SOME PRACTICAL MATHEMATICS OF CANAL WORKS.

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SOILS AND ALLOWABLE PRESSURES ON FOUNDATIONS.

Soils –

The soils met with in the Punjab and North-West Frontier Province are generally alluvial soils formed by the deltas of rivers and, in some cases, have resulted from deposition in lakes long since emptied. Soil formed by wind is readily distinguishable by its looseness, and would instinctively be discarded for foundations.

Where there have been prolonged washings of the soil by fresh water, such as rain or floods, the earthy salts have been largely dissolved out. Those soils in which they are still present in fair quantity are apt to become very soft when wet, and "well" or "spread" foundations will be advisable, although the soil may have an extremely firm appearance, when dry.

Foundations –

The materials usually met with in the foundations of cana works are as follows :—

(1) Loose sand, containin

(2) Firm sand of various ineness.

(3) Wet clay as soft as a b er pudding.

(4) Firm clay of various stiffnesses.

(5) Shingle, or shingle and boulders, admixed with sand.

(6) Shingle, or shingle and boulders, admixed with clay.

(7) Rocks of sedimentary formation.

An important point to bear in mind, in designing canal works, is that percolation from the canal above those works, may in course of time, sodden and soften foundations in clay.

Corthell's re ults—

Corthell in "Allowable pressures on deep foundations," gives data with regard to the foundations of a hundred and seventy-eight works, but admits that the figures supplied to him were in some cases wanting in detail as to whether the pressures were mean or maximum, or whether deductions had been made for the buoyancy of the surrounding water, or the frictional resistance of the sides of the structures. His results* are abstracted below :—

Loose wet sand, if not confined by curtains or sheet piling, will not bear 1·5 tons per square foot, but, if properly confined, will bear $2\frac{1}{2}$ tons per square foot.

Firm dry sand of fine texture will bear 4 tons per square foot. The sand met with in canal works must be regarded as wet, when there is percolation from above.

Dry coarse sand and gravel will bear $4\frac{1}{2}$ tons per square foot. In this case also the material met with in canal construction must be regarded as wet, and as more nearly resembling shingle. It may be noted that a shingle or boulder formation containing sand, is not so safe as one containing a filling of clay in the interstices, because the sand is apt to be carried out by outflowing drainage during construction, whereas the clay usually makes the formation water-tight. The writer has known cases in which foundation trenches twenty feet deep were dug through this formation, close alongside flowing water, without troublesome percolation.

Dry hard clay will bear $4\frac{1}{2}$ tons. Here again percolation from a canal would probably alter the bearing-power of the foundations in course of time.

Very hard clay will bear nearly 8 tons. The kind of clay referred to is, no doubt, that impacted with lime and in process of becoming argillaceous rock. Percolation is capable of softening this to a much less extent than oth[illegible]

With regard to foundations in clays, [illegible] orbent nature makes them treacherous, both from t[illegible] view of bearing power and of resistance against sliding. Clays absorb forty to sixty per cent. of water and the coefficient of friction taken in their case is only ·33.

Skin friction—

The skin friction offered by foundations of canal works seldom needs to be taken into account. Clay offers the least frictional resistance and sharp gravel the most. The friction may range from 200 lbs. to 1,500 lbs per square foot according

* (In English tons of 2,240 lbs).

to the depth of the structure, and the nature of the material. The skin friction in each case can always be determined from the load necessary to sink a well through the material in question.

Rankine—

Rankine deals with foundations on three classes of materials, *viz.* :—

(1) Rock.

(2) Hard clay, gravel or sand.

(3) Soft earth.

For Rock he takes a factor of safety of 8, and mentions that the ultimate crushing strength of sandstone varies from 9,824 lbs. per square inch for strong sandstones, to 3,000 lbs. for weak ones. Similar figures for limestones are 8,528 and 3,050 lbs. per square inch. Applying his factor of 8, we get safe loads of 80 to $18\frac{1}{2}$ tons per square foot for sandstone, and $68\frac{1}{2}$ to $25\frac{3}{4}$ tons per square foot for limestone.

For hard clay, gravel and sand, when safeguarded against water saturation by means of surface or subsoil drains, he recommends a maximum intensity of pressure of $1\frac{1}{2}$ to 2 tons per square foot. These are the pressures usually adopted for canal works founded on materials of that class.

For foundations in soft earth.—He mentions that the weights required to overcome frictional resistance in sinking iron cylinders for piers of bridges was found to be 336 lbs. per square foot in stiff clay, and 224 lbs. per square foot in sand, and stones in mud, but it may be assumed that it would not be permissible to give credit for frictional resistance in reduction of load on foundations, except in rare cases, where the foundations were quite secure from scour.

In certain cases, he recommends the substitution of more stable material, such as sand or concrete, by removing unstable material, and gives the following formula for calculating the thickness of this substituted material :—

$$\text{Depth of stable material required} = \frac{\text{Maximum pressure intensity} \times \left(\dfrac{1-\sin \text{L. of repose}}{1+\sin \text{L. of repose}}\right)^2}{\text{Unit weight of unstable material} - \text{Unit weight of stable material}\left(\dfrac{1-\sin \text{L. of repose}}{1+\sin \text{L. of repose}}\right)^2}$$

If unit weight of unstable material	=	80 lbs.
And unit weight of stable material	=	100 ,,
Maximum pressure intensity	=	2,000 ,,
∠ of repose of unstable material	=	·25° ,,

Then depth of stable material required $= \dfrac{2000 \times \cdot 165}{80 - 100 \times \cdot 165} = 5\cdot 2$ ft.

A table of values of the sine expressions in the above formula is given on page 380 of his Civil Engineering, and a modification of this formula is also useful in ascertaining whether a stratum of firm material (which is underlain by a soft one) is of sufficient thickness to support the designed structure.

Bligh—

Bligh in his "Practical design of Irrigation Works" states that even fine Nile silt will bear one ton per square foot, coarse sand two tons or over, and that on clay four tons is not exceptional.

