PAPER No. 201.

TURBULENT FLOW IN CHANNELS.

(A Development from the Lacey Theory.)

By T. Blench, B.Sc., (Hons.), A.M.I.C.E., I. S. E.

The Chezy equation is deduced conventionally and correctly by equating the body forces on a conveniently shaped mass of fluid to the external resistances; for the internal forces must cancel. There is, however, a fundamental error in assuming that the external forces vary as the square of the mean velocity over a section. It is further assumed that the same law holds whether the channel be formed in its own incoherent transported material (when the resistance is "cushioned"), or have a rigid boundary and no transported material. As a consequence of the first error the Chezy formula is wrong in form, and most hydraulic research has been directed towards finding empirical formulæ to express the parameter C in $V=C\sqrt{RS}$. Of these Kutter's makes C depend on R and S; Bazin's makes C depend on R only; and Mannings makes C such a simple function of R that the formula can be written V=const. $R^{\frac{2}{3}}$ $S^{\frac{1}{2}}$. Barne's has empirical formula for different types of channels giving V=const. R^m S^n, where m and n vary for each case. None of the formulae realize that the physical condition of the fluid may be of importance; and because of the second error it is found (Bib. I. Ch. IX.) that formulae which fit rigid channels fairly well do not fit earth channels at all well.

It must have occurred to many investgators that an investigation on energy lines would be illuminating; but their lack of detailed knowledge of the mechanism of eddy formation has been an insuperable obstacle. The Lacey empirical relations, however, suggest, by their obviously dynamical soundness, that behind them is an energy relation from which they can be deduced by using the proper criterion of self-formation. The writer has worked inversely to find what energy relation would give the Lacey formulae, and used the new knowledge so obtained, to derive a flow formula for rigid channels. This formula tests the analysis and the new energy relation is now available as a test for any subsequent theory of the mechanism of turbulent flow.

I. *Recapitulation of the Lacey Relations.*—The original theory will be found in Lacey's I.C.E. Paper 4893, *and his replies to discussion.* The practical aspect has been given in the present writer's Punjab Engineering Congress Paper 187 "Scientific Irrigation Channel Design." The dimensional development of the subject, which is the easiest for the beginner, is in the writer's article on the Lacey Theory in 'The Indian

and Eastern Engineer' of May 1936. It is assumed that the reader is acquainted with these articles, and knows the conventional symbolism.

The whole of the Lacey relations, barring the rough one involving d, can be considered to be based on the following.

(*i*) $P=2{\cdot}67Q^{\frac{1}{2}}$, which applies to a self-formed section, and will be called the perimeter formula. It may be written by squaring and dividing by A, as $P/R=7{\cdot}12V$, which will be called the shape formula.

(*ii*) $V=16\sqrt[3]{R^2S}$, which applies when the section and slope are finally self-formed, and will be called the Regime Test Formula, because its greatest use is to test a channel for final regime, before attempting to apply formulae involving 'f.'

(*iii*) $V=1{\cdot}151\sqrt{fR}$, which defines 'f' the silt-factor. All it states is that, on a broad average, it has been found that regime channels of the same system, which would be expected to have the same order of "silt content," indicate the law stated with a fixed value of f. Analysis of units of the system gives values of 'f' for each unit not differing more from the general 'f' than would be expected from the obvious physical conditions.

Combinations of these formulae give more convenient formulae for check, and support the definition of 'f.'

By combining (*ii*) and (*iii*) Lacey found a flow formula $V=\text{const.}\ R^{\frac{3}{4}}S^{\frac{1}{2}}$; but it is just has simple to find $V=\text{const.}\ RS$. In both cases the const. depends on 'f' but in a different way for each. For future analysis the writer prefers the latter form.

The term "silt-factor" is not very good, as true similarity of behaviour depends not only on the silt, but also on the viscosity and on 'g' but it is good enough in practice.

For scientific analysis it is most important to realize that the following conditions are necessary for the Lacey formulae to be rigidly applicable. (*i*) The discharge must be fixed (*ii*), the silt load must be fixed, (*iii*) the physical conditions of the fluid must be fixed, (*iv*) the silted condition must be final, (*v*) the channel must be self formed in its own *incoherent* transported material (so mud berms will cause a deviation from perfection), (*vi*) the turbulence must be due to the incoherent boundary alone and not to any extraneous cause such as sharp bends, or nearby falls, or clods in the bed, etc.

That most channels in filling follow the Lacey rules reasonably
ıl should not be allowed to obscure the theoretical points in a rigid
is.

There is also an omission in the Lacey relations, *viz.*, that there is no definite rule on the perfect channel shape. It would appear to be elliptic; but there is nothing definitely known except that channels do tend to some perfect shape. It is for this reason that the relations do not work with small trapezoidal sections, which deviate, because of their smallness, noticeably from nature's perfect shape.

II. *Dimensional Aspect.*—The Lacey relations, being statistically derived, average out variations in v and g and the density of silt relative to water, as they are small compared with the major variables. A dimensional analysis cannot deal with the last term, which is fortunately very slightly variable; but it can deal with the others, and gives the Lacey relations in the following forms:

(*i*). $P=\text{const. } Q^{\frac{1}{2}} \quad (vg)^{\frac{1}{6}}$ or $P/R=\text{const. } V/(vg)^{\frac{1}{3}}$

where the constants are now non-dimensional.

(*ii*) $V=\text{const. } (v^{-\frac{1}{9}} g^{\frac{5}{9}}) \sqrt[3]{R^2S}$, which is better written as

$$V^3=\frac{Cg^2}{(vg)^{\frac{1}{3}}} \cdot R^2S$$

because this indicates the form:—

$$\frac{V^2}{gR}= C.gS/(vg)^{1/3}$$

where the left-hand side gives a ratio of energies, and gS is energy gradient, while $(vg)^{\frac{1}{3}}$ of (*i*) appears as a term with a definite meaning. It has the dimensions of velocity, and occurs in the mathematical analysis of the problem of the wind required to maintain a certain wave motion against viscosity (Bib. 2. paragraph 348). Its appearance in that problem, in the rationally written Lacey relations, and in the sequel, suggests that it is a criterion deciding how boundary action is transmitted into a fluid by eddy formation, and gives an orientation to future research.

