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BEHAVIOUR OF GEOMETRICALLY IMPERFECT SLENDER STEEL COLUMNS

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ABSTRACT

A study is presented on the comparative behavior of slender steel columns with pinned ends. AISC (1) suggests that largest effective slenderness ratio for unbraced compression members of Grade 36 ksi under axial loads is 126 and the maximum slenderness ratio for any type of compression members should not exceed 200, therefore all experimental slender columns specimens had slenderness in this suggested range. Based on the regression analysis of model columns, a new, updated imperfect lion parameter has been suggested, in comparison to previous studies (2), in terms of slenderness ratio. The results would be very useful to the steel column designers in Pakistan.

INTRODUCTION

The ultimate load of a slender steel column is the Euler load. Although some studies indicate that there is a reserve strength of column in the post-buckling region, it is usually neglected because it is very small in comparison to the buckling load. Euler theory applies to ideally straight columns obeying certain assumptions, which rarely occurs in actual practice in which both geometric imperfection and the material imperfections are present. The role of geometric imperfections on the carrying capacity of the long columns has been studied experimentally in the present investigation which gives an updated value of geometric imperfection parameter for locally manufactured slender steel column specimens in terms of the slenderness ratio through linear regression analysis. In the opinion of the Authors, this relationship is sufficiently true and by increasing the number of column specimens with similar properties, this may not be improved further considerably.

PERRY AND ROBERTSON METHOD

A real column may have imperfections induced during manufacturing or erection. As a result, the actual center line of the column may not be coinciding with ideal or that mathematically obtained. Consequently, the actual center line will have lateral deviation from each other. Thus on application of loads along ideal center line of a column having initial imperfections, an excessive moment is induced due to initial imperfections, and the member fails before the critical load calculated by Euler's theory, as the Euler critical load was derived by neglecting the initial imperfections. This fact is incorporated by the Perry-Robertson (3) in formulae for critical load. In Perry Robertson formula, the yield load is

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The Perry Robertson formulae is based on the assumption that ultimate load equals the yield load, measured from tests.
The imperfection parameter used in the Perry Robertson method (3) is given as ,

$\eta = a_o c/r^2$

where a_o = Initial imperfection w.r.t centre-line determined with the help of Southwell plots
c= Distance from neutral axis to the point of maximum stress.
r= Radius of gyration.
The Perry formula (3) is given as

$\sigma^2 - \sigma[\sigma_y + \sigma_e(\eta-1)] + \sigma_y\sigma_e = 0$

With $\sigma < \sigma_e$ and is used for determining the stresses in imperfect columns by incorporating the dimensionless imperfection parameter 'η'. Where σ is actual stress, σ_y is yield stress and σ_e is Euler stress. Various η values are used to determine column stresses at failure. Stability aspects for columns may be seen elsewhere (5,6,7,8).

SOUTHWELL PLOT TECHNIQUE

In the Perry Robertson formula as given above, the effect of initial imperfection 'a_o' has been incorporated but this formula is not workable as the value of initial imperfection 'a_o' cannot be measured accurately. However, Southwell has devised a technique for obtaining 'a_o' and is known as Southwell Plot Technique, which provides a graphical method of determining the initial imperfection.

If a column is loaded, the deflections measured at various values of the load and 'δ/p' plotted against 'δ', then the slope of the resulting straight line graph is equal to the reciprocal of the critical load. Also a quantitative measure of the important imperfection 'a_o' is available from the X intercept of the plot.