Before leaving the subject of soils, their susceptibility to percolation under heads of water will be briefly discussed; because the point is of great importance in the design of canal works.

Bligh gives the following as the lengths of enforced percolation necessary under works founded upon different soils. For light silt, such as that of the Nile, the ratio of length of percolation to head up-stream, should not be less than 18 to 1:—

For fine sand, such as that in Himalayan rivers	15 to 1
For coarse sand, such as that in the Central Provinces	12 to 1
For boulders, shingle, gravel, and sand	from 5 to 1 to 9 to 1

It will be observed that the above ratios relate to various kinds of sand and gravel. Where the foundations of the work rest upon a sufficiently thick layer of *clay*, the percolation ratio is much less than any of the above, and 3 to 1 or 4 to 1 is usually taken in "made" banks, according to the staunchness of the clay. In compact, original soil the gradient may be as steep as $1\frac{1}{2}$ to 1. Boulders, shingle and clay provide a foundation which is one of the most staunch. It has already been mentioned that boulders, shingle and sand is not a good mixture for wet foundations.

Molesworth—

Molesworth * gives the following as safe examples of pressure on foundations :—

Clay ...	... 1·5 to 1·7	tons per	square foot.
Sandy clay ...	... 2·1 to 3·0	,,	,,
Unstable sand	... 1·8	,,	,,
Compact sand	... 2·3 to 4·0	,,	,,
Coarse gravel	... 4·4	,,	,,
Compact stoney clay ...	5·5	,,	,,
Compact sand and gravel	7·5	,,	,,

The foundations were *dry* in many of these instances, *i.e.*, not subject to constant percolation from a canal.

Woods—

Woods gives the following limiting intensities :—

Rock 8 to 12 tons per square foot.

Gravel and clay 2 tons per square foot.

Loamy soil 1 ton per square foot.

Summary—

Figures given by the various authorities greatly vary, but there is good evidence to show that certain soils will safely bear considerably heavier loads than it is the practice in the Irrigation Branch to place upon them : but careful discrimination is necessary, and the best guide would be a series of tests of the bearing power in each case.

Owing to the lightness of the majority of canal works the need of investigating the bearing power of foundations is not often felt. It is generally possible, with due regard to economy, to spread the footings sufficiently to reduce the intensity of pressure on foundations to moderate limits. In the case of heavy dams or regulators however, it pays to ascertain the actual bearing power of the foundations.

* Pocket Book, page 88.

STRENGTH OF MATERIALS.

Rankine—

Rankine gives the following as the safe loads per square foot :—

Sandstone (good)	... ...	80 tons.
Sandstone (weak)	... ...	$18\frac{1}{2}$ tons.
Compressed sand	... ...	2 tons.
Limestones (good)	... ...	$68\frac{1}{2}$ tons.
Limestones (weak)	... ...	$25\frac{3}{4}$ tons.
Brickwork (strong bricks in cement)		7 to $8\frac{1}{2}$ tons.
Brickwork (weak bricks) ...	...	4 to 6 tons.
Concrete (good broken stone)	...	$3\frac{3}{8}$ tons.

These are 1/8th of the crushing strengths. On the same basis, mortar made from lime and surkhi would safely stand $5\frac{1}{4}$ tons per square foot one and a half years after mixture

Sir Benjamin Baker—

Sir Benjamin Baker recommends that no material should be used in dams which will not bear fifty tons per square foot without splintering. This would represent a safe load of ten tons per square foot.

Molesworth—

Molesworth gives the following as the working strengths in compression :—

Sandstone ...	...	36 tons per square foot.
Brick in cement	...	$11\frac{1}{2}$ do.
Brick in lime mortar	...	6 to $10\frac{1}{2}$ do. after one and a half years.

Cement concrete 1 : 2 : 4·74...13 tons after three months.

Love's Applied Mechanics—

Love's Applied Mechanics gives the following :—

Best brick masonry in lime	...	about 5 tons per square foot.
Common do.	...	about $2\frac{1}{2}$ do.
Cement concrete	...	$10\frac{1}{4}$ tons per square foot.

Baker—

Baker in his Treatise on Masonry Construction gives the following :—

Ordinary cement concrete 1 : 2 : 4,
sixty-five days old ... 14 tons per square foot.
Portland cement concrete 1 : 2 : 4,
one month old ... $9\frac{1}{2}$ tons per square foot.
Common brick in lime mortar ... About $6\frac{1}{2}$ tons per square foot.

Brickwork in lime mortar is usually regarded as only 67 per cent. as strong as brickwork in Portland cement mortar.

Rubble masonry	... 10 to 15 tons.
Squared stone masonry	... 15 to 20 tons.
Limestone, ashlar	... 20 to 25 tons.

No doubt these values are for masonry in cement.

Kempe—

The following information is taken from experiments made by the Royal Institute of British Architects :—

London stock bricks in lime mortar ... $18\frac{1}{2}$ tons per sq. ft. *ul imate* strength.

London stock bricks in cement ... 39 do. do.

Pressed bricks made from special clays gave very much higher results up to 114 tons per square foot in lime mortar.

A good average quality of brickwork in lime mortar gave 45 tons per square foot *ultimate* strength, say $5\frac{1}{2}$ tons per square foot safe strength.

Buckley.—

Buckley's Irrigation Pocket Book has the following :—

Limestones, average	... 56 tons safe load per square foot.
Sandstones, average	.. 40 tons per square foot.
Bricks	... 13 to 25 tons.
Brickwork in lime	... With light bricks, having a *safe* strength of ten tons and mortar consisting of 1 lime and 1 sand, safe load 3·4 tons per square foot. With bricks having a safe strength of twenty five tons and mort consisting of 1 lime and sand, safe load 8·3 tons.

In this instance 1/10th of the ultimate strength has been taken. If a factor of 1/8th be applied the safe loads would be $4\frac{1}{4}$ tons and 10·4 tons per square foot.

