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Paper No.519

AUTOMATED STRUCTURAL ANALYSIS

ABSTRACT

This paper deals with the fundamental concepts of automated analysis of the structures. The author has explained the automation process with the help of a plan frame analysis programme (FRAP), developed by him. Relevant parts of the FRAP source code have been reproduced to illustrate the suggested algorithm for computerizing each step of the analysis procedure. The analysis is based on the Direct Stiffness Method. Part of the programme computes static equivalent of the lateral forces generated by an earthquake. Lateral forces resulting from wind pressures have also been discussed briefly. Appendix-A consists of a hand analysed small structure (carrying gravity and lateral loads) and the FRAP output of the same structure for illustration and validation purposes. Computation of lateral forces on another structure, due to earthquake and wind are included in Appendix-B.

Paper No. 519

AUTOMATED STRUCTURAL ANALYSIS

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INTRODUCTION

The Computers have revolutionized the science of structural analysis. Today, the engineer does not have to be shackled to a cumbersome process of painstaking hand calculations for the complex strucutres. A number of commercial programmes are available to help him in solving structural engineering problems of fairly complicated nature at an amazing speed. The use of such programmes, however, is not so simple because they heavily depend on the way the structural system is modelled and the output is interpreted. This is why most of the commercial programmes disclaim, one way or the other, the responsibility for the accuracy of the results. Besides the expert use of the programme the present day engineer essentially requires a working knowledge of the algorithim employed in the computer analysis of structures to help him in effectively modelling them and interpreting the results.

The intent of this paper is to go over some fundamental concepts of automated structural analysis with the help of a Frame Analysis Programme (FRAP) developed by the author. FRAP is plane frame analysis programme. Its version of the BASIC Language has been used in the paper in view of its easy to understand syntax as compared with FORTAN. The alogorithm has been discussed in sufficient detail, listing the relevant portions from the FRAP source code to illustrate the programming steps involved. With the help of this paper the engineers with some knowledge of computers and structural engineering are expected to develop an insight required for a thorough understanding of computer oriented structural analysis. Engineers not in touch with the computers and modern computing methods are expected to get a "feel" for the application of digital computer in the structural analysis.

Sections 1 thru 8 of the following discussion cover main steps of structural analysis and a suggested approach to computerize each step. Section 9 deals with computing the static equivalent of the lateral forces induced by the earthwuake while section 10 contains a similar treatment of the wind loads.

Hand solution as well as FRAP analysis of an example structure have been given in Appendix-A to validate the computer programme. Appendix-B contains hand calculated wind and seismic forces on a 17 storeg building. The equivalent static forces generated by FRAP subroutines for same building have also been included in Appendix-B for comparison. Automated analysis has been based on the Director Stiffness Method. Section 1 covers the first step of the analysis procedure which consists of constructing

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Member Stiffness Matrix.

1.0 COMPUTE THE MEMBER STIFFNESS IN MEMBER COORDINATES [S]

Member Stiffness is represented by a 6 x 6 matrix given below. The six degrees of freedom (DOF), three on each end, are shown in figure 1(a) for a member of the plane frame. Forces corresponding to three degrees of freedom F1,F2, and F3 at the left end, are indicated in figures 1(b), (c) & (d). The forces of fig. 1(b), resulting from a unit displacement (rotation in this case) corresponding to Ist degree of freedom, constitute the first row of the Member Stiffness Matrix. Second and third rows of the matrix are obtained from figures I(c) & I(d). The last three rows relate to the DOF on the right end and follow from symmetry. In general an element Sij of the member stiffness matrix represents a force corresponding to jth degree of freedom resulting from a unit displacement at ith degree of freedom.

$$\begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{-6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & 0 \\ 0 & 0 & \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} \\ \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{-6EI}{L^2} & 0 \\ \frac{-6EI}{L^2} & \frac{-12EI}{L^3} & 0 & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & 0 \\ 0 & 0 & \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} \end{bmatrix}$$

MEMBER STIFFNESS MATRIX [S]

