+ 4.7	+ 0.7
-0.2	+ 3.7	- 8.3	+ 0.5	+ 2.3	+ 1.1	- 0.7
-0.1	- 1.4	+ 3.2	+ 0.3	+ 1.0	- 1.1	- 0.2
	+ 1.6	- 0.7		- 0.5	+ 0.5	
-2.6			+12.7			-27.5
	+ 6.5	+48.0		-67.1	+40.4	
-2.6			+12.9			-26.5
-0.1			+ 0.3			- 0.3
-0.2			+ 0.6			- 0.5
-1.0			+ 0.6			+ 1.3
-2.4			+ 1.2			+ 1.4
+0.3			+ 3.4			-10.6
+0.6			+ 6.8			-17.8

MOMENTS DISTRIBUTION—*Concl'd.*

0.13 13 0.13		0.60 14 0.11		0.46 15 0.27	
0.74		0.18			
+0.6	— 4.5	+ 4.5	+ 6.8	—66.0	+66.0
..	+ 3.3	+ 36.9	..	+11.0	—30.4
—2.4	+18.5	+ 1.7	+ 1.1	—15.2	+ 5.5
..	—14.0	+ 6.0	..	+ 1.8	+ 2.3
—0.3	+ 3.0	— 7.0	+ 0.6	+ 1.1	+ 0.9
..	— 1.5	+ 3.2	..	+ 0.9	— 1.0
—2.1	+ 1.6	— 0.7	+ 8.5	— 0.5	+ 0.4
— 1.1	+ 6.4	+44.6	+ 4.3	—66.9	+43.7
— 1.1			+ 0.3		— 8.2
— 1.2			+ 0.6		— 0.3
+ 0.3			+ 3.4		+ 0.7
16			17		18