Austrian practice.—

Figures taken from Austrian practice are :—

Ordinary brickwork in lime mortar		4·6 tons per sq. ft.	
do.	Cement mortar	... 7	do.
do.	Portland cement mortar	... 9	do.

One of the points worth noticing, when abstracting information from various sources is that, unless the cement is described as *Portland*, the results given will generally relate to some other kind of cement.

Concrete—

Cement concrete 1 : 2 : 6, 28 days old	...	$6\frac{1}{2}$ tons per square foot.
Cement concrete 1 :2 : 6, 6 months old	...	$8\frac{3}{4}$ do.

The kind of cement used is not stated, nor is the strength of ordinary concrete in lime mortar given.

Stone Masonry—Stones begin to crack with about half their ultimate crushing strength, and stone masonry in cement mortar is also only about 1/6th of the strength of the stone. Applying this figure we get the following safe loads :—

Limestone masonry	...	... 9	tons per square foot.
Sandstone masonry	...	... 7	do.

There are, however, known pressures on existing limestone masonry of 9 to 19 tons per square foot, and failures have occurred with 36 and 64 tons per square foot.

Experiments in America—

Experiments made upon common brickwork in the United States with mortar composed of one lime to three sand, gave a safe average strength of nearly eleven tons per square foot, but in another set of experiments with brickwork in lime mortar four weeks old, 6·6 tons per square foot was the average working strength.

Experiments made with *brick piers* laid in mortar composed of one Portland cement and two sand, showed that the strength of *brickwork* was only 1/6th of the strength of the bricks. This would not give a greater safe load than four tons per square foot for brickwork in lime.

Woods—

	Safe loads.
Bricks, stocks at 112 lbs. per cubic foot	... 18 tons per square foot.
Bricks, Staffordshire blue at 112 lbs. per cubic foot	... 50 tons per square foot.
Lime concrete, at 120 lbs.	... 3·6 tons to 6 tons, but usually 3 to 4 tons.
Cement concrete at 135 lbs.	... 13 tons per square foot.
Limestone 130 to 160 lbs.	... 60 tons to 72 tons.
Sandstone 135 lbs. ..	... 46 tons to 80 tons.
Brickwork 112 lbs. ...	... 4·8 tons to 9·6 tons.
Stone masonry 116 to 144 lbs	.. 6·4 tons per square foot.

Farrant—

In a pamphlet compiled by Mr. J. T. Farrant, late Chief Engineer, the following information is given regarding the *crushing* strength of building stone used on the canals of the Triple Project, Punjab :—

Baghanwala stone	1·85 to 2·55	tons per square inch.
Taraki ,,	3·22 to 4·32	ditto.
Hasan Abdal ,,	5·57 to 7·35	ditto.

Taking the average of the above, and using a factor of safety of 8, we get the following working strengths :—

Baghanwala stone	39·6	tons per square foot.
Taraki ,,	67·9	ditto.
Hasan Abdal ashlar	116·3	ditto.

Cunningham –

	Safe loads.
Limestone (compact)	... 56 tons to 68 tons per square foot.
Sandstone	... 26 tons to 36 ditto.
Concrete of lime and stone	... $3\frac{1}{3}$ tons per square foot.
Brickwork in cement mortar	... 8 tons ditto.
Bricks, weak	... 4·4 to 6·4 ditto.
Bricks, good	... 6·4 to 8·8 ditto.

Trautwine—

Trautwine states that in practice neither stonework nor brickwork should be trusted with more than 1/6th to 1/10th of the crushing load according to circumstances. For brickwork he recommends 1/8th. Some absorbent sandstones, when wet, lose fully half their strength.

		Safe loads.
Limestone	...	78 tons per square foot as an average.
Sandstone	...	32 to 105 tons per square foot.

Stones begin to crack and splinter at about half their crushing strength.

Ordinary brickwork in lime mortar, maximum 6 tons. It begins to crack with 25 tons and crushes with 30 to 100 tons. With *very good bricks* the safe strength is 10 tons. Cement brickwork is considerably stronger than brickwork in lime.

Bricks.—The safe strength varies from 5 to 37 tons per square foot according to the methods of manufacture and the nature of the clay.

The weight of brickwork, with pressed brick, is 129 lbs. per cubic foot, and of ordinary brickwork 116 lbs. per cubic foot.

Portland cement concrete—

1 month old	...	... about	2	tons per square foot.
6 months old	...	... ,,	8	ditto.
12 months old	...	... ,,	12	ditto.
Brickwork in lime		... ,,	3	ditto.

Specifications of several railway companies take a safe load of 2 tons per square foot ; others take $2\frac{1}{2}$ to 3 tons.

Lime mortar	...	$1\frac{1}{4}$ to $2\frac{1}{2}$ tons per square foot.

Rivington –

Limestone, granular	... ...	64·8	tons per square foot.
Sandstone, ordinary	... ...	36	ditto.
Lime concrete	... ...	3·6	ditto.
Portland cement concrete, 5 to 1	...	14·4	ditto.
Ordinary stock bricks	... ...	5·75	ditto.
Brickwork in good mortar	.. ...	3·6	ditto.
Brickwork in cement	.. ...	5·75	ditto

Unwin—

The testing of materials of construction—

Limestone ... ... 40 to 80 tons per square foot.
Sandstone .. ... 33 to 61 ditto.

Rubble limestone masonry, three months old, the mortar consisting of 1 lime and 2 sand, 5·8 tons per square foot.

Bricks.—London stocks weighing 6·8 lbs. each, pressed bricks, 10 lbs., and ordinary red bricks, 7 lbs. Size about 9" × 4" × 2½".

Common red, safe load 15 tons to 24 tons per square foot.
London stocks (yellow), safe load 13 tons to 23 tons per square foot.

Lime Brickwork (3 months old) 8¾ tons, constructed with bricks of 20 tons working strength and mortar of 1·4 tons working strength (consisting 1 lime, 2 sand).

Cement Brickwork (3 months old) 12½ tons, constructed with mortar composed of 1 cement and 3 sand, having a working strength of 24 tons, and bricks of working strength 20 tons.