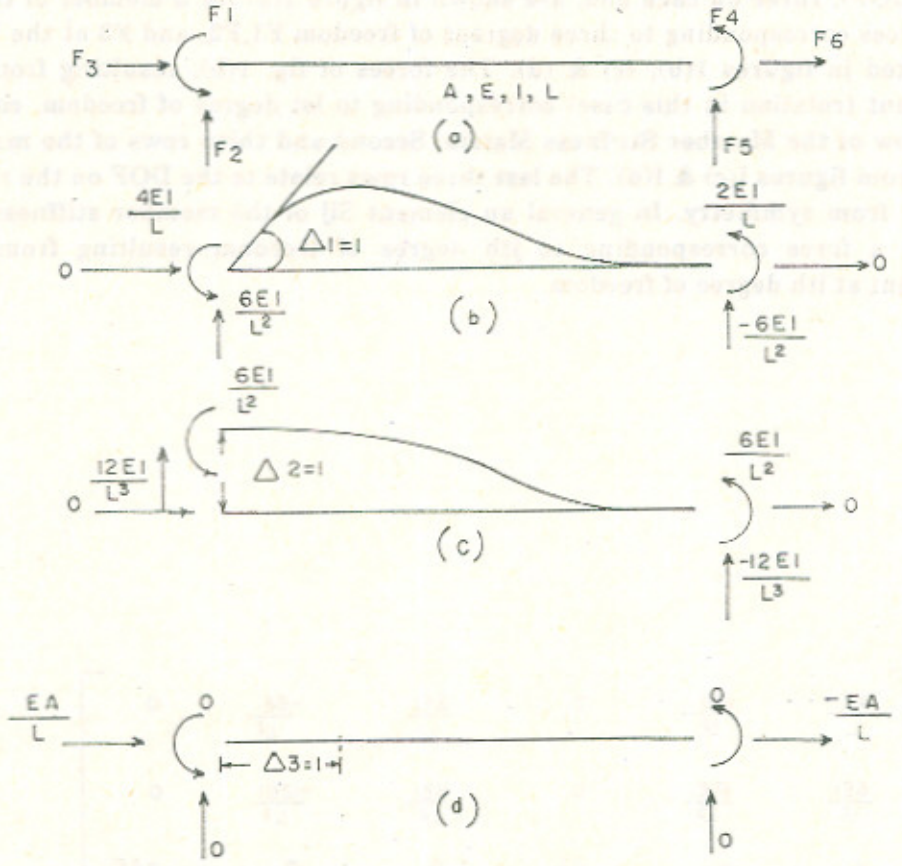


FIG.1 ELEMENTS OF THE MEMBER STIFNESS MATRIX

FIG.1 ELEMENTS OF THE MEMBER STIFFNESS MATRIX

FRAP statements 5200 to 5370 accomplish the simple task of constructing the member Stiffness Matrix. Upper half of the matrix is computed from the entered member properties and calculated member lengths. The lower half is generated from symmetry.

```
5200 REM SUBROUTINE FOR GENERATING STIFFNESS MATRIX IN MEMBER COORDS
5205 REM UPPER HALF OF THE MEMBER STIFFNESS MATRIX
5210 E=E(I)
5215 M=MIR(I)
5220 A=A(I)
5225 L=12*L(I)
5230 S(1,1)=4!*E*M/L
5235 S(1,2)=6*E*/L^2!
5240 S(1,3)=0!
5245 S(1,4)=2!*E*M/L
5250 S(1,5)=S(1,2)
5255 S(1,6)=0
5260 S(2,2)=12*E*M/L^3!
5265 S(2,3)=0!
5270 S(2,4)=S(1,2)
5275 S(2,5)=S(2,2)
5280 S(2,6)-0!
5285 S(3,3)=A*E/L
5290 S(3,4)=0!
5295 S(3,5)=0!
5300 S(3,6)=-S(3,3)
5305 S(4,4)=S(1,1)
5310 S(4,5)-S(1,2)
5315 S(4,6)=0!
5320 S(5,5)=S(2,2)
5325 S(5,6)=0!
5330 S(6,6)=S(3,3)
5335 REM GENERATE LOWER HALF OF THE MEMBER STIFFNESS MATRIX
5340 FOR J=1 TO 6
5345 FOR K=1 TO 6
```

```
5350 IF K < J OR K=J THEN 5360
5355 S(K,J)=S(J,K)
5360 NEXT K
5365 NEXT J
5370 RETURN
```

2.0 TRANSFORM THE MEMBER STIFFNESS TO GLOBAL COORDINATES

The member stiffness relations in the member coordinates are rotated to the global coordinates with the help of a transformation matrix [T]. The transformation matrix for the considered degrees of freedom is as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 & 0 & 0 \\ 0 & \sin\theta & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$$

THE TRANSFORMATION MATRIX [T]

The horizontal inclination of each member is computed by the programme along with member lengths from the input coordinates. The Member Stiffness Matrix, in global coordinates, [KM], is obtained from following relations

[KM] = [TT] [S] [T]

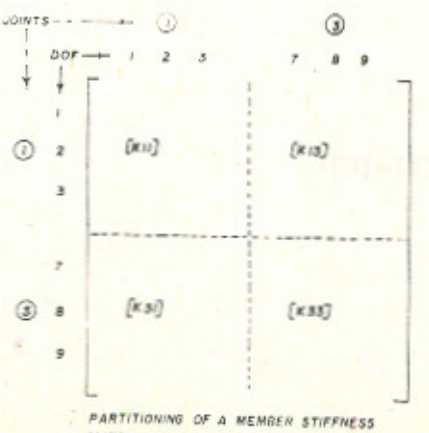
The subroutines 5500 to 5670 and 6000 to 6060 construct [T] and [TT] (transpose of [T] respectively, while subroutines 6500 to 6600 and 7000 to 7090 perform matrix multiplications in two stages to obtain [KM] for individual members.

```
5500 REM GENERATE TRANSFORMATION MATRIX [T]
5510 ANG=AGL (I)
5520 FOR J=1 TO 6
5530 FOR K=1 TO 6
5540 T(K,J)=0!
5550 NEXT K
5560 NEXT J
5570 T(1,1)=1!
5580 T(2,2)=COS(ANG)
5590 T(2,3)=-SIN(ANG)
5600 T(3,2)=SIN(ANG)
5610 T(3,3)=COS(ANG)
5620 T(4,4)=1!
5630 T(5,5)=COS(ANG)
5640 T(5,6)=-SIN(ANG)
5650 T(6,5)=SIN(ANG)
5660 T(6,6)=COS(ANG)
5670 RETURN
6000 REM GENERATE T-TRANSFORMED MATRIX [TT]
6010 FOR J=1 TO 6
6020 FOR K=1 TO 6
6030 TT(K,J)=T(J,K)
6040 NEXT K
6050 NEXT J
6060 RETURN
6500 REM COMPUTE [S]*[T]=[C]
6510 REM C(6,6)=[S]*[T]
6520 FOR N=1 TO 6
6530 FOR J=1 TO 6
```

```
6540 C(N,J)=0
6550 FOR K=1 TO 6
6560 C(N,J)=C(N,J)+S(N,K)*T(K,J)
6570 NEXT K
6580 NEXT J
6590 NEXT N
6600 RETURN
7000 REM COMPUTE MEMBER STIFFNESS IN GLOBAL COORDS.
7010 REM [KM]=[TT]*[C]
7020 FOR N=1 TO 6
7030 FOR J=1 TO 6
7040 KM(I,N,J)=0
7050 FOR K=1 TO 6
7060 KM(I,N,J)=KM(I,N,J)+TT(N,K)*C(K,J)
7070 NEXT K
7080 NEXT J
7090 NEXT N
```

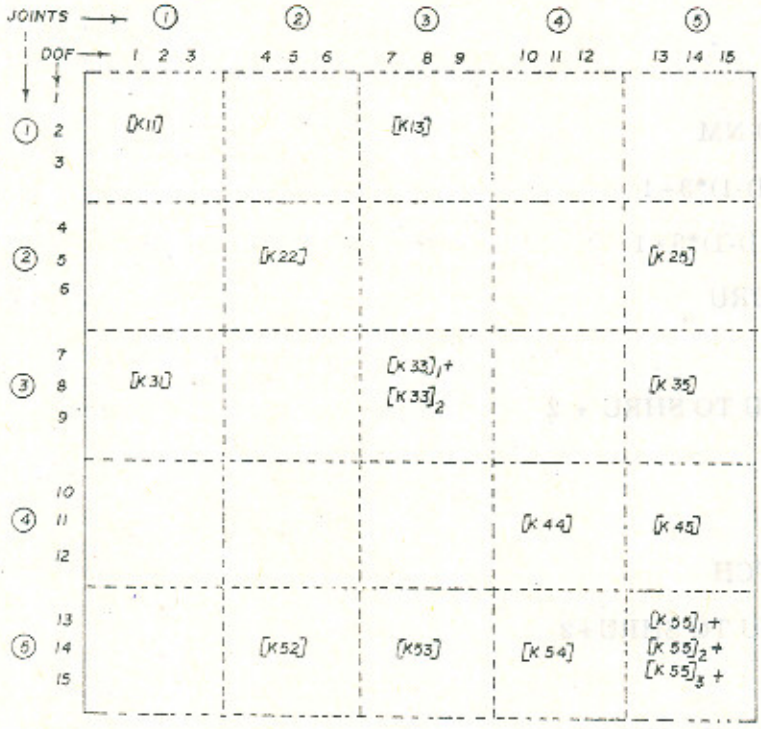
3.0 ASSEMBLE THE GLOBAL STIFFNESS MATRIX [KG]

The next step is to assemble the Member Stiffness Matrices into the Global Stiffness Matrix of the structure. As the example structure of Appendix-A consists of 5 joints with three degrees of freedom at each join the Assembled Stiffness Matrix will be 15 x 15 in size. The assembling process aims at re-arranging the individual Member Stiffness Matrices so as to form the appropriate rows and columns of the Assembled Stiffness Matrix. Member no.1, for instance, spans from joint no.1 to joint no.3, its 6 x 6 stiffness matrix is partitioned as under:



PARTITIONING OF A MEMBER STIFFNESS MATRIX

The sub matrices, each 3x3 in size, must appear at their shown locations in the Assembled Stiffness Matrix. Joint no.3 is common between member 1 and 2, therefore we get two [K33] matrices each from members 1 and 2. These matrices are added and placed at the indicated location in the Assembled Matrix. Joint no.5 is common between three members, there are, as such, three [K55] matrices added together in their final destination.