$$\begin{array}{r} (75.00) \\ 1.238 \\ \hline 0.500 \\ 0.738 \end{array}$$

$$\begin{array}{r} (71.0) \\ 1.175 \\ \hline 0.369 \\ 0.806 \end{array}$$

$$\begin{array}{r} 0.262 \\ 0.106 \\ \hline 0.368 \\ (126.0) \end{array}$$

$$\begin{array}{r} 1.000 \\ 0.369 \\ \hline 1.369 \\ (83.0) \end{array}$$

$$\begin{array}{r} 0.738 \\ 0.403 \\ \hline 1.141 \\ (69.2) \end{array}$$

$$\begin{array}{r} (78.0) \\ 1.287 \\ \hline 0.500 \\ 0.787 \end{array}$$

$$\begin{array}{r} (118.0) \\ 0.344 \\ \hline 0.131 \\ 0.213 \end{array}$$

$$\begin{array}{r} (75.0) \\ 1.238 \\ \hline 0.393 \\ 0.845 \end{array}$$

$$\begin{array}{r} 0.213 \\ 0.267 \\ \hline 0.480 \\ (50.50) \end{array}$$

$$\begin{array}{r} 1.000 \\ 0.393 \\ \hline 1.393 \\ (84.5) \end{array}$$

$$\begin{array}{r} 0.787 \\ 0.422 \\ \hline 1.209 \\ (73.5) \end{array}$$

$$\begin{array}{r} 0.155 \\ 0.220 \\ \hline 0.375 \\ (39.4) \end{array}$$

$$\begin{array}{r} (58.2) \\ 0.965 \\ \hline 0.500 \\ 0.465 \end{array}$$

$$\begin{array}{r} (65.5) \\ 0.641 \\ \hline 0.106 \\ 0.535 \end{array}$$

$$\begin{array}{r} (48.0) \\ 0.792 \\ \hline 0.232 \\ 0.560 \end{array}$$

$$\begin{array}{r} 1.000 \\ 0.232 \\ \hline 1.232 \\ (74.3) \end{array}$$

$$\begin{array}{r} 0.465 \\ 0.280 \\ \hline 0.745 \\ (45.2) \end{array}$$

FACTOR METHOD FOR LATERAL LOADING

			(27.7) 0.100		(19.7) 0.230	
			0.040		0.030	
			0.060		0.200	
1.75T	1		2		3	
(30.1) 1.351		0.080		0.060		
0.431		0.030		0.100		
0.920		0.110		0.160		
		(30.5)		(13.7)		
			(30.8) 1.385		(25.1) 1.130	
			0.445		0.330	
			0.940		0.800	
			(49.3) 0.178		(33.6) 0.394	
			0.069		0.054	
			0.109		0.340	
			0.891		0.660	
			0.470		0.400	
			1.361		1.060	
			(30.2)		(23.5)	
2.5T	4		5		6	
(28.7) 1.293		0.138		0.109		
0.431		0.054		0.170		
0.862		0.192		0.279		
		(53.0)		(23.8)		
			(29.6) 1.336		(22.0) 0.990	
			0.445		0.330	
			0.891		0.660	
			(49.3) 0.178		(33.6) 0.394	
			0.069		0.054	
			0.109		0.340	
			0.891		0.660	
			0.445		0.330	
			1.336		0.990	
			(29.6)		(21.9)	
2.5T	7		8		9	
(28.0) 1.265		0.138		0.109		
0.403		0.054		0.170		
0.862		0.192		0.279		
		(53.0)		(23.8)		
			(29.2) 1.313		(20.9) 0.940	
			0.422		0.280	
			0.891		0.660	
			(86.1) 0.252		0.517	
			0.097		0.077	
			0.155		0.440	
			0.845		0.560	
			0.445		0.330	
			1.290		0.890	
			(28.6)		(19.7)	
2.5T	10		11		12	

INTERNAL SWAY DUE TO UNBALANCED MOMENTS BY
FACTOR METHOD

(1) Unbalanced moment in 4th Storey :

$$-1.7-1.9+5.2+5.9-12.1-14.4=-19.0.$$

(2) 3rd Storey = $-21-2.2+6.5+6.6-16.6-16.6=-24.4.$

(3) 2nd Storey = $-2.0-1.6+5.9+5.5-14.7-12.8=-19.7.$

(4) 1st Storey = $-2.6-2.6+12.7+12.9-27.5-26.5=-33.2.$

(5) Basement = $-2.1-1.1+8.5+4.3-16.9-8.2=-15.5$

RATIO OF ACTUAL AND RELATIVE MOVEMENTS OBTAINED BY
FACTOR METHOD

4th Storey = $30.1+29.5+30.8+30.2+25.1+23.5=169.2$

$$-\frac{19}{169.2}=-0.112.$$

3rd Storey = $28.7+28.7+29.6+29.6+22.0+21.9=160.5$

$$-\frac{24.4}{160.5}=-0.151.$$

2nd Storey = $28.0+27.4+29.2+28.6+20.9+19.7=154.2$

$$-\frac{19.7}{154.2}=-0.127.$$

1st Storey = $71.0+69.2+75.0+73.5=48.0+45.2=381.9.$

$$-\frac{33.2}{381.9}=-0.087.$$

Basement = $75.0+83.0+78.0+84.5+58.2+74.3=453.0$

$$-\frac{15.5}{453.0}=-0.034.$$

Factor Method—*Joint 1*

$$g_{12} = \text{Girder Factor} = \frac{22.2}{299.2} = 0.08.$$

$$c_{14} = \text{Column Factor} = 1 - 0.08 = 0.92.$$

$$\text{Relative Girder Moment} = 0.11 \times 277 = 30.5$$

$$\text{Relative Column Moment} = 1.351 \times 22.2 = 30.1$$

Joint 2

$$g_{21} = \frac{22.2}{384.6} = 0.06.$$

$$g_{23} = \frac{22.2}{384.6} = 0.06.$$

$$c_{25} = 1 - 0.06 = 0.94.$$

$$\text{Relative Girder}_{21} \text{ Moment} = 0.10 \times 277 = 27.7.$$

$$\text{Relative Girder}_{23} \text{ Moment} = 0.16 \times 85.4 = 13.7.$$

$$\text{Relative Column}_{25} \text{ Moment} = 1.385 \times 22.2 = 30.8$$

Joint 3

$$g_{32} = \frac{22.2}{107.6} = 0.2.$$

$$c_{36} = 1 - 0.2 = 0.8$$

$$\text{Relative Girder Moment} = 0.23 \times 85.4 = 19.7.$$

$$\text{Relative Column Moment} = 1.13 \times 22.2 = 25.1.$$




Similarly girder moments and column moments are calculated for all other joints.