(*iii*) V^2/gR=a criterion of a similar behaviour of silt bearing channels in final uniform turbulent flow. This criterion is an energy ratio and dimensionless—in fact the Froude number.

III. *The New Energy Theory.*—Consider Fig. 1, which shows unit length of a channel, not necessarily self-formed, but the flow is uniform. The energy relation is, in words, "Rate of working by gravity on unit length equals (rate at which the boundary resistance does work) plus (rate at which energy is dissipated internally)."

In viscous non-turbulent flow it is known that there is no boundary slip (Bib. 2 paragraph 331), so the first term on the right drops out and all the energy is dissipated internally. When the flow is turbulent there

is slip, and the first term must be considered. Our problem is to find such expressions for the two terms that, when the resulting equation is treated by the principle of self-formation, Lacey relations will result.

To find the first term is simple, for we should expect the boundary resistance law to be *linear* in V_b the perimetral velocity which should be uniform over the perimeter in a self-formed channel, and also in a rigid one of smooth shape (*e.g.*, a horse-shoe conduit, not a rectangular flume), *vide.*, Bib. 3. Chapter on 'Open Channels.' The reason for the linear law is that the active bed silt provides a cushioned resistance and removes the element of shock. It is admitted that this reason is not final ; but it does show why a quadratic law should not be likely, and the only simple alternative suggested by general hydraulics is linear. The interested reader can try an n^{th} power law in the subsequent analysis and verify that n=1 if the Lacey relations are to hold.

Assuming the linear law we have boundary resistance equals $\Phi V_b P$ for the unit length of fluid considered where Φ is the resistance per unit area per unit velocity. The question arises whether Φ depends only on the nature of the active silt layer, or on that and the nature of the fluid. The answer to this cannot be certain, and the question forces us to admit that nothing is accurately known about the inner mechanism of boundary resistance, whether for a rigid or a self-formed channel ; and that the use of the term " boundary resistance " is highly artificial, especially for a self-formed channel where the boundary consists of sand mixed with water, and moving on sand. It would be more satisfactory to drop the idea of boundary resistance (in spite of its conventional usefulness) and, instead of saying that the boundary resistance law is linear, or quadratic, say that the boundary energy transformation law is quadratic or cubic. This permits one to proceed rigidly and tacitly points out that the exact mechanism of boundary resistance is a matter for research.

It would appear that the bulk of the boundary energy loss is due to internal sand friction, and some is possibly due to eddies which form and pass into the body of the fluid, being finally dissipated by viscosity. The internal loss, as accounted for above, will then be that portion due to viscous laminar slip, as modified by the presence of eddies. There are other possibilities ; but provided we separate the energy transformations in some definite manner we can proceed with analysis without knowing their exact nature—we know one has its seat in the boundary and the other in the fluid and the exact mechanism of transformation can be left for future research. For the present we shall assume that Φ does not depend on the nature of the fluid, and in interpreting the results will bear that in mind.

The rate of boundary working is then $\Phi V^2 b P$ ft. pdls———(1)

The K. E. is not the K. E. found from the mean velocity but from the means of the resultant velocities throughout the fluid. But if V be the mean forward velocity at a point, the actual velocity will have components u, $V+v$, w where u, v, w are small random variables The K. E. per lb. at a point at an instant will be $\frac{1}{2}(u^2+(V+v)^2+w^2)$; and we have to sum such terms over the fluid and over a period of time. The theory of error tells us that ΣVv must be zero, and Σu^2, v^2, w^2, minima. So we are left with the K. E. reckoned on the mean V as a minimum also. And as we consider a fixed discharge Q, and K. E. $\propto$ QV it follows that the channel must adjust itself so that V is a minimum.

So all that is required is to differentiate (4a) first with reference to R and then with reference to S and omit all terms containing dV/dR and dV/dS as factors. There will then be two new relations which solved with the original one should lead to Lacey result if our fundamental assumptions were correct.

On performing these differentiations we find:

$$dS/dR=0=(\rho g/\Phi).(S^2/V) \quad (2k/pg).(V^3/R^3) \qquad \text{——————(5)}$$

$$dS/dS=1=(2\rho g/\Phi).(RS/V) \qquad \text{——————(6)}$$

Divide and simplify, getting,

$$V^3=(\rho g/4k)\,R^2S \qquad \text{——————(7)}$$

which is the Lacey Final Regime Test Formula.

Use this result in (3a) and get:—

$$V=4/3\,(\rho g/\Phi).\,RS \qquad \text{——————(8)}$$

which is the simple flow formula found in section I by solving Lacey's (*ii*) and (*iii*) together.

Reverse the process, *i.e.*, solve (7) with (8) and get

$$V^2/R=3\Phi/16k \qquad \text{——————(9)}$$

which is Lacey's formula defining 'f.'

That Lacey's formulæ (*ii*) and (*iii*) follow in this manner is strong evidence that the energy relation (3) is correct. A more stringent test will be given later.

IV. Deductions from the New Theory.

(*i*) The conventional force relation is—

$$\Phi V_b=\rho gRS \qquad \text{——————(10)}$$

Divide this by (8) and we get:—

$$V_b=0{\cdot}75V \qquad \text{——————(11)}$$

This remarkable relation can be tested easily. It should be remembered, however, that it is not impossible that V_b as used in this analysis may be an intermediate variable, and not the true bed velocity, *e.g.*, it is possible to derive the *form* of the equation for pure viscous flow by assuming a linear resistance law, when there is actually no boundary velocity at all.

(*ii*) The new theory shows that the two energy manifestations bear a simple ratio to each other, as we should expect.

Thus use (8) in the first term on the right of (4a) and that term becomes three-quarters of S. As the left side is S the remaining term on the right must be $\frac{1}{4}$S; *i.e.*, the boundary and internal energy rates are in the ratio of 3 : 1 if the channel is self-formed.