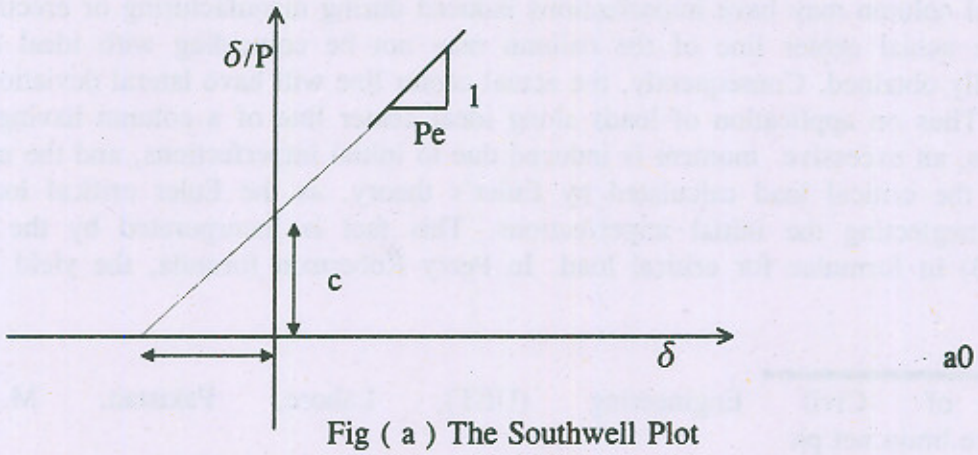


Fig (a) The Southwell Plot

From Perry's equation

$\delta\ (P_e/P - 1) = a_o$

$\delta/P_e\ (P_e/P - 1) = a_o/P_e$

$\delta/P - \delta/P_e = a_o/P_e$

$\delta/P = 1/P\ (\delta) + a_o/P_e$

This is analogous with equations of a straight line

$y = mx + c$

Here $y = \delta/P$, $m = 1/P_e$ and $c = a_o/P_e$

The graph is also helpful in determining in advance , the critical actual load P before actual tests , $P < P_e$

MANUFACTURE OF MODEL COLUMNS

Four model columns were manufactured having an average cross-section of 2.5" x 0.375". These columns were obtained after cutting, shaping and finishing the available MS sheets of nearest dimensions. Therefore, the actual dimensions of each of the experimental model columns was slightly different than the above dimensions which were used for theoretical analysis. The dimensions reported in Table 1 correspond to average dimensions taken at different heights. The initial crookedness was induced in model columns by gradually applying load in such a way to generate a curve along the length of the column.

SMITH-SOUTHWELL PLOT OF MODEL COLUMNS

Smith Southwell plot technique is universally applied to study the behavior of geometrically imperfect slender columns. The initially crooked column is loaded very slowly and corresponding out-of-plane displacements are recorded for every load level. After obtaining this information δ versus δ/p graphs are plotted. The vertical intercept is equal to a_o/p_{cr}. This process requires very careful attention. Its accuracy increases with the number of observations. This is specially true if a regression analysis has to follow in order to fit the data in a linear relationship. Basically, it has to be a linear relationship but due to slight variations in flexural stiffness along the column height, the actual plot approximates to linear behavior after a few readings. If the vertical intercept a_o/p_{cr} and a_o are known, then, the value of Young's Modulus of Elasticity can be computed indirectly even for composite materials.

TEST RESULTS

Tables 1 and 2 gives the dimensions and properties of modal pin-ended columns for experimental and theoretical analysis respectively.

Fig. 1(a) through 1(d) represents the theoretical Smith-Southwell plots for model pin-ended slender steel columns with different slenderness ratios and initial imperfections whereas 2(a) through 2(d) represents the experimental Smith-Southwell plots for the same.

IMPERFECTION PARAMETER

In energy approach methods, the total Potential Energy functions are set up in terms of an imperfection parameter η, where $\eta = a_0 c/r^2$ where c is the extreme fibre distance to the axis of zero strain (N.A), about weaker axis, r is the least radius of gyration and a_0 is the initial imperfection. The maximum values of this imperfection have been proposed by various researchers.

Perry (3) proposed $\eta = 0.0015kl/r$ where kl/r is the slenderness ratio. Robertson (3) modified $\eta = 0.003kl/r$. Duthiel (4) suggested $\eta = 3x10^{-5}$ $(kl/r)^2$. Earlier investigation (2) suggested $\eta = 1.24x10^{-4}(kl/r)^2$. The authors believe after careful test procedures, Smith Southwell plots and regression analysis based on a number of observations that $\eta = 1.68x10^{-4}$ $(kl/r)^2$.

Fig. 3(a) through (d) represents the actual load vs. central out-of-plane deflections for various model slender steel columns with different slenderness ratios and imperfections.

REGRESSION ANALYSIS

Linear regression analysis (9) also termed as least -squares regression was used to inspect the plotted data on Smith- Southwell diagrams to sketch a "best" line through the points. The linear regression method was chosen due to visual inspection of the Southwell plots, which suggested a linear relationship between δ and δ/p.

Some assumptions inherent in the linear least - squares procedures are;

1) x has a fixed value, it is not random and is measured without error.
2) The y values are independent random variables and all have the same variance.
3) The y values for a given x must be normally distributed.