DISTRIBUTED MOMENTS DUE TO INTERNAL SWAY TO BE ADDED IN ORIGINAL MOMENTS WITH
REVERSE SIGN

(Factor Method)

	+3.38	+2.31	+1.14	+2.82
-3.38			-3.45	-2.82
-3.3	+7.64	+5.32	-3.4	-2.64
-4.34			-4.5 +2.58	-3.33
-4.34	+7.91	+5.52	-4.5 +2.70	-3.32
-3.57			-3.72	-2.66
-3.5	+9.7	+7.0	-3.65 +3.16	-2.52
-6.2			-6.51	-4.17
-6.05	+8.61	+6.34	-6.4 +2.73	-3.94
-2.56			-2.67	-1.99
-2.84			-2.89	-2.54

FINAL MOMENTS WITH SWAY (BY FACTOR METHOD)

	-0.18	+30.59	-40.94	+10.08	
+1.68		+ 8.65		- 9.22	
+1.14	-1.14	+49.48	-72.18	+25.73	-11.76
+2.24		+ 11.0		- 13.27	
+2.14	-1.31	+49.18	-72.3	+25.82	-13.28
+1.57		+ 9.62		- 12.04	
+1.9	-3.2	+41.0	-70.26	+33.71	-10.28
+3.6		+ 19.21		- 23.33	
+3.45	-2.21	+38.26	-69.63	+37.77	-22.56
+0.46		+ 11.17		- 14.91	
+1.74		+ 7.19		- 5.66	
					

Seismic Loads**(a) Weight of top half Storey**

Column	$= 3/2 \times 2290$	$= 3440$
Wall (External)	$= 3/2 \times 10820$	$= 16200$
Wall (Partition)	$=$	$= 11300$
Beam (Main)	$= 2 \times 5700$	$= 11400$
Beam (Secondary)	$= 2 \times 1230$	$= 2460$
Roof	$= 115 \times 8.5 \times 34$	$= 33300$
	Total	$= 78100 \text{ lbs.} = 35 \text{ tons}$

Seismic Force top Storey $= 35/20 = 1.75 \text{ tons.}$

(b) Weight of One inter Storey

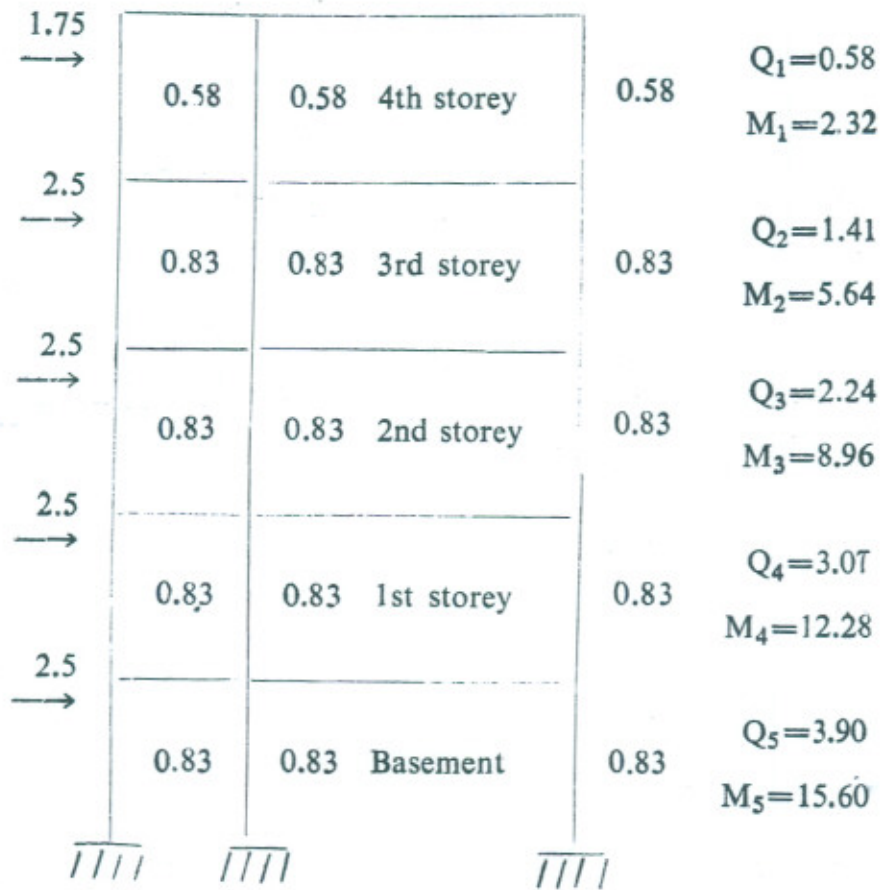
Column	$= 3 \times 2290$	$= 6890$
Wall (External)	$= 3 \times 10820$	$= 32460$
Wall (Partition)	$= 2 \times 11300$	$= 22600$
Beam (Main)	$= 2 \times 5700$	$= 11400$
Beam (Secondary)	$= 2 \times 1230$	$= 2460$
Floor	$= 130 \times 8.5 \times 34$	$= 37500$
	Total	$= 113310 \text{ lbs.} = 50.5 \text{ tons}$