(*iii*) We can now get a better idea of the meaning of 'f.' Eqn. (9) gives Lacey's 'f' as $3\Phi/16k$. But (7) which is identical with the Section II (*ii*) form $V^3=(Cg^2/(vg)^{\frac{1}{3}})$. R^2S shows that $1/k$ is $4Cg/(p.(vg)^{\frac{1}{3}})$; whence we have:—

$$V^2/R=\tfrac{3}{4}\cdot\frac{\Phi}{p}\cdot\frac{gc}{(vg)^{\frac{1}{3}}} \qquad \text{——————————(12)}$$

The derivation of C showed that it is either an absolute constant or one multiplied by some function of the relative density of sand. Φ depends on the nature of the boundary, and may include the nature of the fluid. Whichever view is taken V^2/R definitely depends both on the nature of the boundary and the nature of the fluid. This, along with various other reasons, explains why attempts to link 'f' directly with d, and to find a constant 'f' from very exact observations over a long period have failed. Viscosity is very variable with temperature, and 50% throughout a year would not be remarkable. 'f' values plotted against $v^{-\frac{1}{3}}$ would be illuminating, and might show the nature of C and Φ definitely. For practice, of course, the value of the mean 'f' from surveys still remains unimpaired.

Although there is no point in dropping 'f,' it is interesting to note that the most logical criterion of similarity of silt content might be the rate of working by gravity per pound, *viz.*, gVS. For transposing all terms but S from the right of (7) to the left, multiplying both sides by gV, and using the values we already know for V^2/R we have:—

$$gVS= g(V^2/gR)^2.(vg)^{\frac{1}{3}}/C$$

$$=\frac{9}{16}\cdot\frac{\Phi^2}{p^2}\cdot\frac{gC}{(vg)^{\frac{1}{3}}} \qquad \text{——————————(13)}$$

which shows that, in all channels of the same boundary nature with fluid of the same physical properties and sand of the same density, and g the same, the rate of working per lb. is fixed.

A little algebraic manipulation will also show that
$gVS=2.(C'/C)\,(V^2/gR)\,(V/R)\,(R/P)\,(V^2/2)$. ————(14)

where C′ is the constant in $P/R=C'V\,(\upsilon g)^{\frac{1}{3}}$

Admitting that all these factors must occur it would be difficult to find a simpler formula.

The ratio C′/C is of interest. We have, using the suffix 0 to denote the mean value for the whole of the data Lacey analysed to find his formulæ:

$$V^3=C\,(g^2_0)/(\upsilon_0 g_0)^{\frac{1}{3}}\,R^2S=16^3\,R^2S \qquad \text{————(15)}$$

$$P/R=C'V/(\upsilon_0 g_0)^{\frac{1}{3}}\;=\;(64/9).V \qquad \text{————(16)}$$

whence, if g be taken as 32 exactly we get C′/C=16/9. Whether this simple figure is accidental, or has a definite meaning will be known when the new energy relation has led to more research into the physics of the problem.

(*iv*). The reason that the P, Q relation is not derived from the analysis is seen from (4) where P is eliminated, and Q cancels out. This is sufficient to indicate that a P, Q relation exists. Physically the reason seems to lie in the fact that a channel, in addition to obeying the dynamical laws must develop its section in such a way as to carry the silt load. It is possible to develop the formula from a fairly plausible argument; but that would be a digression. The argument leads one to consider that $\Phi \propto m\sqrt{gd}$ where m is the mass of incoherent *bed* silt per second per cusec. From this 'f' can be studied in terms of d by using (12).

V. *Extension of the New Theory to Rigid Channels.*—If we take skin friction experiments to show that the boundary resistance law in a rigid channel is quadratic than the boundary term in the energy equation (3) will require to be altered but the internal energy term, which depends on the nature of the fluid will remain unchanged in form, although k may have a different meaning.

In fact the force relation will now be:—

$$\Phi\,V^2{}_b P=\rho g PRS$$

and the rate of boundary work $\Phi V^3{}_b P=\Phi\;(\rho g\,\Phi)^{\frac{3}{2}}.\;(RS)^{\frac{3}{2}}.\;Q/RV$ so the energy equation will be:—

$$\rho g.QS=\Phi\,(\rho g/\Phi)^{\frac{3}{2}}\,R^{\frac{1}{2}}\,S^{\frac{3}{2}}Q/V+KQV^3/R^2 \qquad \text{————(17).}$$

reducing to $S=(\rho g/\Phi)^{\frac{1}{2}}\,R^{\frac{1}{2}}\,S^{\frac{3}{2}}/V+(K/\rho g).\;V^3/R^2$ ————(17).

The only self-adjusting dimension is R, so differentiating reference to R and omitting all terms with dV/dR as factor we get

$$0 = \tfrac{1}{2}(\rho g/\Phi)^{\frac{1}{2}}.\ S^{\frac{3}{2}}./(VR^{\frac{1}{2}}) - 2kV^3/(\rho g R^3) \qquad \text{——————(18)}$$

This simplifies to an equation of the form :—

$$V = \text{const.}\ R^{\frac{5}{8}}\ S^{\frac{3}{8}} \qquad \text{——————(19)}$$

which is the *dynamically sound form of Manning's formula.* The constants depend on the boundary and on the nature of the fluid. The formula applies to rigid boundaries and is quite wrong for regime channels, to which V=const. RS=another constant $R^{\frac{3}{4}}\ S^{\frac{1}{2}}$ applies. It is interesting to quote Bib. 1, Chap. IX which says of Bazin's points that they " do not always, even approximately, fall on the lines representing these classes ;——and on natural river channels the points fall in a very different manner."

To test the new formula the writer referred to Barne's " Hydraulic Flow Reviewed " and drew Figs. 2, 3, 4, on which sufficient references are given for the reader who wants to check the diagrams. It was most difficult to get data where R, S both varied for the one work, the roughness would be uniform over the section, the shape should be smooth, and the depth should be great enough to eliminate surface ripple Fig. 2 was for almost perfect conditions, the authority being Ftealey and Stearne. The crosses are for observations for one year, and the dots for another. The two dots badly off line are for very low velocities, presumably below critical. The agreement with formula is almost perfect. Fig. 3 is for almost negligible depths and velocities up to 18 feet per sec. Surface effect would drop the lower points below theoretical, so the fit is good under the circumstances. Fig. 4 is for four different channels, small and rectangular. The shape would distort velocity contours in such a way as to drop the lower points, so the fit is good.