Brickwork in lime (1 lime, 2 sand) is only ·44 of the strength of the bricks themselves.

Brickwork in cement (1 cement, 3 sand) is 0·63 the strength of the bricks themselves. This is about 50 per cent. stronger than brickwork in lime.

Portland cement concrete (proportions not given) one to two months old, 14½ tons.

Tensile strength of mortars.

Buckley's *Pocket Book*. Portland cement briquettes should bear on the average 400 lbs. per square inch of section after seven days, and 500 lbs. after twenty-eight days.

It is stated (page 281 and page 283) that pouring lime *from above* on to a screen of mesh 8, placed at an angle of 45° is equivalent to a screen of mesh 20 (used horizontally).

Lime mortars.—

1 Sutna lime, 2 surkhi, 2 sand gave 39 lbs. per square inch after one week.

1 Kunkar lime, ½ surkhi, 1½ sand gave 77 lbs. per square inch after one week.

The lime was ground by a Lucops disintegrator.

TABLE I.

Safe strength of materials, and safe loads for concrete, brickwork and stone masonry, allowing a factor of safety of 8 in each case.

AUTHORITIES.	MATERIALS.			CONCRETE AND MASONRY.					
	Limestone.	Sandstone.	Bricks.	Concrete in lime.	Concrete in Portland cement.	Brickwork in lime.	Brickwork in Portland cement.	Limestone masonry.	Sandstone masonry.
	In tons per square foot.								
Rankine	25·7 to 68·5	18·5 to 80		...	3·4	...	4 to 8·5	...	...
Molesworth	...	36	...	...	13 (3 mos.)	6 to 10·5 (18 mos.)	11 5	...	...
Love	...	...	...	...	10	2·5 to 5	...	...	...
Baker	...	...	...	...	19½ (1 month)	6·5	...	10 to 25 according to dressing and bedding.	
Kempe	...	...	...	...	...	5·5	...	...	...
Buckley	56	40	13 to 25	...	6·5 (1 m nth) to 8·7 (6 mos.)	4 to 10	9	9	7
Woods	60 to 72	46 to 80	18 to 50	3·6 to 6	13·5	4·8 to 9·6	...	...	...
Cunningham	56 to 68	26 to 36	4·4 to 8·8	3·3	...	...	8	...	...
Trautwine	78	32 to 105	5 to 37	...	8 (6 mos) 12 (12 mos)	6 to 10	9 to 12	...	...
Rivington	65	36	5·7	3·6	14	3·6	5 7	...	...
Marryat's specifications ...	...	...	...	3 to 3·5	25 to 35	2 to 5	...	...	...

Summary—

There is considerable want of uniformity in the results given by authorities. All the safe loads just tabulated have been arrived at by using 8 as a factor of safety. The need of departmental research is great.

As far as can be judged from information at present available, the following maximum safe loads should not be exceeded in canal works. The constant pressure of water in the canal above works has a softening influence on some materials, although the hardening of the mortar is increased.

		Safe loads.
Concrete in lime	...	$3\frac{1}{2}$ tons per square foot.
Do. cement	...	8 ditto.
Brickwork in lime	...	$6\frac{1}{2}$ ditto.
Brickwork in cement with very good bricks	...	$8\frac{1}{2}$ ditto,
Rubble stone masonry	...	$6\frac{1}{2}$ ditto.
Good stone masonry	...	$8\frac{1}{2}$ tons to $10\frac{1}{2}$ tons per square foot, according to the crushing strength of the stone used.

FOUNDATIONS AND FOOTINGS.

Depth of foundations and splay of footings.—The maximum intensity of pressure must not be such as will heave wet soil in the vicinity of the work. In order to avoid settlement from this cause, Rankine gives the following formula for the depth to which foundations must be carried in wet or weak soils. The maximum intensity of pressure on foundations (which will call into play the whole of the resisting power of the earth against upheaval) is considered.

$$\text{Depth necessary} = \frac{\text{maxm. intensity of pressure in lbs.}}{\text{unit weight of earth concerned}} \times \left(\frac{1 - \text{sin. L. of repose}}{1 + \text{sin. L. of repose}}\right)^2$$

from which x, the depth necessary for foundations can be found.

Splay or spread of footings.—Rankine gives the rule that ordinary walls should be splayed by footings till the breadth of the base is $1\frac{1}{2}$ times the thickness of the wall in compact gravel and twice that thickness in sand and stiff clay.

Baker gives the following formula for calculating the greatest possible projection for each footing, regarded as a cantilever.

$$\text{Greatest offset} = \tfrac{1}{7} \times \text{thickness of course in inches} \times \sqrt{\frac{\text{Transverse strength of material}}{\text{Pressure intensity at bottom of course} \times 10}}$$

Abutments.

Thickness of abutment—

Bligh uses Trautwine's rule that the thickness at springing $= \cdot 2R + \cdot 1V + 2$, where R = radius, V = versed sine.

For rear slope, use the ratio $\dfrac{\cdot 02 \text{ span}}{\cdot 5 \text{ versed sine}}$

This neglects assistance from earth backing. If a heavy load of water is carried, add 1/5th of depth of water to above thickness.

Graphically : find resultant of R the reaction at skewback, W the weight of abutment, and E the earth thrust.

Economise by using buttresses ; regard weight of buttress as distributed over intervening area, thus increasing vertical load proportionately.

Mr. J. T. Farrant in his note of 1906 refers to the difficulty of ascertaining and balancing the upward thrust transmitted from the adjacent piers by inverts when these are used. He assumes that the stability of existing structures evidences that the abutting power of the earth backing counteracts this thrust. This abutting or resisting power, exerted at any plane whose top and bottom are distant h_2 and h below the terrain line, is given by the formula :—

$$\text{Abutting power} = \frac{4 \times \text{unit wt. of earth} \times \sin.\ \text{L. o repose}}{\text{Cos.}^2\ \text{L. of repose}} \times \frac{h^2{}_1 - h^2{}_2}{2}$$

Mr. Farrant assumes that this resisting power is exerted at a plane equal to the depth of the concrete base only, and takes it and the thrust of the inverts into account when considering the stability of abutments, but regards them as equal and opposite.