Say 50 tons.

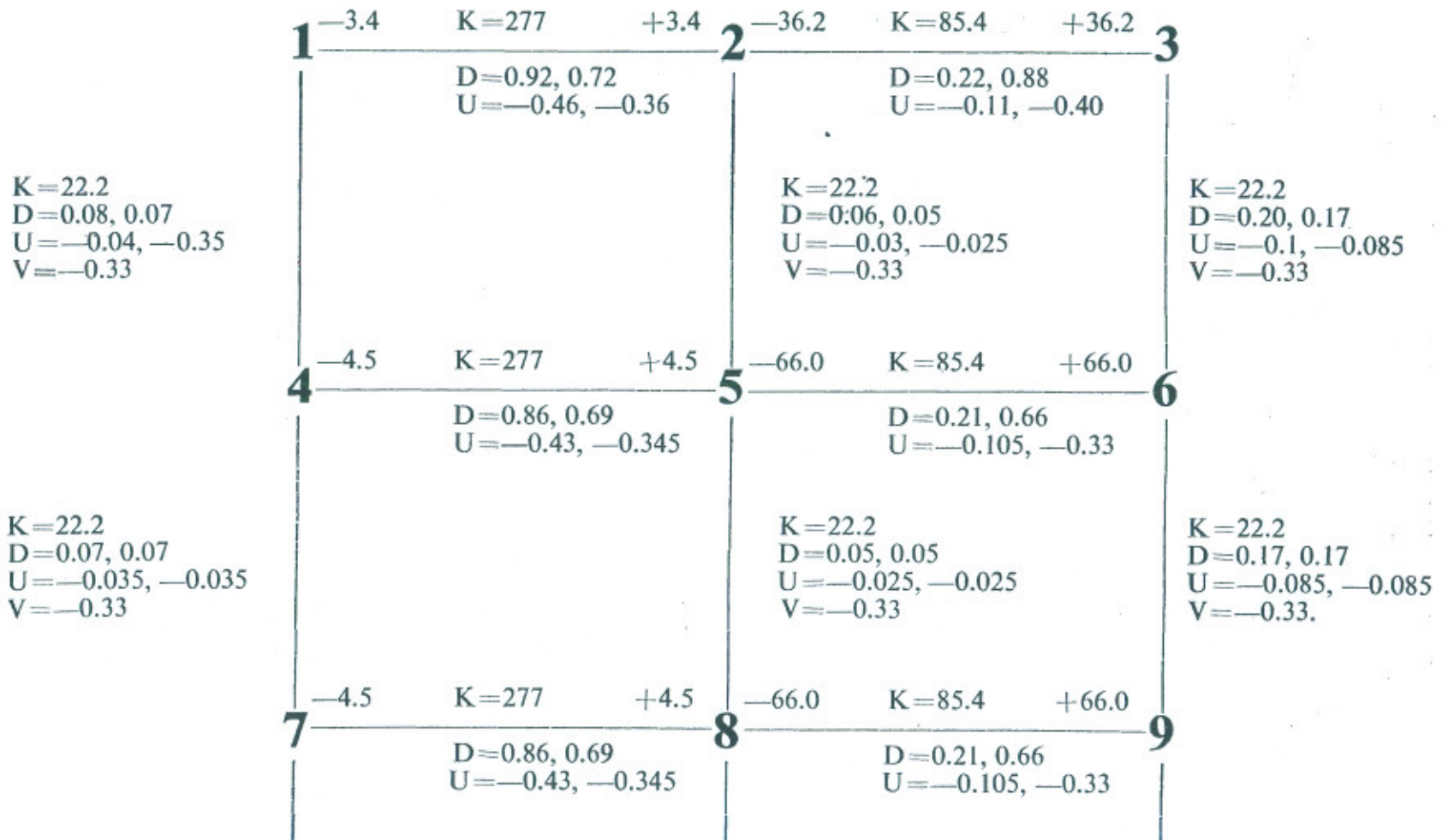
Seismic force inter floor $50/20 = 2.5 \text{ tons}$