FORM OF THE ASSEMBLED STIFFNESS MATRIX OF THE EXAMPLE STRUCTURE

Subroutine 7500 to 7750 constructs Assembled Stiffness Matrix of the structure. A member has a starting joint (ST) and an ending joint (ED) input as member incidences. Begining with member no.1 (statement 7570) the programme determines lowest degrees of freedom (SHRU and AKHR) at starting and ending joints respectively, and updates the relevant rows of the Assembled Stiffness Matrix for all the three DOF at the two ends of the member. Some DOF are encountered more than once by the programme. The DOF U7, U8, U9 (Appendix-A) occur at the ending joint of member no.1 and also at the starting joint of member no.2. The relevant degrees of freedom are updated by required additions when member no.2 is considered, thus obtaining the sum [K33] + [K33]2 as indicated in the Assembled Matrix. The algorithm fits the submatrices of the individual members in their appropriate places allowing for effect of the members on common DOF. The procedure devised by the author is one of the many possible.

```
7500 REM STATEMENTS UPTO 7750 CONSTRUCT THE SIGULAR STIFFNESS
 MATRIX
7520 FOR J=1 TO NJ*3
7530 FOR K=1 TO NJ*3
7540 KG(K,J)=0
7550 NEXT K
7560 NEXT J
7570 FOR I= 1 TO NM
7580 SHRU=(ST(I)-1)*3+1
7590 AKHR=(ED(I)-1)*3+1
7600 SWITCH=SHRU
7610 C1=0!
7620 FOR J=SHRU TO SHRU + 2
7630 C1=C1+1
7640 C2=0!
7650 SHRU=SWITCH
7660 FOR K=SHRU TO SHRU+2
7670 C2=C2+1
7680 KG(J,K)=KG(J,K)+KM(I,C1,C2)
7690 NEXT K
7700 IF K=AKHR+3 THEN 7720
7710 SHRU=AKHR : GOTO 7660
7720 NEXT J
7730 IF J=AKHR+3 THEN 7750
7740 SHRU=AKHR:GOTO 7620
7750 NEXT I
```

4.0 GENERATE THE RIGHT HAND SIDE LOAD VECTOR

The Right Hand Side Load Vector is assembled in following three steps:

i) Generate Fixed-End Forces Vector for each member. The Vector consists of six reactions at the two ends of a member. Fixed end moments alongwith vertical and horizontal reactions can be easily computed for a given set of loading. For combination of point loads and (UDL) the FRAP statemens 1250 to 1355 compute fixed end force vector and store it in a two dimensional array (FM) for each member. The Fixed End Force Vector can be optionally entered from the key board for a member with an unusual loading.

```
1250 REM COMPUTE FIXED END FORCES
1255 FOR I=1 TO NM
1260 N=NPL (I)
1265 W=UDL (I)
1270 L=L(I)
1275 FM (I,1)=W*L^2 : REM WL^2/12*12(IN - KIPS)
1280 FM(I,4)=-FM(I,1)
1285 FM(I,2)=W*L/2
1290 FM(I,5)=W*L/2
1295 FM(I,3)=0
1300 FM(I,6)=0
1305 IF N=0 THEN 1355
1310 FOR J=1 TO N
1315 P=PL(I,J)
1320 X1=XPL(I,J)
1325 X2=L-X1
1330 FM(I,1)=FM(I,1)+P*X1*X2^2/L^2*12
1335 FM(I,4)=FM(I,4)-p*X2*X1^2/L^2*12
1340 FM(I,2)=FM(I,2)+P*X2/L
1345 FM(I,5)=FM(I,5)+P*X1/L
1350 NEXT J
1355 NEXT I
```

ii) Rotate the Fixed End Force Vector to global coordinates and assemble the equivalent Joint Load Vector. Following relation is used in rotating the Fixed End Force Venctor to global coordinates.

$$\{FG\} = [TT]\{FM\}$$

Statements 8000 to 8130 rotate the fixed end forces to global coordinates and assemble the equivalent Joint Load vector.

```
8000 REM CONVERT FIXED END FORCES VECTOR OF INDIVIDUAL MEMBERS IN
8010 REM GLOBAL COORDS. AND ASSEMBLE THE EQUIVALENT JOINT LOAD VECTOR
8020 SHRU=(ST(I)-1)*3+1
8030 AKHR=(ED(I)-1)*3+1
8040 K=SHRU
8050 FOR J= 1 TO 6
8060 FOR L=1 TO 6
8070 FG(K)=FG(K)+TT(J,L)*FM(I,L)
8080 NEXT L
8090 K=K+1
8100 IF K=AKHR+3 THEN 8120
8110 IF K=SHRU+3 THEN K=AKHR
8120 NEXT J
8130 RETURN
```

iii) Assemble the Right Hand Side Load Vector. The R.H.S Load Vector is sum of the entered joint loads FJ and the equivalent joint loads {FG}, with a negative sign. Statements 1450 to 1470 do this simple job.

```
1450 REM INCLUDE JOINT LOADS IN [FG]
1455 REM [FG] INCLUDES RESTRAINED DOF
1460 FOR I=1 TO NJ*3
1465 FG(I)=-FG(I)+FJ(I)
1470 NEXT I
```

5.0 MODIFY THE SYSTEM FOR RIGID SUPPORT CONDITIONS

The Assembled Stiffness Matrix [KG] (Section 3.0) is singular because it contains restrained DOF at supports. The author has used method of partitioning to modify the Stiffness Matrix for rigid support conditions. Columns and rows corresponding to restrained DOF are deleted from [KG] to obtain the non-singular stiffnes matrix {PP}

which gives a unique set of displacements when solved. Similarly a modified R.H.S Load Vector {FF} is obtained from {FG} by deleting rows corresponding to support reactions. The FRAP statements 7760 to 7890 modify [KG] and statements 1500 to 1535 modify (FG).

```
7760 REM OBTAIN NON SINGULAR STIFFNESS MATRIX BY DELETING
7770 REM COLUMNS AND ROWS CORRESPONDING TO FIXED DEGREES OF
FREEDOM FROM [KG]
7780 C3=0!
7790 FOR J=1 TO NJ*3
7800 IF U(J) < 0 THEN 7880
7810 C3=C3+1
7820 C4=0!
7830 FOR K=1 TO NJ*3
7840 IF U(K) <0 THEN 7870
7850 C4=C4+1
7860 PP(C3,C4)=KG(J,K)
7870 NEXT K
7880 NEXT J
7890 RETURN
1500 REM COMPUTE [FF] FROM [FG] BY DELETING ELEMENTS
CORRESPONDIG
1505 REM TO RESTRAINED DOF
1510 C5=0
1515 FOR K=1 TO NJ*3
1520 IF U(K) < 0 THEN 1535
1525 C5=C5+1
1530 FF(C5)=FG(K)
1535 NEXT K
```