The moments considered are—

(1) Horizontal thrust at crown × distance from springing of arch, or invert, or foundation line (according to the moment required).

(II) Weight of half arch above abutment line × distance from crown line to C. G.

(III) Weight of half invert × distance from crown line to C. G.

(IV) Earth pressure above springing of arch × ht. of C. P. of plane above springing.

This is expressed as follows :—

$$\frac{1}{2} \text{ unit wt. of earth} \times \frac{1 - \text{sin. L. repose}}{1 + \text{sin. L. repose}} \left\{ (\text{dh. to base of plane})^2 - (\text{dh. to top of plane})^2 \right\} \times (\text{dh. to base of plane})$$

$$- \frac{2}{3} \times \left(\frac{(\text{dh. to base of plane})^3 - (\text{dh. to top of plane})^3}{(\text{dh. to base of plane})^2 - (\text{dh. to top of plane})^2} \right)$$

(V) Earth pressure above springing of invert × ht. of C. P. of plane above invert springing.

Formula similar to IV above for bottom depth of plane to invert springing, and top depth of plane as in IV.

(VI) Earth pressure above foundations × ht. of C. P. of plane above foundation.

Formula similar to IV above for bottom depth of plane to foundations, and top depth of plane as in IV.

Top width of abutment.—To determine this, moments are taken about the outer edge of the middle third.

The positive moments are—

(*a*) Load over abutment (*i. e.* top width ht. to load line) $\times \frac{\text{top width of abutment}}{6}$

(*b*) Weight of half arch $\times \frac{2}{3}$ top width of abutt. + horl. distance to its line of action.

(*c*) Earth pressure above springing of arch. × ht. of C. P. of plane above arch springing.

Formula given under IV above for *c*.

The negative moments are—

(*d*) Horizontal thrust × vertical distance from springing to its line of action.

By equating *a*, *b*, *c*, *d* to zero, the top width of abutment is found by solving for x (the top width).

To find maximum intensity of pressure on abutment at arch springing.

Add load over abutment to weight of half arch, and divide by top width of abutment ; this will give the mean intensity. Then, as resultant passes through the outer edge of the middle third, the maximum intensity will be twice the mean intensity.

To find width of abutment at springing of invert.

Again take moments about the outer edge of the middle third.

The positive moments are—

(*a*) Load over abutment (*i. e.* bottom width $x \times$ ht. to load line) $\times \dfrac{\text{bottom width of abutt.}}{6}$

(*b*) Wt. of half arch $+\ \frac{2}{3}$ bottom width + horl. distance to its line of action.

(*c*) Earth pressure above springing of invert $\times$ ht. of C. P. of plane having its base at invert springing.

Formula given under V above for *c*.

The negative moments are—

(*d*) Horizontal thrust $\times$ ht. from invert springing to its line of action.

By equating *a*, *b*, *c*, *d* to zero, and solving for *x* the width of the abutment at invert springing is found.

To find maximum intensity of pressure on abutment at invert springing.

Add load over abutment, and weight of abutment itself, to weight of half arch—then divide by width of abutment at invert springing. This will give mean intensity, and maximum intensity will be twice mean intensity.

Stability against sliding—

The masonry will not slide until the resultant pressure makes an angle with the vertical to the base which is greater than the angle of friction between the materials concerned. In the case of masonry and brickwork in mortar this angle is $36\frac{1}{2}°$ and its tangent is ·74. In the case of structures on moist slippery clay however, the angle is $18\frac{1}{4}°$ and its tangent ·33.

The angle made by the resultant with the vertical to the base can either be measured graphically, or its tangent may be calculated.

$$\text{Tangent} = \frac{\text{horizontal component of all overturning force}}{\text{vertical component of all loads}}.$$

This tangent must not be greater than the tangent of the friction angle, and it is usual to apply a safety factor of 1·2 to the resulting widths for abutments.

Stability against crushing.—

The maximum intensity of pressure should not exceed the safe load per square foot for the materials concerned.

Deviation of line of resultant from centre of base.—As far as possible foundations should go to the *same* depths where the load to the borne is the same, and the intensity of pressure on the base should be uniform, but in practice it is frequently not possible to obtain these desirable conditions. In order, therefore, to control the unevenness of pressure on foundations which occurs in practice, and to prevent upheaval of adjacent soil, Rankine limits the extent to which the line of the resultant may deviate from the middle of the base of a structure (or rather from the C. G. which, in symmetrical structures, is coincident with the centre of the base). One-sixth of the base is the maximum deviation he recommends when the base is rectangular, and-one eighth of the diameter, when the base is circular, and so on for various shapes of bases.

His general rule for a rectangular base is—

$$\text{Maximum deviation} = \frac{\text{base}}{6} \times \frac{2 \sin \text{L. of repose}}{1 + \sin^2 \text{L. of repose}}$$

Although the angle of repose of earth is usually taken as 33°—42', giving a slope of $1\frac{1}{2}$ to 1, it is safer, in designing canal works subject to constant percolation from above, to take a flatter angle say 30° for good earth, and 20° for very light soil. This will have the effect of somewhat reducing the extent of deviation allowable by Rankine's rule.

Thus, when the repose angle is 20°, the extent of deviation allowable will be $\frac{\text{base}}{6} \times \cdot 612$. It is only in the case of a repose angle of about 50°, that the allowable deviation, by Rankine's rule, $= \frac{\text{base}}{6}$, which corresponds to the resultant passing through the outer edge of the middle third.

In calculations for canal works however, the resultant is usually given the extreme position, and security is obtained (especially in the case of retaining walls) by taking a low value for the repose angle.

In the case of abutments, or walls subject to external pressure as well as pressure from behind, it is very necessary to pack in very tightly the *filling* behind the structure, especially from the natural surface downwards, because the abutting power of the earth is then fully brought into play.