STRESS ANALYSIS BY KANTS METHOD

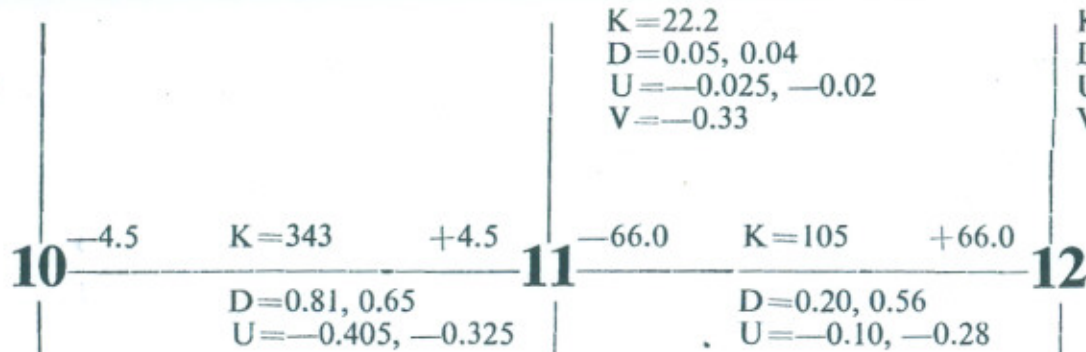
Seismic loads, horizontal shear and storey moments



STRESS ANALYSIS BY KANI'S METHOD (SEISMIC LOADS) VALUES OF K, D, U AND V OF DIFFERENT MEMBERS OF THE FRAME



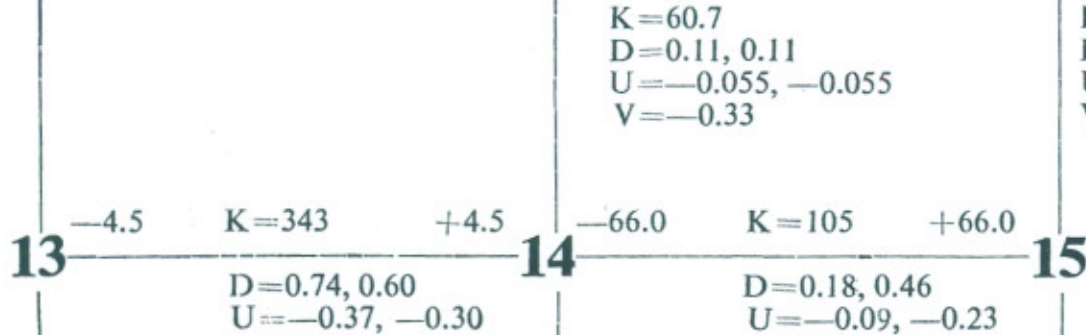
$K=22.2$
 $D=0.07, 0.05$
 $U=-0.035, -0.025$
 $V=-0.33$