In the writer's opinion the excellence of fit with accepted data justifies the New Energy Theory.

VI. *Summary.*—(*i*) By combining Hamilton's Principle with simple hydraulic concepts an energy equation has been found from which two of Lacey's relations follow.

(*ii*) This energy relation divides the total rate of working into a part arising at the boundary and another related purely to the interior of the fluid.

(*iii*) Further research, probably mathematical, may determine the nature of boundary resistance and eddy formation, give a direct method of

deriving the energy relations, and show how to sub-divide the two portions already determined.

(*iv*) In the process of deriving the energy relation it was found that, for a final regime channel, the two parts are in the ratio 3 : 1; that $V_b = \frac{3}{4}V$, where V_b may be an intermediate variable; that Lacey's 'f' depends on viscosity, and not only on silt load; and that the true criterion of similarity of behaviour should be that the rate of working per lb. is the same. Also, this rate of working can be expressed entirely in terms of physical constants, or entirely in terms of the usual measurable quantities of surveys.

(*v*) By extending the new energy relation to rigid channels an exponential formula $V = \text{const.}\ R^{\frac{5}{8}}\ S^{\frac{3}{8}}$ is obtained, where the constant depends on boundary and fluid natures.

(*vi*) This formula of (V) applies rigidly under limited conditions, of which one of the most important is that the channel shape should be such that the velocity is uniform over the perimeter; and another is that the channel should not be so small and shallow that surface ripple shows a noticeable transformation of energy at the surface.

(*vii*) One reason why all existing empirical flow formulæ are unsound is that they try to represent two dynamically different problems *viz.*, channels which can form their own S and channels which cannot.

(*viii*) The new exponential formula fits suitable data with very considerable accuracy.

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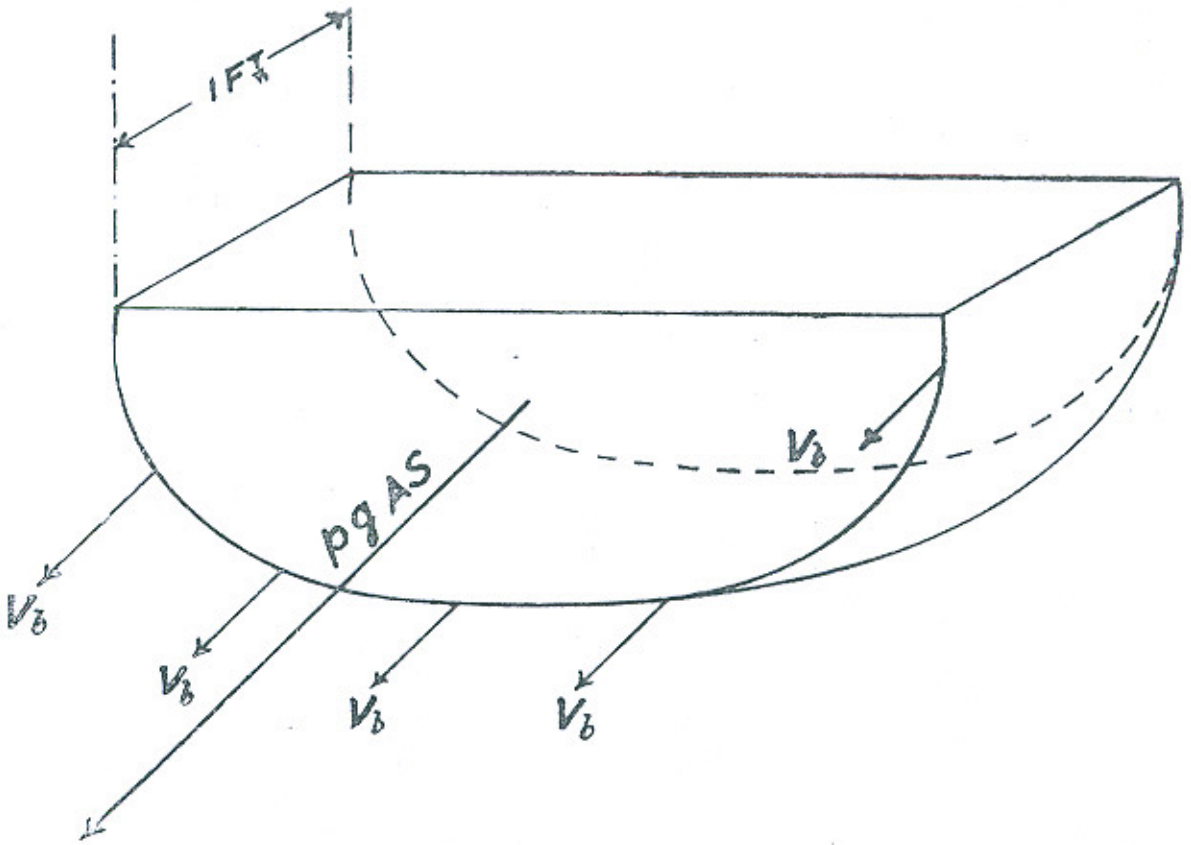