6.0 SOLVE FOR THE GLOBAL DISPLACEMENTS {UG}

The basic relation [PP]{UG} = {FF} is now ready. The next step is to solve for the global displacements {UG}. A number of matrix techniques are available to solve the given set of simultaneous equations. Cholesky's Method, used by the author, is very

economical of the computer time and has been used extensively in some larger structural analysis programmes. Upper and lower triangular matrices can be overlaid on the augmented matrix in the same memory location to make an efficient use of the core storage of the computer. Scope of this paper does not permit a more detailed theoretical treatment of the method. The FRAP statements 8500 to 8700 compute Global Displacement Vector {UG} by using Cholesky's Method.

```
8500 REM SOLVE SIMULTANEOUS EQUATIONS BY CHOLESKY'S DECOMPOSITION
8505 REM READ AUGMENTED MATRIX (N ROWS, M COLUMNS)
8510 REM CALCULATE FIRST ROW OF UPPER UNIT TRIANGULAR MATRIX
8515 REM L AND U MATRICLES ARE LAID OVER THE AUGMENTED MATRIX
8520 FOR J=2 TO M
8525 PP(1,J)=PP(1,J)/PP(1,1)
8530 NEXT J
8535 REM CALCULATE OTHER ELEMENTS OF L AND U MATRICES
8540 FOR 1=2 TO N
8545 J=I
8550 FOR TU=J TO N
8555 SUM=0!
8560 JM=J-1
8565 FOR K=1 TO JM
8570 SUM=SUM+PP(TU,K)*PP(K,J)
8575 NEXT K
8580 PP(TU,J)=PP(TU,J)-SUM
8585 NEXT TU
8590 AP=I+1
8595 FOR JJ=AP TO M
8600 SUM =O!
8605 AM=I-1
8610 FOR K=1 TO AM
8615 SUM=SUM+PP(I,K)*PP(K,JJ)
8620 NEXT K
8625 PP(I,JJ)=(PP(I,JJ)-SUM)/PP(I,I)
```

```
8630 NEXT JJ
8635 NEXT I
8640 REM SOLVE FOR DISPLACEMENTS (UG) BY BACK SUBSTITUTION
8645 UG(N)=PP(N,N+1)
8650 L=N-1
8655 FOR NN=1 TO L
8660 SUM = O!
8665 I=N-NN
8670 AP=I+1
8675 FOR J=AP TO N
8680 SUM = SUM + PP(I,J)* UG(I)
8685 NEXT J
8690 UG(I)=PP(I,M)-SUM
8695 NEXT NN
8700 RETURN
```

7.0 SOLVE FOR THE FINAL MEMBER FORCES

The Final Member Forces are computed in the following steps:

i) Construct the displacements vector (UFG) from {UG}. Elements of {UFG} equal the number of DOF of the structure. The vector {UFG} consists of global displacements of all the joints corresponding to the considered DOF. FRAP statements 1595 to 1635 contruct the vector {UFG}.

```
1595 REM CONSTRUCT DISPLACEMENT VECTOR UFG(NJ*3)FROM UG {C5}
1600 C6=0
1605 FOR K=1 TO NJ*3
1610 IF U(K)< 0  THEN 1630
1615 C6=C6+1
1620 UFG(K)=UG(C6)
1625 GOTO 1635
1630 UFG(K)=0!
1635 NEXT K
```

ii) Extract Vector of Member Displacements {MUG} in global coordinates from {UFG}. The vector {UGF} contains the global displacements at all the joints of the structure including zeros for fixed DOF. The member no.4, for instance, spans from join no.5 to the joint no.2. The vector {MUG} for member no.4 consists of displacements corresponding to the DOF 13, 14, 15 and 4, 5, 6. The FRAP statements 1645 to 1750 extract {MUG} for each member from {UGF}

```
1645 FOR I=1 TO NM
1650 REM STEP1 EXTRACT MEMBER DISPLACEMENTS {MUG} IN GLOBAL
COORDINATES
1660 REM FROM {UFG}
1670 SHRU=(ST(I)-1)*3+1
1680 AKHR=(ED(I)-1)*3+1
1690 K=SHRU
1700 FOR J=1 TO 6
1710 MUG(J)=UFG(K)
1720 K=K+1
1730 IF K=AKHR+3 THEN 1750
1740 IF K=SHRU+3 THEN K=AKHR
1750 NEXT J
```

iii) Rotate the member disfplacements to member coordinates by the following relation.

{MD} = [T] {MUG}

{MD} is vector of member displacements in member coordinates and [T] is transformation matrix for the member. The FRAP statements 1760 to 1840 do the above mentioned job.

```
1760 GOSUB 5500:REM GENERATE [T]
1770 GOSUB 6000:REM GENERATE [TT]
1780 REM STEP2 COMPUTE MEMBER DISPLACEMENTS IN MEMBER
COORDS. =[T] * {MUG}
1790 FOR J=1 TO 6
1800 MOD(J)=0!
1810 FOR L=1 TO 6
1820 MD(J)=MD(J)+T(J,L)*MUG(L)
1830 NEXT L
```

```
1840 NEXT J
```

iv) Compute the final member forces by the following relation:

{MF} = [S] {MD} + {FM},

Where:

{MF} is the vector of member forces in member coordinates;

{MD} and {FM} are the vectors of member displacements and fixed end forces in member coordinates. The statements 1850 to 1960 compute the final member forces.

```
1850 REM STEP 3 COMPUTE MEMBER FORCES IN MEMBER COORDS. {MF}
1860 REM {MF}= [S]*{MD}+{FM}
1865 GOSUB 5200:REM GENERATE MEMBER STIFFNESS IN MEMBER COORDS.
1870 FOR J=1 TO 6
1880 SMD(J)-0!
1890 FOR L=1 TO 6
1900 SMD(J)=SMD(J)+S(J,L)*MD(L)
1910 NEXT L
1920 NEXT J
1930 REM FINAL MEMBER FORCES
1940 FOR J=1 TO 6
1950 MF (J)=SMD(J)+FM(I,J)
1960 NEXT J
```

8.0 COMPUTE SUPPORT REACTIONS

Support reactions are computed from the following relation:

D = [RP] {UG} + {FR}

D and {UG} are vectors of support reactions and global displacements respectively. The matrix [RP] is partitioned from the singular Assembled Stiffness Matrix of the structure [KG] such that an element [RPij] is the reaction contribution on ith DOF (restrained) resulting from the unit displacement corresponding to jth DOF (unrestrained). {FR} is vector of fixed end forces corresponding to fixed DOF in global coordinates. The FRAP statements 9000 to 9060 construct [RP] from [KG] while statements 9065 to 9100 and 4880 to 4990 compute and print support reaction.

```
9000 REM SUBROUTINE FOR SUPPORT REACTIONS
9005 D1=0!
9010 FOR J=1 TO NJ*3
9015 IF U(J) < 0 THEN 9025
9020 GOTO 9060
9025 D1=D1+1
9030 D2=0!
9035 FOR K=1 TO NJ*3
9040 IF U(K) < 0  THEN 9055
9045 D2=D2+1
9050 RP(D1,D2)=KG(J,K)
9055 NEXT K
9060 NEXT J
9065 REM SUPPORT REACTIONS
9070 FOR J=1 TO D1
9075 D(J)=0
9080 FOR L=1 TO D2
9085 D(J)=D(J)+RP(J,L)*UG(L)
9090 NEXT L
9095 NEXT J
9100 RETURN
4880 FOR I=1 TO RJ
4885 SRJ=SNRJ(I)
4890 K1=(SRJ-1)*3+1
4895 IF U(K1)  > -1 THEN 4915
4900 SA=SA+1
4905 M1=D(SA)-FG(K1)+FJ(K1)
4908 MOM=INT(M1*100)/100
4910 GOTO 4920
4915 MOM=0!
4920 K2=K1+1
```

```
4925 IF U(K2) >  -1 THEN 4945
4930 SA=SA+1
4935 V1=D(SA)-FG(K2)+FJ(K2)
4938 VERT=INT(V1*100)/100
4940 GOTO 4950
4945 VERT=0!
4950 K3=K2+1
4955 IF U(K3) > -1 THEN 4975
4960 SA=SA+1
4965 H1=D(SA)-FG(K3)+FJ(K3)
4968 HORIZ=INT(H1*100)/100
4970 GOTO 4980
4975 HORIZ-0!
4980 LPRINT SNRJ (I) TAB (15) MOM TAB(30) VERT TAB(45) HORIZ
4985 NEXT I
4990 RETURN
```

9.0 SEISMIC ANALYSIS

The term dynamic analysis as applied to structures subjected to earthquakes is associated with two slightly differet analytical approachses. The first approach, known as 'Mode Superposition' uses response spectra to determine minimum response parameters for different modes of vibration each treated as a single degree of freedom system with a characteristic frequency. The modal contributions are combined by various techniques (most common technique being square root of the sum of squares) considering the fact that modal maxima may not occur simultaneously. The approach is applicable only to linearly elastic systems. The second approach determines the time history of the response of the mathematical model of a structure to a given earthquake. Modal superposition or direct numerical integration of the equations of motion can be used in this analysis.

The above noted approaches involve some of the advanced concepts of Mathematics and Dynamics. Only a separate paper can adequately describe the theoretical basis of the procedures involved. The computer programming of the two approaches is an equally specialised job. The analytical proceedures, therefore,have not been discussed in further detail. An alternative procedure involving the static analysis has been discussed below. The approach, known as STATIC EQUIVALENCE, is accepted by all leading codes.

9.1 THE STATIC LOAD EQUIVALENCE

The current provisions of the US seismic codes consist of the following principal steps:

i) Determination of lateral static forces on a building resulting from the earthquake. The base shear is calculated corresponding to a computed or estimated foundamental period of vibration of the structure. The base shear is then distributed over the height of structure. Provisions of ANSI, UBC, and BCP are applicable.

ii) Analysis of the structure under a critical combination of lateral forces, wind loads and gravity loads. ANSI-82 and ACI 318-83 specify slightly different load factors for various load combinations.

iii) Designing members and joints for selected load combination. For reinforced concrete ACI 318-83 lays down special provisions for detailing of members and their connections to ensure their ductile behaviour and thus allow the structure to sustain without collapse the distortions in a seismic event. The ACI provisions have not been discussed here because they do not relate to the topic of the paper.

The design base shear,V, represents the total seismic force acting parallel to the horizontal axis of the structure. A similar force acting parallel to the other horizontal axis of the structure is assumed to act nonconcurrently, and thus may be considered separately. The base shear is calculated by the following formula.

$V = ZIKCSW$

The empirical modifiers used in the above formula have been briefly described below, details may be obtained from ANSI-82 or BCP.

Z= A numerical coefficient whose value ranges from 3/32 to 1 depending upon the seismic zone the structure is located in.

I= Occupancy importance coefficient representing a greater conservatism for buildings like hospitals and other essential facilities that must remain operational after a major earthquake. Its value varies from 1 to 1.5.

K= The framing coefficient reflecting the ductility and as such, the ultimate damping of the structure. Value of K ranges from 0.67 for a ductile moment resisting frame to 2.5 for an elevated tank.

C= A coefficient representing the flexibility of the structure.

$C = 1/15\sqrt{T}$, but not greater than 0.12

T is the elastic undamped fundamental period of vibration of the structure, estimated as follows in absence of a precise determination.

i) For shear walls or exterior frames with deep beams or wide piers or both:

$T = 0.05\, Hn\sqrt{D}$

ii) For shear walls not interconnected by frames.

$T = 0.05 Hn\ Ds$

iii) Moment resisting space frames capable of resisting 100% of the required lateral forces.

$T = CT\ Hn^{3/4}$

Where:

Hn = height of the structure above base

D = building dimension parallel to the applied frces.

Ds = Longest dimension of the shear wall or braced frame parallel to the applied force.

CT = 0.025 for concrete frames and 0.035 for steel frames.

S = Soil factory reflecting the possible magnification of dynamic structural response due to adverse soil conditions. The value of S, as assigned by ANSI for three types of soils are 1.0, 1.2 and 1.5;

W = T tal dead load of the structure.

9.2 BASE SHEAR DISTRIBUTION

The total horizontal force, V, is divided in two parts for structures with regular shapes of framing systems. First part consists of a concentrated load (Ft) applied at the top of the structure to account for the whiplash effect. The second part includes the balance load, (V-Ft) distributed triangularly along the entire height of the structure with loads concentrated at each storey level.

The top load, Ft, is determined as:

$$Ft = \begin{cases} 0.07\ TV \leqslant 0.25v & \text{When } T > 0.7 \text{ Sec} \\ 0 & \text{When } T < 0.7 \text{ Sec} \end{cases}$$

The lateral force at level x, Fx, is given by:

$$Fx = \frac{(V-F_t)W_x\ h_x}{\sum_{i=1}^{n} Wihi}$$

Where: 1+1

Fx = lateral force at level x above the base;

Wx,Wi = part of the total load at level x and i respectively;

hx,hi = height above base upto level x and i respectively; and

n = number of floors above base.