$K=22.2$
 $D=0.05, 0.04$
 $U=-0.025, -0.02$
 $V=-0.33$

$K=22.2$
 $D=0.17, 0.12$
 $U=-0.085, -0.06$
 $V=-0.33$

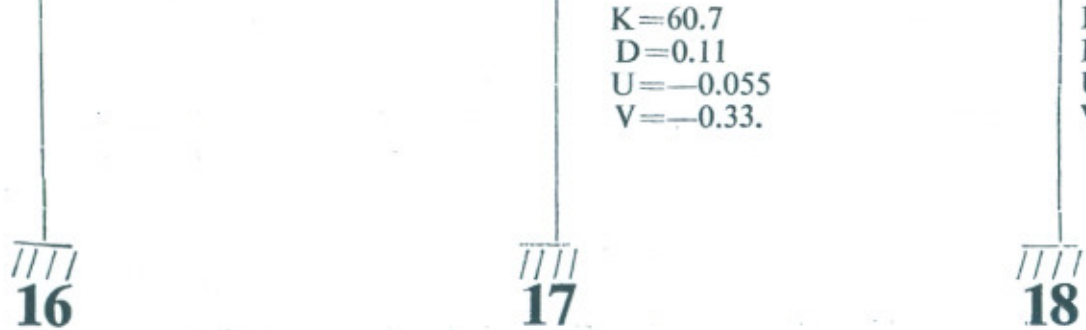
$K=60.7$
 $D=0.14, 0.13$
 $U=-0.07, -0.065$
 $V=-0.33$



$K=60.7$
 $D=0.11, 0.11$
 $U=-0.055, -0.055$
 $V=-0.33$

$K=60.7$
 $D=0.32, 0.27$
 $U=-0.016, -0.135$
 $V=-0.33$

$K=60.7$
 $D=0.13$
 $U=-0.065$
 $V=-0.33$



$K=60.7$
 $D=0.11$
 $U=-0.055$
 $V=-0.33$

$K=60.7$
 $D=0.27$
 $U=-0.135$
 $V=-0.33$

ANALYSIS OF STRESS BY KANI'S METHOD

	1 (0.0)		2 (0.0)		3 (0.0)
		+ 1.605	+ 1.254	+ 0.384	+ 1.395
		+ 1.3	+ 0.425	+ 0.13	+ 1.31
+ 0.141			+ 0.105		+ 0.348
+ 0.113			+ 0.036		+ 0.328
		- 3.32		- 3.32	- 3.32
2.32		- 4.39		- 4.39	- 4.39
+ 0.38			+ 0.19		+ 1.03
+ 0.30			+ 0.21		+ 0.72
	4 (0.0)		5 (0.0)		6 (0.0)
		+ 3.63	+ 2.94	+ 0.888	+ 2.79
		+ 4.70	+ 2.62	+ 0.8	+ 4.0
+ 0.30			+ 0.21		+ 0.72
+ 0.38			+ 0.19		+ 1.03
		- 8.46		- 8.46	- 8.46
5.64		- 10.05		- 10.05	- 10.05
+ 0.70			+ 0.36		+ 1.89
+ 0.471			+ 0.336		+ 1.14
	7 (0.0)		8 (0.0)		9 (0.0)
		+ 5.79	+ 4.62	+ 1.41	+ 4.41
		+ 8.60	+ 5.05	+ 1.54	+ 7.3

	+ 0.471 + 0.70		+ 0.336 + -0.36		+ 1.14 + 1.89
8.96		-13.44 -15.38		-13.44 -15.38	
	+ 0.785 + 0.459		+ 0.51 + 0.369	+ 2.0 + 1.11	
10 (0.0)			11 (0.0)		12 (0.0)
		+ 7.38 +12.5	+ 6.0 + 8.2	+ 1.842 + 2.51	+ 5.16 + 9.30
	+ 1.29 + 2.2		+ 1.014 + 1.38		+ 2.94 + 5.3
12.28		-18.42 -24.03		-18.42 -24.03	
	+ 2.73 + 1.53		+ 1.44 + 1.29		+ 6.10 + 3.15
13 (0.0)			14 (0.0)		15 (0.0)
		+ 8.64 +15.6	+ 7.02 + 7.85	+ 2.1 + 2.35	+ 5.37 +10.40
	+ 1.53 + 2.73		+ 1.29 + 1.44		+ 3.15 + 6.10
15.60		-23.40 -26.38		-23.40 -26.38	
16			17		18

ANALYSIS OF STRESSES BY KANI'S METHOD FINAL MTS.