Abutting power of earth.—This is the resistance offered by the earth to tangential force, *i.e.*, to pressure tending to push it back. Unlike water, earth offers considerable resistance to tangential force, if it is compact; hence the need for close, firm, contact between masonry and the sides of foundation trenches.

To find Centre of Resistance (i. e. point of action of resultant pressure) at foundation level.

First take moments about the vertical through the crown to ascertain the C. G. line of all vertical loads. The sum of the moments of all the vertical loads, about any vertical line, will be equal to the moment of the total vertical load about the same line, *i.e.* the distance to the C. G. of the total vertical load

$$= \frac{\text{sum of moments of vertical loads as above}}{\text{total vertical load}}$$

= distance to C. G. of total vertical load from vertical line through crown.

The positive and negative moments may now be equated by taking moments about the Centre of Resistance, distant x from C. G. line of vertical loads.

The positive moments are :—

(*a*) The total vertical load	$\times\ x$
(*b*) Earth pressure on plane having its bottom at foundation level and its top at intrados of arch	$\times$ distance above foundation level to line through C. P.
(*c*) Weight of invert	$\times$ horizontal distance to its CP.
(*a*) Abutting power of earth	neglected ;
(*e*) Horizontal thrust of *arch*	$\times$ distance from foundation level to its line of direction.

(*f*) Horizontal thrust of *invert* × distance from foundation level to its line of direction.

From which x may be found.

The above method does not take into account the support afforded by wingwalls, because the case of lengthy culvert barrels was under consideration. Mr. Farrant has given tables in his note for facilitating the calculations for abutments of long culverts.

Kennedy's method for a. up to 100 *ft. span with abutments up to* 25 *ft. in height* is as follows :—For 60°, 75°, 90° and 120° arches the joints of rupture are taken as being at the springing. For semi-circular arches the joints of rupture are taken as subtending an angle of 100° at the centre, the rest of the arch acting as an abutment.

To obtain dimensions of abutments with greater accuracy, assume a joint of rupture and find the thrust on it ; draw out the curve of resistance for the whole arch; where this curve is nearest to the intrados or extrados is the *true* joint of rupture. Find the horizontal thrust by taking moments round this true joint ; then deduce the dimensions of the abutment by taking moments of all the forces round the outer edge of the middle third for the top width, and again for the bottom width of abutment.

$$\text{The maximum intensity of pressure} = \text{mean intensity} \times \left(\frac{1 + 6 \times \text{deviation}}{\text{mean intensity}}\right)$$

PIERS.

Bligh—

Bligh gives a thickness 1/10th to 1/6th of span in fair sized spans. For spans 40' to 80' batter sides of piers to give uniformity of pressure. Small spans are more economical than large, especially in head regulators. In small regulator spans of 10 to 25 feet, make the pier 1/3 to 1/4 the span. Design for (*i*) load, (*ii*) weight of masonry, (*iii*) water pressure transverse to length of pier.

In calculating the weight, omit the portion of the pier upstream of the grooves of the regulator, as it lies beyond the plane of pressure, and is subject to flotation. Resultant should fall within the middle third as usual, and the maximum pressure will be twice the mean pressure when it passes through the outer edge of the middle third.

Widen pier foundation to obtain allowable pressure on foundations. Increase thickness of piers when head of water is great. Footings are usually splayed at $\frac{1}{2}$ to 1.

For aqueducts carrying five or six feet of water, and having spans 25 feet and over, make the pier equal to $\sqrt{\text{span}}$ in thickness at the springing. For spans less than 25 feet, make it $\cdot 2s$, and batter the sides according to Molesworth's formula.*

Mr. Stoddard's Note.

The horizontal thrust on a pier, from the weight of half the arch alone, is found by equating the moment of the horizontal thrust about the springing, with the moment of the half arch plus other loads. This is counterbalanced by an opposite thrust from the adjoining arch, except when a heavy load such as a steam roller arrives, when there is a net horizontal thrust equal to the *difference* between the horizontal thrust *with* the roller and the horizontal thrust *without* the roller. The other forces acting on the pier are the vertical load of the roller, and the vertical load of the arch and backing. Also the weight of the pier itself.

Taking moments about the point at which the resultant cuts the base of pier—the distance from the edge of the pier being denoted by x,—the forces in equilibrium are—

(*i*) The net horizontal thrust.

(*ii*) The vertical component of the crown thrust, equal to $7\frac{1}{2}$ tons or half the weight of a roller.

(*iii*) The roller (in two equal loads of $7\frac{1}{2}$ tons each acting at different points.)

(*iv*) The dead load, consisting of two half arches and backing plus the load of the pier.

The value of x is then found by equating the moments of these forces. The position of the resultant being known, the mean pressure on the foundations, and the maximum and minimum intensities can be found.

* Molesworth's Pocket Book, page 89.

Mr. Stoddard also investigated the load caused by a crowd at 120 lbs. per square foot, and the result was almost the same as that obtained for a steam roller.

Rankine fixes various limits for the extent to which the resultant may deviate from the centre of the base, according to the angle of repose of the foundation soil. In wet unstable soils the amount of deviation permitted is small, while in firm soils a greater deviation is permissible. His rule for deviation is—

$$\text{Deviation} = \frac{\text{base}}{6} \times \frac{2 \sin \text{L. of repose of soil}}{1 + \sin^2 \text{L. of repose of soil.}}$$

The deviation must be within Rankine's limit for the soil concerned, and the maximum pressures on masonry and on foundations must not exceed the allowable pressures for the masonry and the soil.

The minimum thickness of the top of a pier is twice the depth of arch at crown $\times$ sin $\frac{1}{2}$ angle at centre of arch.

ARCHES.

Bligh—

Horizontal thrust depends on weight and load overlying arch between crown and joint of rupture. The weight and load beyond the joint of rupture does not affect the horizontal thrust, as it really forms a projecting part of the abutment ; but in segmental arches the joint of rupture is at the springing. Horizontal thrust $= w.\ r.\ t.$

$w.$ = unit weight of material of arch and filling.