The lateral forces placed at storey levels are treated as joint loads in the analysis of the structure. The static analysis thus performed conservatively reflects the dynamic effects induced by the earthquake. The FRAP statement 10000 to 10170 compute the lateral forces resulting from th base shear. Part of the programme accepting the input data and computing some of the empirical modifiers has not been reproduced in view of its simple nature.

```
10000 REM COMPUTE BASE SHEAR
10010 V=Z*O*K*C*S*WT
10020 IF T>.7 THEN FT=.07*T*V
10030 IF FT>.25*V THEN FT=.25*V
10040 IF T=.7 OR T<.7 THEN FT=0
10050 V1=V-FT
10060 DENOM=0
10070 HX1=0
10080 FOR J=1 TO NF
10090 HX1=HX1+H(J)
10100 DENOM=DENOM + W(J)*HX1
10110 NEXT J
10115 REM COMPUTE LATERAL FORCES
10120 HX=0
10130 FOR J=1 TO NF
10140 HX=HX+H(J)
10150 FX(J)=V1*W(J)*HX/DENOM
10160 NEXT J
10170 RETURN
```

10.0 WIND FORCES

The pressure that wind exerts on buildings is dynamic in nature. An equivalent static pressure (in lbs/ft2) can be obtained by the following UBC formula adopted by BCP.

$P = 0.00256\ Cp\ Cf\ Vs^2$

Where:

$Vs = V \times S$

V = Wind velocity

S, Cp & Cf are the ground roughness, pressure and force coefficients respectively.

The wind pressure increases with the height of the building. The increase in pressure is accounted for by an incrased speed Vz at a height Z given by the following relation.

$Vz = V30\ (hz/h30)^{1/7}$

Where:

Vz = datum air velocity at elevation h30;

h30 = 30 ft;

hz = height at which velocity is required.

The varying wind pressure along the height of a building can be easily computed by a simple computer programme. The codes generally contain a table giving velocity or pressure values for different height zones, which effectively replaces a computer programme. The wind pressures shown in Appendix-B have been obtained from UBC table 24-F for a datum air velocity of 30 mph. Advantage of the computer usage lies in handling the lateral loads generated by wind (FRAP accepts wind loads as uniformaly distributed loads in different height zones). Another advantage of using a computer is the ease with which Dead, Live, Earthquake and Wind loads can be combined with various load factors in one or more than one runs.

Computation of overturning movements and forces on parts and portions of the structures generated by lateral loads, have not discussed in the paper. Codes provide an elaborate treatement of such forces independent of the overall analysis of the structures.

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5. M.L.JAMES, G.M. SMITH and J.C. WOLFORD, "Applied Numerical Methods for Digital Computer".
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7. ACI COMMITTEE 442, "Response of Buildings to Lateral Forces."
8. AMERICAN NATIONAL STANDARDS A 58.1 - 1982.
9. AMERICAN CONCRETE INSTITUTE 318-83.
10. BUILDING CODE OF PAKISTAN.

INPUT DATA

JOINT COORDINATES AND JOINT LOADS ***

JOINT	X[ft]	Y [ft]	MLOAD	VLOAD	HLOAD
1	0	0	0	0	0
2	28	0	0	0	0
3	0	9	0	0	10
4	16	18	0	0	0
5	16	9	0	0	0

MEMBER INCIDENCES AND MEMBER PROPERTIES***

MEMBER	FROM	TO	L[FT]	A [IN^2]	E [KSI]	I [IN^4]	ANG. [deg]
1	1	3	9	12.96	30000	349.92	90
2	3	5	16	12	30000	600	0
3	4	5	9	12.96	30000	349.92	-90
4	5	2	15	14.4	30000	810	-36.87

MEMBERS LOADS ***

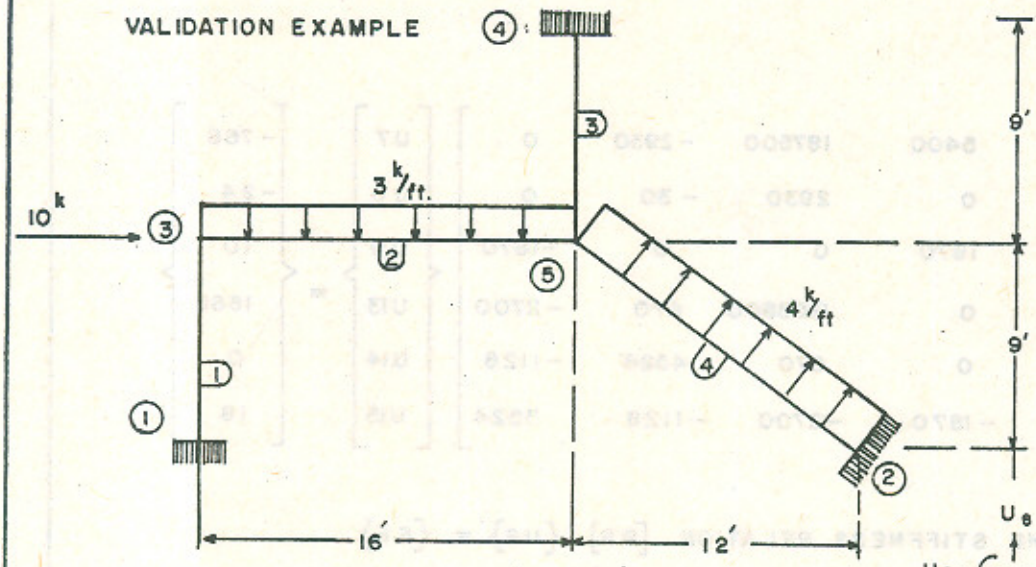
MEMBER	UDL [k/ft]	NO.	PLS	PL [k]	DIST. [ft]
1	0	0			
2	3	0			
3	0	0			
4	-4	0			

FIXED END FORCES (computed)***

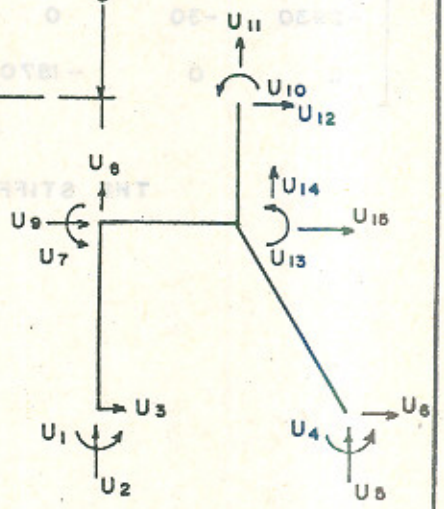
NUMBER	JOINT	M(in-k)	VERT.(k)	HORIZ.(k)
1	1	0	0	0
	3	0	0	0
2	3	768	24	0
	5	-768	24	0
3	4	0	0	0
	5	0	0	0
4	5	-900	-30	0
	2	900	-30	0