	1					2					3
+ 0.11		+ 1.3	+ 0.42	+ 0.04	+ 0.13	+ 1.31				+ 0.328	
+ 0.49		+ 1.72	+ 1.72	+ 0.23	+ 1.44	+ 1.44				+ 1.35	
- 4.39				- 4.39						- 4.39	
		<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>				<u> </u>	
- 3.79		+ 3.02	+ 2.14	- 4.12	+ 1.57	+ 2.75				- 2.71	
- 3.52				- 3.97						- 2.0	
				<u> </u>						<u> </u>	
- 4.39				- 4.39						- 4.39	
+ 0.49				+ 0.23						+ 1.36	
+ 0.38				+ 0.19						+ 0.103	
	4					5					6
+ 0.38		+ 4.7	+ 2.62	+ 0.19	+ 0.8	+ 4.0				+ 1.03	
+ 1.08		+ 8.3	+ 8.3	+ 0.55	+ 4.8	+ 4.8				+ 2.92	
- 10.05				- 10.05						- 10.05	
		<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>				<u> </u>	
- 8.59		+ 13.0	+ 10.92	- 9.31	+ 5.6	+ 8.8				- 6.1	
- 8.27				- 9.04						- 5.24	
				<u> </u>						<u> </u>	
- 10.05				- 10.05						- 10.05	
+ 1.08				+ 0.55						+ 2.92	
+ 0.70				+ 0.36						+ 1.89	
	7					8					9
+ 0.7		+ 8.6		+ 0.36	+ 1.54	+ 7.3				+ 1.89	
+ 1.48		+ 13.6		+ 0.87	+ 8.84	+ 8.84				+ 3.89	
- 15.38				- 15.38						- 15.38	
		<u> </u>		<u> </u>	<u> </u>	<u> </u>				<u> </u>	
- 13.2		+ 22.2		- 14.38	+ 10.38	+ 16.14				- 9.6	

-13.44
 -15.38
 $+ 1.48$
 $+ 0.46$

10

-24.03
 $+ 2.2$
 $+ 4.9$
 -16.9

-16.4
 -24.03
 $+ 4.90$
 $+ 2.73$

13

-26.38
 $+ 5.46$
 -20.92

16

$+12.5$
 $+20.7$
 $+33.2$

$+ 8.2$
 $+20.7$
 $+28.9$

$+15.6$
 $+23.45$
 $+39.05$

$+ 8.75$
 $+23.45$
 $+31.3$

-14.0
 -15.38
 $+ 0.87$
 $+ 0.51$

11

$+ 1.38$
 $+ 2.82$
 -24.03
 -19.83

-19.77
 -24.03
 $+ 2.82$
 $+ 1.44$

14

$+ 1.44$
 $+ 1.44$
 -26.38
 -23.5

17

$+ 2.51$
 $+11.81$
 $+14.32$

$+ 2.35$
 $+12.75$
 -15.1

12

$+ 9.30$
 $+11.81$
 $+ 21.11$

15

$+10.40$
 $+12.75$
 $+23.15$

18

-10.38
 -15.38
 $+ 3.89$
 $+ 1.11$

$+ 5.30$
 $+11.40$
 -24.03
 $- 8.33$

$- 8.53$
 -24.03
 $+11.40$
 $+ 6.10$

$+12.2$
 -26.38
 -14.18

PROPOSED POLYTECHNIC INSTITUTE AT LAHORE
STUDY-CUM PRACTICAL AND ADMINISTRATION BLOCK.

SCALE 1"=32'