FIG. 1

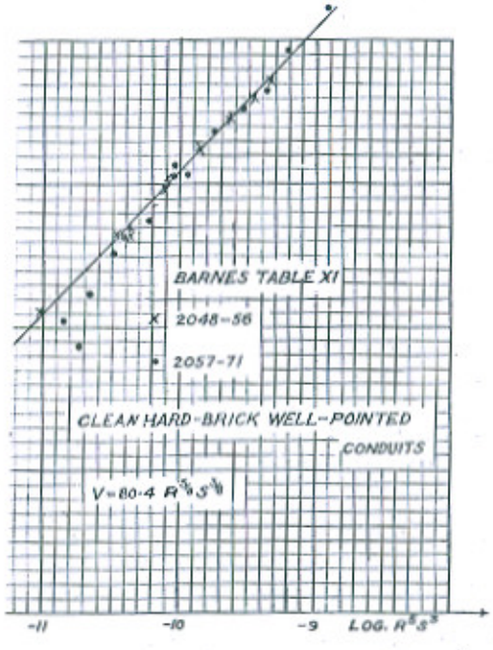


FIG. 2

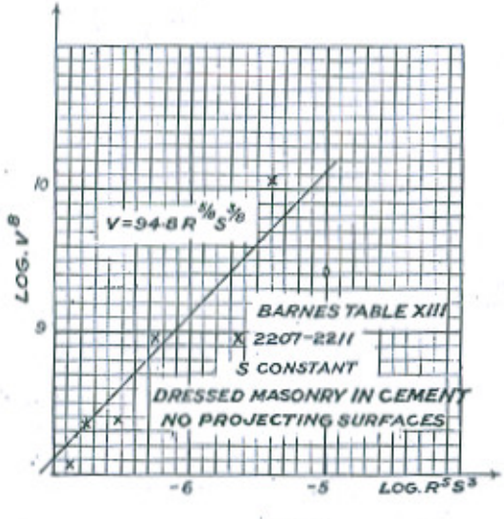


FIG. 3

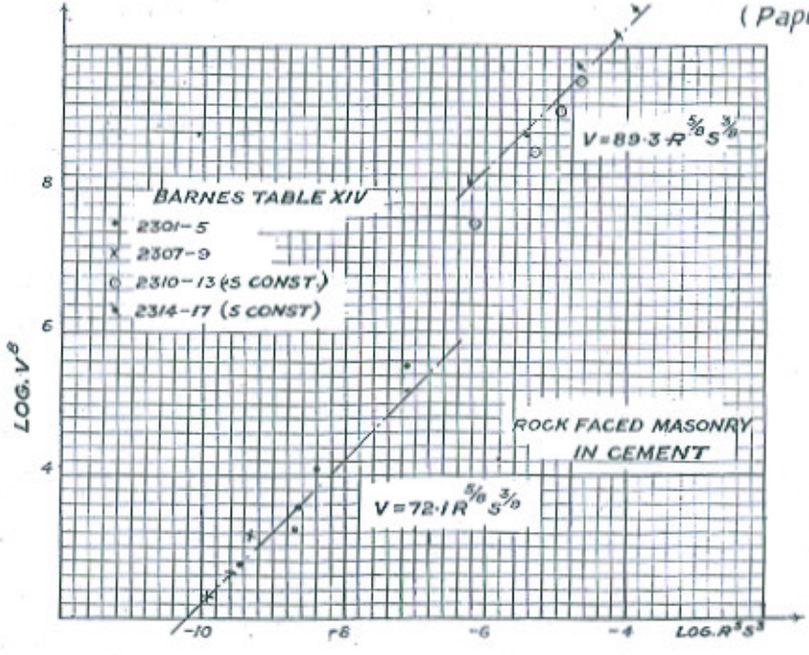


FIG. 4

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equations were familiar to many members of this Congress. Of this theory the speaker said that it had now been adopted in at least three Provinces in India, and that in the Punjab one important fundamental equation had been entirely confirmed; the second was now in very fair agreement, and the third linking rugosity in terms of the size of silt particle with the turbulence was still under investigation. That this should be so was not very surprising when it was realized that of the inner mechanism of turbulence in open channels people knew little. Mr. Lacey thought that he was correct in stating that the greater part of the theory which he published some years ago on flow in open channels had come to stay, subject only to minor modifications, such as a rugosity co-efficient based on actual measurements of silt size rather than a secondary characteristic such as the silt factor.

The speaker said that he would ask his mathematical colleagues in discussing Mr. Blench's Paper to remember the circumstances in which it was written. The engineer who bore the heat and burden of the day engaged in routine duties could not of necessity be aware of all the work that has been done in the field of his research elsewhere. He thought that this was of advantage, and that the worker who accepted the *non-possimus* of superior authority in matters of pure science lacked the initiative essential to original work.

The fresh implications of Mr. Blench's theory he proposed discussing in a written communication. With regard to Mr. Blench's remark that no definite shape was assigned to the channel in the Lacey relationships the speaker said that he had stated on many occasions that he considered the semi-ellipse as the perfect shape, and this shape allied with velocity contours should prove a useful field of research into the mechanism of turbulence. It would also be observed that Mr. Blench treated only R and S as variable, and dealt with unit length of wetted perimeter, when associated with unit discharge, as a constant. He assumed therefore implicitly that the ratio of the wetted perimeter to the product RV was constant in all turbulent channels. This law which he proposed some years ago had been abundantly proved in the Punjab and Sind by subsequent statistical analysis. The reason for this particular relationship was yet to be found in terms of turbulence. He considered Mr. Blench's theory of cushioned resistance, associated with a boundary layer, original and most valuable; his expression for turbulence, although it produced a relationship gratifying to the speaker, did not explain the mechanism and was a "locked box" with unknown forces in it. That did not mean that the expression was of necessity incorrect.

Mr. **Radha Kishan** said that Mr. Blench had discovered an easy method of evolving a new energy theory of flow of water, by simply working inversely to find what energy relation would give the Lacey Formulae, instead of starting from first principles. In the words of Mr. Blench, lack of detailed knowledge of the mechanism of eddy formation presented insuperable obstacles to all previous investigators, in their arriving at right

conclusions. Mr. Blench, by his great skill at 'algebraic manipulations', as he said, had been able to short-circuit that great obstacle; and without saying a word about the mechanism of eddy formation, had succeeded in perfecting his theory by simply resorting to a few multiplications and divisions.

He would not venture to offer any criticism of Mr. Blench as some people conversant with algebraic manipulations might be tempted to do, on his having accepted without argument the dynamical soundness of the Lacey empirical relations, on which assumption rested the entire structure of his theory. But it would perhaps be pertinent to enquire where to find a perfect Lacey channel to which his empirical relations were applicable. Mr. Blench had laid it down as most important to be realized that the following conditions were necessary for the Lacey formulae to be rigidly applicable: (*i*) The discharge must be fixed, (*ii*) the silt load must be fixed, (*iii*) the physical conditions of the fluid must be fixed, (*iv*) the silted condition must be fixed, (*v*) the channel must be self-formed in its own incoherent transported material (so mud berms would cause a deviation from perfection), (*vi*) the turbulence must be due to the incoherent boundary alone, and not to any extraneous cause, such as sharp bends, or nearby falls, or clods in the bed, etc. Also there was no definite rule to perfect channel shape. It would appear to be elliptical; but there was nothing definitely known except that channels, according to Mr. Blench, did tend to some perfect shape.