FIXED DEGREES OF FREEDOM * fixed=-1 : free=0**

JOINT NO.	ROTATION	VERTICAL	HORIZONTAL
1	-1	-1	-1
2	-1	-1	-1
4	-1	-1	-1



EXAMPLE STRUCTURE



GLOBAL DEGREES OF FREEDOM

MEMBER	I, in^4	A, in^2	ORIGIN
1	349·92	12·96	1
2	600	12·0	3
3	349·92	12·96	4
4	810	14·40	5

MEMBER PROPERTIES

$$\begin{bmatrix} 763800 & 2930 & 5400 & 187500 & -2930 & 0 \\ 2930 & 3630 & 0 & 2930 & -30 & 0 \\ 5400 & 0 & 1970 & 0 & 0 & -1870 \\ 18700 & 2930 & 0 & 1303800 & 670 & -2700 \\ -2930 & -30 & 0 & 670 & 4526 & -1128 \\ 0 & 0 & -1870 & -2700 & -1128 & 3524 \end{bmatrix} \begin{Bmatrix} U7 \\ U8 \\ U9 \\ U13 \\ U14 \\ U15 \end{Bmatrix} = \begin{Bmatrix} -768 \\ -24 \\ 10 \\ 1668 \\ 0 \\ 18 \end{Bmatrix}$$

THE STIFFNESS RELATION $[PP]\ \{UG\} = \{FF\}$

U7 = -0·001589

U8 = -0·006556

U9 = 0·034276

U13 = 0·001574

U14 = 0·005218

U15 = 0·26172 = ·026172

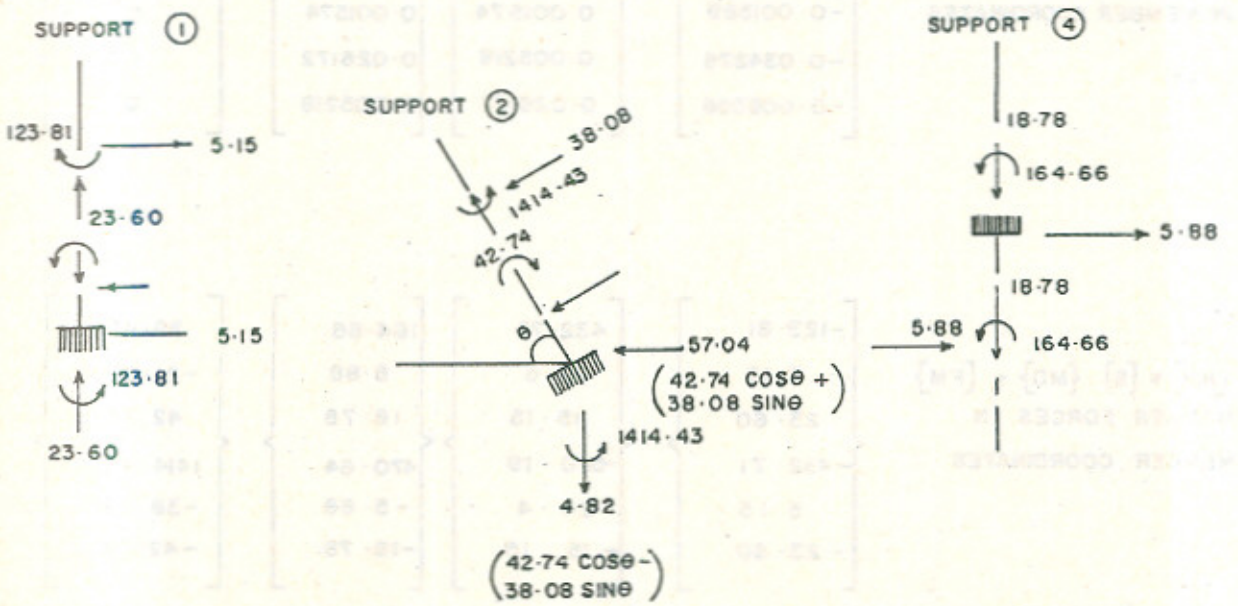
GLOBAL DISPLACEMENTS OF THE STRUCTURE

MEMBER DISPLACEMENTS AND FORCES

MEMBER	(1)	(2)	(3)	(4)
$\{MUG\}$ MEMBER DISPLACEMENTS IN GLOBAL COORDINATES	0 0 0 -0·001589 -0·006556 0·034276	-0·001589 -0·006556 0·034276 0·001574 0·005218 0·026172	0 0 0 0·001574 0·005218 0·026172	0·001574 0·005218 0·026172 0 0 0
$\{MD\} = [T]\ \{MUG\}$ MEMBER DISPLACEMENTS IN MEMBER COORDINATES	0 0 0 -0·001589 -0·034276 -0·006556	-0·001579 -0·006556 0·034276 0·001574 0·005218 0·026172	0 0 0 0·001574 0·026172 0·005218	0·001574 0·019878 0·017807 0 0 0
$\{MF\} = [S]\ \{MD\} + \{FM\}$ MEMBER FORCES IN MEMBER COORDINATES	-123·81 - 5·15 23·60 -432·71 5·15 - 23·60	432·75 23·6 15·15 -510·19 24·4 - 15·15	164·66 5·88 18·78 470·64 -5·88 -18·78	39·41 -21·92 42·74 1414·43 -38·08 -42·74

$$\begin{bmatrix} 194400 & 0 & 5400 & 0 & 0 & 0 \\ 0 & -3600 & 0 & 0 & 0 & 0 \\ -5400 & 0 & -100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 270000 & 3600 & 2700 \\ 0 & 0 & 0 & -3600 & -896 & 1128 \\ 0 & 0 & 0 & -2700 & 1128 & -1554 \\ 0 & 0 & 0 & 194400 & 0 & -5400 \\ 0 & 0 & 0 & 0 & -3600 & 0 \\ 0 & 0 & 0 & 5400 & 0 & -100 \end{bmatrix} \begin{Bmatrix} -0.001589 \\ -0.006556 \\ 0.034276 \\ 0.001574 \\ 0.005218 \\ 0.026172 \\ \\ \\ \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 900 \\ -24 \\ -18 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -123.81 \\ 23.60 \\ 5.15 \\ 1414.43 \\ -4.82 \\ -57.04 \\ 164.66 \\ -18.78 \\ 5.88 \end{Bmatrix}$$

SUPPORT REACTIONS $[RP]\ \{UG\}+\{FR\}=\{D\}$



SUPPORT REACTIONS FROM JOINT EQUILLIBRIUM

INPUT DATA

JOINT COORDINATES AND JOINT LOADS ***

JOINT	X [ft]	Y [ft]	MLOAD	VLOAD	HLOAD
1	0	0	0	0	0
2	28	0	0	0	0
3	0	9	0	0	10
4	16	10	0	0	0
5	16	9	0	0	0

MEMBER INCIDENCES AND MEMBER PROPERTIES ***

MEMBER	FROM	TO	L [ft]	A [in^2]	E [ksi]	I [in^4]	ANG. [deg]
1	1	3	9	12.96	30000	349.92	90
2	3	5	16	12	30000	600	0
3	4	5	9	12.96	30000	349.92	-90
4	5	2	15	14.4	30000	810	-36.87