$r.$ = radius of intrados.

$t.$ = vertical height to equivalent load line.

The resultant of the horizontal thrust, and the weight and load on half the arch (between crown and face of abutment) must fall within the middle third of the arch at the skewback. Taking the horizontal thrust as starting at the centre of the arch at the crown, according to Molesworth's empirical formula, the crown thickness = ·4 radius, for ordinary bridges, and ·5 radius, for aqueducts ; the thickness at the haunches must then be increased so that the resultant may not pass outside the middle third.

The effect of a fifteen ton steam roller has been investigated by Mr. A. A. Stoddard, Executive Engineer in connection with the design of bridges on the Upper Chenab Canal. The roller is taken as the heaviest part of the plant, and he shows that the crown thrust is greatest when the roller is over the crown. $7\frac{1}{2}$ tons is assumed as the load on the roller itself, and $7\frac{1}{2}$ tons on the two driving wheels.

Mr. Stoddard shows that it is allowable to assume that the curve of equilibrium (line of thrust) passes through the centre of the arch ring at the crown, through the upper limit of the middle third at the springing (when the roller is over that half of arch), and through the lower limit of the middle third at the springing in the case of the vacant half arch.

Taking a roller six feet wide, and situated two feet above the crown, the load is distributed over $6+2+2=10$ feet, of arch, and the intensity per foot width $=\frac{7\cdot5}{10}=\cdot75$ ton per foot assuming load to be distributed at 45° as usual. Rankine requires that the resultant thrust shall be within the middle third of the arch both at the crown and at the springing, and Mr. Stoddard shows that this is equivalent to saying that

$$\frac{\text{Radius to middle arch}\left(1-\text{cosine}\frac{\text{central angle}}{2}\right)-\text{distance of resultant vertical load from crown}\times\sin\frac{\text{central angle}}{2}}{1+\text{cosine}\frac{\text{central angle}}{2}}$$

must not be greater than $\frac{\text{thickness of arch ring at crown}}{6}$

This is a convenient practical rule for finding the thickness of an arch at the crown by trial.

Mr. Stoddard does not take the thrust at crown as horizontal when the steam roller is in transit, hence both the vertical and horizontal components of the crown thrust are considered. The former is taken as acting upwards on the roller side of the arch and downwards on the vacant side. The vertical component produces the shear, and its amount is calculated by equating the moments of vertical component, horizontal component, and the load. The mean intensity of the shear is found by dividing this amount by the thickness of the arch; while the maximum shear intensity at the centre is $1\frac{1}{2}$ times the mean.

The maximum pressure intensity per square foot at the crown is as follows :—

$$\frac{\text{Intensity of horizontal thrust}}{2} + \sqrt{(\text{Maximum shear intensity})^2 + \left(\frac{\text{Intensity of horizontal thrust}}{2}\right)^2}$$

The vertical reaction at springing, on the roller side, is found by taking moments about the springing.

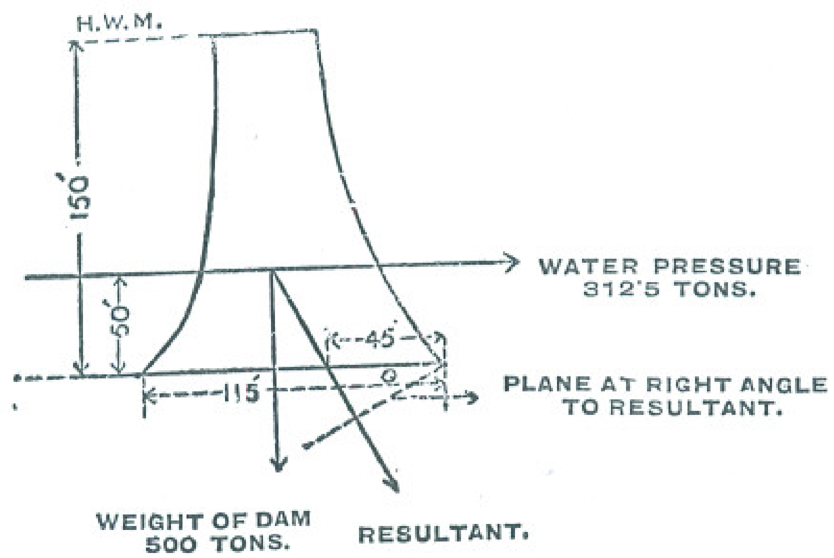
The resultant thrust $= \sqrt{\text{vertical reaction} + (\text{thrust at crown})^2}$ and its *mean* intensity is found by dividing the result by the thickness of the arch at springing. Then, since the resultant passes through the outer third of the arch, the maximum pressure intensity at springing will be twice the mean intensity.

The graphic method is as follows :—From the centre of the arch ring at the crown, draw a horizontal line to cut the resultant vertical load line, and from the point of intersection thus obtained, draw a line parallel to the inclination of the soffit of the arch at springing, *i.e.*, parallel to tangent at soffit. If this line intersects the skewback within the middle third the arch is stable.

DAMS.

Dams—

The following example is taken from Woods' "Strength and elasticity of structural members." Take a dam weighing 500 tons per linear foot and of section sketched.



Water pressure $= \frac{w.\ h^2}{2} = \frac{1}{36} \times \frac{150^2}{2} = 312\cdot5$ tons, the weight of water being $\frac{1}{36}$th of a ton per cubic foot.

$$\text{Maximum intensity of pressure on base} = \frac{\text{twice weight of dam}}{\text{base of dam}}$$

$$\times \left(2 - \frac{3 \times \text{distance of resultant from outer toe}}{\text{base of dam}}\right)$$

$$= \frac{2 \times 500}{115}\left(2 - \frac{3 \times 45}{115}\right) = 7{\cdot}18 \text{ tons per square foot.}$$

$$\text{Minimum intensity of pressure} = \frac{\text{twice weight of dam}}{\text{base}}$$

$$\times \left(\frac{3 \times \text{distance of resultant from toe}}{\text{base}} - 1\right)$$

$$= \frac{2 \times 500}{115}\left(\frac{3 \times 45}{115} - 1\right) = 1{\cdot}5 \text{ tons per square foot.}$$

$$\text{Mean intensity} = \frac{7{\cdot}18 + 1{\cdot}5}{2} = 4{\cdot}34 \text{ tons per square foot.}$$

The intensities on a plane at right angles to the resultant must, however, be found, as they are the true intensities.