Perhaps the need for brevity had compelled Mr. Blench to cut short his list of conditions necessary for the Lacey formulae to be applicable to flow in any channel. But even so, the conditions were all-embracing, and had reduced the existence of a perfect Lacey channel to a purely imaginary conception. It could be argued that all higher sciences and great theories are based on imaginary conceptions. But the science of river engineering had not yet, unfortunately, advanced to such a stage as to benefit by symbolic representations of natural forces, by means of complex mathematical formulæ, the authority for which was derived not from the actual behaviour of flow in natural and artificial channels, but from their extremely nice fit with the Lacey formulæ, which in turn were based on the imaginary conception of a perfect channel. Mr. Blench would have therefore done some useful work in the sphere of Hydraulic Science, if instead of working inversely to find what energy relation would give the Lacey formulæ, he had tried to acquire some knowledge of the mechanism of eddy formation, and had discovered for himself the dynamical unsoundness of the Lacey empirical relations, on the study of which he had spent eight long years without arriving at any results. If instead he had spent only eight hours, with an open mind on the study of the speaker's 50-paged pamphlet, entitled "Silt Theory of Flow of Water," he would have gained a clear insight into all the fundamental facts of flow of water in rivers and streams. He would not only have found out what energy relation would give correct formulæ, but he would also have realized the function of silt in flowing water, on the study and

experimenting of which the Punjab Engineers had spent ages of time and treasuries of money, and had still not got beyond the stage of collecting data about the dimensions of silt particles measured in millimetres and fractions of millimetres.

He would have realized that the function of silt, from the heaviest rocks and boulders to the finest of sand and powder, was to maintain the regime or the existing condition of any channel, to conform with the energy of flowing water. If the differences between the various sizes of silt particles present anywhere in a channel were great, as in the hills, and at the foot of the hills, and we could manage to extract the heaviest grades of particles from the channel at any point of its flow, then the channel below that point would be scoured out by the removal of bed silt, in order that it might acquire a flatter slope of flow such that the particles of silt that were left in it after silt extraction were heavy enough or large enough to preserve or maintain the changed regime of the channel. If, on the other hand, the differences in sizes of silt particles present anywhere in a channel were not very marked, as was the case in all flat sloped channels in the plains, then the extraction of the heaviest grades of silt did not help appreciably to scour the bed below the point of silt extraction, simply because the silt that remained in the channel was heavy enough to maintain the channel in more or less its old regime. Arguing on the basis of this function of silt in flowing water, it was possible to explain all the known facts, without a single exception, about the presence of different grades of silt in different channels, or in different reaches of the same channel. By the same argument it could be said with certainty whether silt extraction from any channel served the purpose for which it was intended. If, for example, the extraction of silt from the Upper Bari Doab Canal at or near its head were intended for the purpose of increasing the capacity of the canal below the point of extraction, then such extraction would be eminently useful. If, on the other hand, it were for the purpose of reducing the silt trouble in branches and distributaries, then silt extraction at the head of the canal would be perfectly useless. Similarly, if there were considerable differences in the grades of silt in the Upper Jhelum Canal, then the working of silt extractors would increase the capacity of the canal ; otherwise they would be more or less ineffective.

Mr. **M. D. Mithal** said he would like the Author to clear some inconsistencies in the arguments in the Paper.

In the second paragraph on page 69, the Author said that Lacey's empirical relations were of " obviously dynamic soundness "; but on page 71 he turned a somersault when he said that these formulæ were " statistically derived." Could he explain how they can be dynamic and static at one and the same time !

The speaker said that he asked himself why it was that Mr. Blench had taken pains to find exact mathematical relations to justify Lacey's

formulae when on page 70 he declared that the formulae would hold only under the following conditions :—

(*i*) the discharge must be fixed,

(*ii*) silt load must be fixed,

(*iii*) physical conditions of the flow must be fixed,

(*iv*) the silt conditions must be final,

(*v*) the channel must be self-formed in its own coherent transported material,

(*vi*) and there should be no bends and mud berms.

These were impossible conditions for any channels belonging to the Irrigation Branch to fulfil. Could the Author produce a channel of this type?

Obviously these formulæ had no use for irrigation engineers who are on earth and have to work with varying discharges, varying silt charges, varying physical conditions like temperature and ghats due to cattle and jungle floating in the stream, and berms and bends. The silt conditions changed seasonally in the best of channels.

Dr. **N.K. Bose** said that as the whole Paper of Mr. Blench was based on Eq. (3) he would like to examine it carefully. The Author had put this as " Rate of working by gravity on unit length equals the rate at which the boundary resistance does work plus the rate at which the energy is dissipated internally." So that—

$$\rho g Q S = \phi \rho V_b^2 + K Q V^3 / R^2.$$

The first term on the right-hand side he derived from the following expression for the boundary resistance

$$\phi V_b P$$

The usual law of turbulent boundary resistance as derived by Prandtl and Von Karman for the case of flow over plates was, he said—

$$D_f = \lambda_s \frac{\rho}{2} \bar{u}^2.$$

where $s = Cl$ the surface, $\bar{u}$ the undistributed velocity and

$\lambda = \frac{0{\cdot}072}{\sqrt[5]{R}}$, R being the Reynold's Number $\frac{l \bar{u}}{R}$

Of course this law held when the boundary was fixed, whereas in the case dealt with by Mr. Blench though the ultimate boundary was fixed, the boundary layer through which the momentum passed from the boundary to the free liquid above was composed of silt in suspension.