MEMBERS LOADS ***

MEMBER	UDL [k/ft]	NO. PLS	PL [k]	DIST.[ft]
1	0	0		
2	3	0		
3	0	0		
4	-4	0		

FIXED END FORCES (computed) ***

MEMBER	JOINT	M(in-k)	VERT.(k)	HORIZ.(k)
1	1	0	0	0
	3	0	0	0
2	3	768	24	0
	5	-768	24	0
3	4	0	0	0
	5	0	0	0
4	5	-900	-30	0
	2	900	-30	0

FIXED DEGREES OF FREEDOM *** fixed=-1 : free=0

JOINT NO.	ROTATION	VERTICAL	HORIZONTAL
1	-1	-1	-1
2	-1	-1	-1
4	-1	-1	-1

A-5

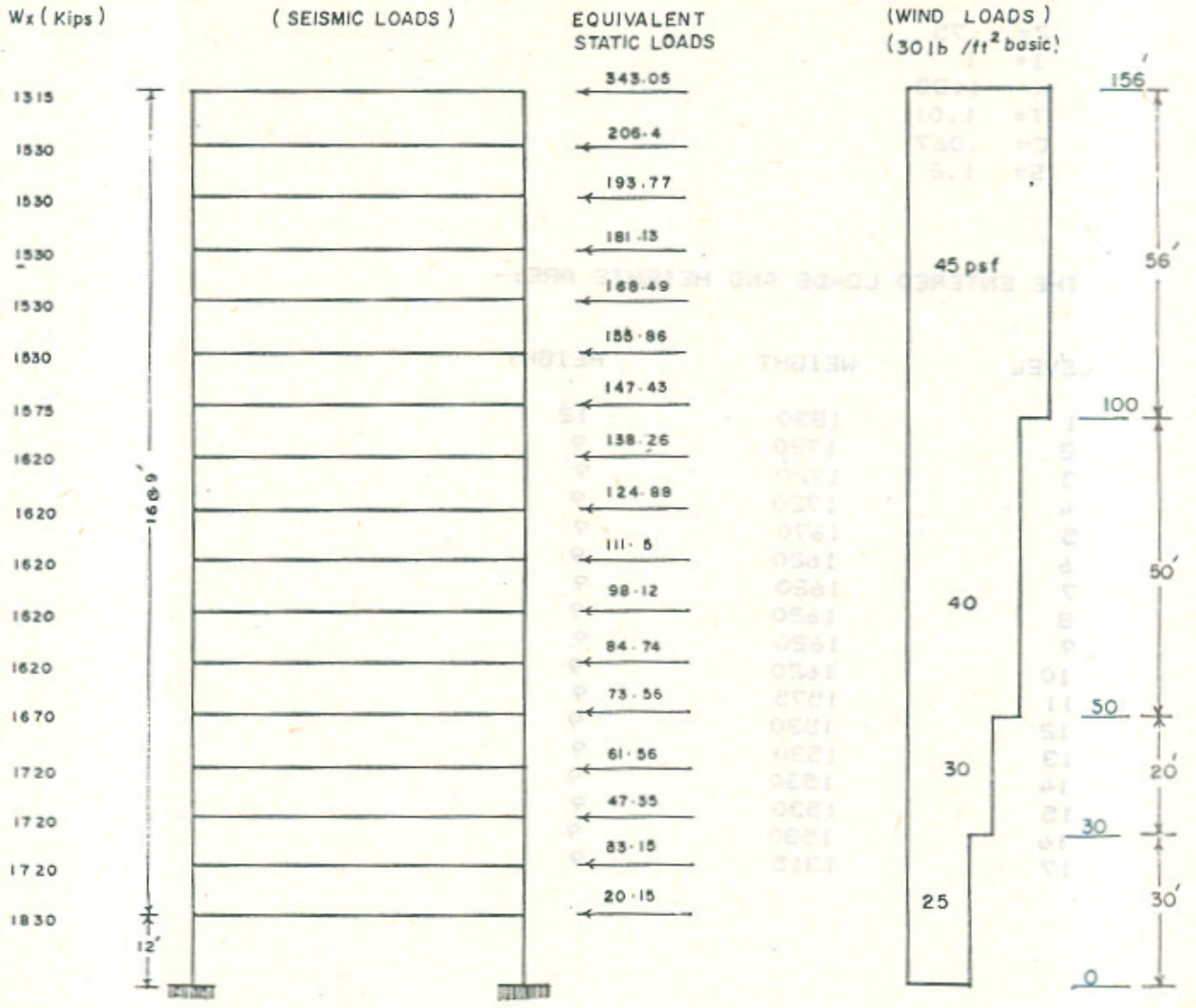
OUTPUT DATA

MEMBER FORCES AND DISPLACEMENTS IN LOCAL COORDS. *

MEMBER	JOINT	MON(in-k)	VERT.(k)	AXIAL-(k)	ROT.(rad)	VD.(in)	HD.(in)
1	1	-123.92	- 5.16	23.59	0	0	0
	3	-432.81	5.15	-23.6	-.002	-.035	-.007
2	3	432.8	23.59	15.15	-.002	-.007	.034
	5	-510.13	24.4	-15.16	.001	.005	.026
3	4	164.67	5.88	18.77	0	0	0
	5	470.67	- 5.89	-18.78	.001	.026	-.006
4.	5	39.45	-21.93	42.73	.001	.019	.017
	2	1414.44	-38.08	-42.74	0	0	0

SUPPORT REACTIONS *

JOINT No.	MOM(in-k)	VERT.(k)	HORIZ.(k)
1	-123.92	23.59	5.15
2	1414.44	- 4.82	-57.04
4	164.67	-18.78	5.88



LATERAL LOADS

SEISMIC DESIGN PARAMETERS

Z= .75
I= 1
K= 1.33
T= 1.01
C= .067
S= 1.2

THE ENTERED LOADS AND HEIGHTS ARE:-

LEVEL	WEIGHT	HEIGHT
1	1830	12
2	1720	9
3	1720	9
4	1720	9
5	1670	9
6	1620	9
7	1620	9
8	1620	9
9	1620	9
10	1620	9
11	1575	9
12	1530	9
13	1530	9
14	1530	9
15	1530	9
16	1530	9
17	1315	9

EQUIVALENT STATIC LOADS

LEVEL	Fx	SHEAR
17	154.79 => (Ft)	
17	188.26	154.79
16	206.40	343.05
15	193.77	549.46
14	181.13	743.23
13	168.49	924.36
12	155.86	1092.85
11	147.43	1248.71
10	138.26	1396.14
9	124.88	1534.40
8	111.50	1659.28
7	98.12	1770.79
6	84.74	1868.91
5	73.56	1953.65
4	61.56	2027.22
3	47.35	2088.78
2	33.15	2136.13
1	20.15	2169.28
0.	0.00	2189.43