Resultant $= \sqrt{(500)^2 + (312{\cdot}5)^2} = 589$ tons.

Divide the former intensities by $\cos \Theta$, where Θ is the angle made by the resultant with the vertical, and $\cos \Theta = \frac{500}{589}$, therefore the maximum intensity $= 7{\cdot}18 \div \frac{500}{589} = 8{\cdot}48$ tons and the minimum intensity $= 1{\cdot}5 \div \frac{500}{589} = 1{\cdot}75$ tons.

Upward pressure from beneath the dam, wind pressure, the effects of flotation on the downstream side, and the vertical component of the water pressure acting on the curved face upstream, have not been taken into consideration in this example. Neglecting these forces, the rule for the thickness of the base of a wall of triangular section (with vertical face) to withstand water pressure alone is—

$$\text{Base width} = \frac{\text{height to water surface}}{\sqrt{\text{S. G. of masonry}}}$$

The resultant will fall within the middle third by this rule, and the weight of wall situated beyond the triangular section will be a reserve of strength. Taking the S. G. of brickwork as 2, and of stone work as 2·4, this is equivalent to saying that for triangular walls under water pressure alone—

Width of base for brickwork = height from floor to water surface × ·7

Do. stonework = height from floor to water surface × ·65

It should be noted that the heights are *from floor to water surface* only, and that the wall is assumed to be triangular in section.

If the top width of the wall at water surface be 4 ft. and the height to water surface be 20 ft. and the S. G. of masonry 2·0, then (height to W. S.)^2 = S. G. (base^2 + top × base − top^2)

$$400 = 2\,(\text{base}^2 + 4\ \text{base} - 16)$$

$$266 = \text{base}^2 + 4\ \text{base}$$

$$\text{base} + 2 = \sqrt{220} = 14{\cdot}83$$

∴ base = 12·83 which is about ·64 of height to water surface. If S. G. = 2·4, the base would equal 11·5' which is about ·57 of height to water surface. In practice,* it is usual to take the *total* height to extreme top of wall and to apply the following proportions:

Base width for stone masonry wall to resist water pressure only = ·5 × total height.

Do. do. for brickwork wall to resist water presssure alone = ·6 × total height.

Recent practice and experience in the construction of gravity dams shows that in *uncertain* strata, it is necessary to allow for an upward pressure under the base of the dam equal to that caused by the whole static head and extending over the whole base. In less uncertain strata the full static head is allowed at the heel, diminishing to zero at the toe.

It is only in the case of a deep foundation on good impervious rock without horizontal or vertical seams, or with all fissures and seams closed by grouting, that upward pressure under the base need not be considered. Similarly, in calculating the resistance to sliding in uncertain strata, the weight of masonry should be reduced by the upward pressure, and a low co-efficient of friction for sliding should be taken. A dam built on a foundation of porous sandstone lying in horizontal strata, underlaid by shale, with fissures filled with clay, sand, and gravel, would be regarded as "uncertain," and in such a case an upward pressure, equal to the full static head, would be taken. In such cases a hollow dam of the Ambursen or Ransome type would be preferable to a solid dam. The interior of a solid dam must be efficiently drained.

The discussion of hollow dams and of curved masonry dams is deferred, to avoid making this paper too long.

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* Engineering News 30-11-11.

Dams or walls under water pressure alone, overturning effect.— Baker's method is to imagine the pier or dam to be a cantilever fixed into the ground at its base, and acted upon *across* its height, by fluid, or wind pressure, or both (as the case may be) ; and *parallel to its height* by its own weight, plus the live or dead load from without the work.

The bending moment of the fluid (or wind pressure) is unit pressure $\times \frac{\text{height}^2}{2} \times$ distance of centre of pressure above the base ; and the stress which this will cause on the outer extremity of the base $= \frac{\text{Fluid moment}}{\text{Moment of Inertia}} \times \frac{\text{thickness of base.}}{2}$ This formula is equivalent to dividing the bending moment by the moment of inertia, less the stress intensity—the distance of the extreme fibres from the natural axis being $\frac{\text{thickness of base.}}{2}$

Every engineer is familiar with the formula for the transverse strength of beams ; *viz.*, by equating bending moment to resistance moment—

$$\text{Bending moment} = \frac{\text{Moment of Inertia}}{\text{extreme fibre distance}} \times \text{working stress.}$$

$$\text{From which working stress} = \frac{\text{bending moment}}{\text{inertia moment}} \times \text{fibre distance,}$$

as in the above formula for strength of walls and dams under fluid pressure. The moment of inertia of a rectangular base is $\frac{1}{12} \times \text{thickness} \times (\text{length of base.})^3$ To the above would be added the weight of the structure per square foot to ascertain the total pressure (maximum pressure) at the outer edge of the base, just as in the case of beams, the weight of the beam is included.

To find the pressure (minimum pressure) at the inner edge of the base, subtract from the weight of the structure per square foot the value of the above expression, *i.e.* $\frac{\text{fluid moment}}{\text{Moment of inertia}} \times \frac{\text{thickness of base.}}{2}$

If the surface of the structure be an inclined one, the fluid moment equals the moment of the horizontal component minus the moment of the vertical component. The horizontal component is the same as for a vertical wall, *viz.*, height $\times$ pressure per sq. ft. $\times \frac{\text{height,}}{2}$ and the arm of its moment is $\frac{\text{height.}}{3}$