Professor Schmidt of the University of Vienna had tried to solve this problem of turbulent boundary resistance on the same lines as he had followed in the case of atmospheric turbulence. What was required for canal flow was the necessity to determine the fine structure of turbulence so that it would be possible to correlate the different states of regime. So long as this was not done, advance must be on purely empirical lines.

Attempts like those of Mr. Blench to use his own words " to work inversely to find what energy relation would give Lacey formulæ " would lead nowhere.

The second term in his Eq. (3) which was the rate at which the energy was dissipated internally

$$\left(\frac{KQV}{R}\right)^2$$

might give the required result but it was not verified by any previous experiments. To take V/R as the average velocity gradient was not at all justified in such calculations where Hamilton's principle was invoked. V/R as the average velocity gradient might be quite good for the average engineer, but should not find a place in such equation as Equation (3).

Hamilton's Principle stated that—

$$\int L\ dt \text{ has a stationary value.}$$

Where L=T--V=Kinetic Energy—Potential Energy.

For the case when the forces are not all conservative such as occur in this problem this integral reduced to

$$\int_{t_1}^{t_2} (\delta T + \Sigma Q_v\ \delta q_r) dt = 0$$

so that Mr. Blench's reduction of law to " the K. E. being a minimum " was not correct.

At the end the speaker quoted the following remarks of the Author : " an investigation on energy lines would be illuminating but the lack of detailed knowledge of the mechanism of eddy formation has been an insuperable obstacle."

Mr. **A. R. Thomas** contributed the following remarks on the Paper :

The Author was to be congratulated on his bold attempt to produce a rational theory of flow in alluvial channels. On page 70, among the conditions necessary for the Lacey formulæ to be rigidly applicable, he mentioned that the silt load must be fixed. The silt load was surely implicit in regime conditions, being the load gradually attained in a long length of channel flowing for a long period. If a silt load other than the regime load were imposed a different section would result. If the water surface level were free to adjust itself this section would tend to revert to the regime section by a raising or lowering of the bed until the silt load became the regime quantity. If however the water surface level were at the same time controlled, as might occur for example above a river barrage or canal regulator, the section would approach a limit different from the Lacey regime section. This may explain many apparent departures from the regime formulæ.

The Author had mentioned that there was no definite rule on the perfect channel shape. It seemed clear, by visualizing the process of distribution of sand grades in the channel section—coarse in the bed and fine in the berms—that the shape of the channel was a function of the size distribution curve of the sand. The greater the proportion of particles of extreme sizes the more rectangular would be the section and conversely the greater the proportion of medium particles the more gradual the merging of berm and bed.

He would not expect the Froude Number V^2/gr to appear in the flow equation (p. 71), as it was a gravity function concerned with the shape of the free surface and not with turbulence, frictional resistance or other internal phenomena. The Reynold's Number may be expected to appear in any expression for turbulence in channels. The effect of gravity was seen in the longitudinal slope and in the sectional shape and its influence could be isolated. Firstly, it was a source of energy, represented in the energy gradient g S. Secondly, it exerted a downward pull on the sand particles, represented in the apparent weight of the particles in the fluid $g(\sigma-\rho)$ where σ is the density of the sand material and ρ that of the fluid. Provided that changes in shape of the free surface were very small compared with the main dimensions of the channel, gravity had no further practical effect and we could expect g to occur in the flow equations only in combination with S and $(\sigma-\rho)$.

It was thus possible to represent the density of sand in the flow formulæ, assuming that the viscosity of the moving sand, due to internal friction, was implicit. In the Lacey regime formulæ

$$P/R=K_1V$$
$$V=K_2\sqrt[3]{R^2S}$$

and $$V=K_3\sqrt{f\,R}$$

Dimensional analysis gave

$$K_1 = \text{Constant} \times \frac{\rho}{\mu^{\frac{1}{3}} g^{\frac{1}{3}} (\sigma-\rho)^{\frac{1}{3}}}$$

and $$K_2 = \text{Constant} \times \frac{g^{\frac{5}{9}} (\sigma-\rho)^{\frac{2}{9}}}{\mu^{\frac{1}{9}} \rho^{\frac{1}{9}}}$$

It was not possible to obtain K_3 unless the relationship between f and the diameter of sand particle was known. If $f \propto \sqrt{d}$ given by Mr. Lacey only as a very rough qualitative formula, then

$$K_3 \sqrt{f} = \text{Constant} \times \frac{g^{\frac{7}{12}} (\sigma-\rho)^{\frac{7}{12}}}{\rho^{\frac{5}{12}} \mu^{\frac{1}{6}}} d^{\frac{1}{4}}$$

Taking the values of the constants given by Mr. Lacey in his 1934 paper "Uniform Flow in Alluvial Rivers and Canals" and equating them to the above expressions with $g=32{\cdot}2$, $\rho=1{\cdot}94$, $\sigma=5{\cdot}1$ and $\mu=23{\cdot}3\times10^{-6}$ (the value for a temperature of 16°C) the equations became

$$\frac{P}{R} = 0{\cdot}599 \frac{\rho^{\frac{2}{3}}}{\mu^{\frac{1}{3}} g^{\frac{1}{3}} (\sigma-\rho)^{\frac{1}{3}}} V$$

or $$P = 0{\cdot}744 \frac{\rho^{\frac{1}{2}}}{\mu^{\frac{1}{6}} g^{\frac{1}{6}} (\sigma-\rho)^{\frac{1}{6}}} \sqrt{Q}$$

$$V = 0{\cdot}594 \frac{g^{\frac{5}{9}} (\sigma-\rho)^{\frac{2}{9}}}{\mu^{\frac{1}{9}} \rho^{\frac{1}{9}}} \sqrt[3]{R^2 S}$$

It was to be remembered that the above constants contained factors representing the shape of the sand particles and their size distribution.

The Author mentioned boundary slip. The slip was of course not between water and sand but was a sliding motion in the sand. If as the Author suggested this was independent of the properties of the water the co-efficients derived by dimensional analysis would require modification; though the principle that g appears only in company with S or $(\sigma-\rho)$ would always apply.

It was to be hoped that the Author would produce a rational derivation of his expression for the dissipation of energy by turbulence $KQV(V/R)^2$. The absence however of one link in the chain of reasoning was in itself no condemnation of the theory presented in this interesting paper.

Mr. **Gerald Lacey** replied to the discussion on behalf of the author and said that Mr. Khanna had criticized at great length the Lacey theory. That theory had been derived from a great mass of data and the empirical equations fitted them. It was true that a channel in perfect regime was seldom encountered and the scatter in plotted observations made that very clear. A regime channel in perfect regime was possibly an "imaginary" mathematical concept, but that did not render it illusory.

Mr. Khanna had advanced a theory of his own but observational data were lacking. A theory based on imaginary data was of all theories most objectionable. Mr. Khanna thought that if Mr. Blench had devoted eight hours to studying the Khanna theory instead of eight years to study the Lacey theory he would have obtained a clear insight into the fundamental facts. Mr. Lacey had devoted much more than eight hours to studying Mr. Khanna's pamphlet; it was debatable whether the time had been well spent, and he freely confessed that he had been unable to detect any degree of coherence in the arguments put forward. Other members of the Congress might, or might not, have been more fortunate.

The objections raised by Mr. Mithal were objections that could be raised to any mathematical presentation of flow in channels. The equations rigidly applied when there were no disturbing factors such as bends, cattle ghats, floating jungle, etc. The practical engineer knew without telling that if he did not take the trouble to maintain his channels they would demand a greater slope.

Mr. Thomas's valuable constructive criticism he would leave Mr. Blench to answer.

Dr. Bose had correctly represented the attitude of the mathematician in the remarks he had made on Mr. Blench's paper.

It was by no means clear however that the attempt to find an energy relation underlying the Lacey empirical equations would "lead nowhere." Engineering practice would never have advanced if empirical equations based on reliable data had not been freely employed.

The attitude of Dr. Bose was one of pure negation. In Mr. Lacey's view Mr. Blench's paper put forward a line of attack that would prove very valuable. To what final conclusion it would lead was not at present known. The theory was a distinct advance in the treatment of the problem.

Correspondence.

Mr. **Blench** in correspondence, regretted that illness had prevented his presence; but felt it was fully made up by Mr. Lacey's introduction and answer on his behalf. It was pertinent that Mr. Lacey should

introduce an extension of his own theory, and gratifying that he had expressed so clearly the author's method of attack and motive. To a purely engineering or purely mathematical audience the analysis must have seemed unconventional although the atomic physicist, who had also to deal with phenomena of which he did not know the details, would proceed even more unconventionally because he had learnt the value of advancing with available weapons, rather than stagnating until the perfect weapon arrived. The author was most indebted to Mr. Lacey for having explained his viewpoint perfectly, and glad that he considered it of potential value.

Mr. Khanna had been adequately dealt with by Mr. Lacey.

Mr. Mithal made the propositions (*i*) that a statistically derived formula could not be dynamically sound, an obvious *non sequitur* and (*ii*) that the Lacey formulae lacked practical value because they would be *rigidly* applicable only under restricted conditions. This being in disagreement with practice, where the Lacey formulae fitted very well indeed, could be dismissed immediately.

Mr. A. R. Thomas's constructive remarks were much appreciated by the author. They covered ground rather too extensive to reply to in detail, and he hoped he would have further correspondence with him. An abridged answer was—

(*i*) The general term "silt load" was defined from $V=1{\cdot}151\sqrt{fR}$, so that there was no need for unmeasurable terms, such as the diameter of silt in the active incoherent boundary layer. Theoretically, at least, it should be possible as an alternative to express f in terms only of those quantities which define the nature of the boundary layer and fluid. That these quantities varied rapidly from place to place, and from time to time, was known; and f found from formulæ involving only the measurable quantities of a channel was doubtless a mean. For this reason the slope discharge formula was more useful in channel analysis than is the fundamental $V=1.151\sqrt{fR}$, *vide* P. E.C. Paper No. 184. Likewise, as pointed out by Mr. Inglis in his comments on Dr. Bose's P. E.C. Paper No. 192, the designing of channels from silt samples was not normally practicable unless, as in designing a new canal system, there was no alternative. Fluctuating discharges controlled by a weir might quite well set up pseudo-regime. This was not, of course, Uniform Flow. The point had been mentioned by Mr. Inglis in his comments on I.C.E. Paper No. 4893.

(*ii*) Mr. Thomas's views on the mechanism of section formation, although unavoidably speculative, were interesting. It would be useful were sand analysis done across some of the approximately elliptic shapes found in practice. The author was inclined to blame the coherent material for departure of shape from the semi-ellipse. The Froude

Number, in steady flow, was the ratio of inertia force to gravity force; the Reynold's Number the ratio of inertia force to frictional force. In turbulent flow in rough pipes viscosity did not appear at all in the flow equation, so there could be no doubt that V^2/R was the criterion representative, in some way, of turbulence. The writer had gone into the meaning of V^2/R, which appeared in all turbulent flow formulæ, whether for rough or smooth rigid channels and pipes, or for Lacey channels, in a Paper filed with the Central Board of Irrigation. Mr. Thomas's introduction of ρ and σ was interesting because the writer's recent researches indicated that $(\nu g)^{\frac{1}{3}}$ contained g because of the fact that the upper part of the boundary layer contains material of density different from that of water, and because the P/R formula seemed to be linked with silt carrying power. (ν represented kinematic viscosity).

Replying to Dr. Bose, the Author pointed out that he (Dr. Bose) had quoted a formula for *plates* derived from Blasius's formula for *smooth* pipes to disprove the author's formula for *rough* pipes. Prandtl's latest pipe formula for rough conditions was $V = \sqrt{8g}\ RS\ (2{\cdot}0\log_{10} 2R/l + 1{\cdot}74)$ which agreed almost exactly with Manning's formula over the range for which it could be justified.

The Author had since advanced considerably and correlated existing authoritative formulae for smooth and rough channels and pipes, with an improved flow formula of his own, and with the Lacey formulæ. There seemed therefore no reason to doubt the correctness of application of Hamilton's Principle. The new work was to be found in the Note "Turbulent Flow within Rigid and Incoherent Boundaries" filed with the Central Board of Irrigation, Simla.