The mathematical expression for a straight line is

$$y = mx + c + E$$

Where m and c are coefficients representing the slope and the intercept, respectively, and E is the error, or residual between model and observations, which can be represented by rearranging the equation as

$$E = y - c - mx$$

Thus, the error, or residual, is the discrepancy between the true value of y and the approximate value, mx + c, predicted by the linear equation.

Linear regression analysis was applied using commercially available spreadsheet software, which outputs the values of the y - intercept c, the slope m, square of the residuals,

standard error of estimate and standard error of coefficient, as can be observed in Appendix B. Borland's Quattro Pro Spreadsheet for DOS version 4.0 was used for the regression analysis. The values of the c and m obtained were than used to calculate the points for the "best" fit line.

DISCUSSION ON EXPERIMENTAL RESULTS

1) Based on Tables 1 and 2 and Fig 1, the δ/p values for different slender steel columns is presented in Table 3.
It can be clearly seen that for a given a_0 value and for increasing kl/r ratios, the δ/p values increases almost linearly with kl/r. This is due to the fact, that the basis of the Smith Southwell plot is a linear formula in terms of δ and δ/p even on a normal graph. The same arrangement is true for all imperfections.
2) Experimental Smith Southwell plots represented in Fig. 2 indicate that in most of the cases, the regression line is on top of the experimental line when these are plotted as δ/p ordinate and δ abscissa. This may be due to the fact that initial imperfections induced in the column - center never produced a true Sine wave, which is obtained when the load is about 40 % of the ultimate load where ultimate load is defined as the one where out-of-plane displacement at column - centre keeps on increasing while the load is either stationary or decreasing.
3) By increasing kl/r, the maximum out-of-plane displacement increases at ultimate load for an almost same geometrical imperfection.
4) The above argument seems to be true for Fig 3, with the exception that by increasing kl/r with almost same initial crookedness, the slope of the curve gets minimized and it is expected that near kl/r=200, the slope may be zero or negative. This was exhibited by columns with slenderness ratio 106, 132 and 150.
5) The values of Young's Modulus of Elasticity E calculated indirectly from Smith Southwell plots are almost exactly the same as is the original value of 29 x 10^6 psi. The calculated values are also reported in Appendix .
6) The above comments are valid even for composite materials behaving elasticity based on an earlier investigation (10).

CONCLUSIONS

A more accurate value of imperfection parameter η is reported as 1.68×10^{-4} $(kl/r)^2$ which may be safely used for analysis of pin ended initially crooked columns.

The load carrying capacity gets reduced by increasing initial imperfection and the slenderness ratio. Therefore, in actual steel column construction, straight columns should be used and long columns may be braced at suitable intervals to increase the capacity of these columns very economically. Every effort must be made not to 'shrink-fit' slender column in steel construction to avoid second order P-Δ deformations which reduce the axial load carrying capacity of steel columns.

The actual imperfection provided and that obtained after linear regression are slightly different in the third decimal position. This maybe due to the method of inducing

imperfection and a variable column stiffness resulting from machining, shaping and cutting processes.

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Table 1. Theoretical dimensions and properties of Model Slender Steel Columns

Length (in.)	X-Section (in.)	I_{min} (in^4.)	r_{min} (in.)	L/r		a_0 (in.)	
11.4748	2.5x0.375	0.010986	0.10825	106	0.125	0.25	0.35
11.2894	2.5x0.375	0.010986	0.10825	132	0.125	0.25	0.375
16.23798	2.5x0.375	0.010986	0.10825	150	0.125	0.25	0.375
19.37732	2.5x0.375	0.010986	0.10825	179	0.125	0.25	0.375

Table 2. Experimental dimensions and properties of Model Slender Steel Columns.

Length (in.)	X-Section (in.)	I_{min} (in^4.)	r_{min} (in.)	L/r	a_0 (in.)	Ultimate Load (lb)
12.0472	2.5x0.3937	0.0127	0.114	106	0.1614	25071.478
14.25196	2.5098x0.374	0.0109	0.108	132	0.1929	15419.949
16.5354	2.5x0.3819	0.0116	0.1102	150	0.15748	12146.172
19.1	2.5196x0.37007	0.0106	0.1068	179	0.13385	8349.8453

Table 3. δ and δ/p values for different slenderness ratios and imperfections

KL/r	Initial Crookedness (in.)	δ/p at 1" deflection (in./lb)	δ/p at 1.5" deflection (in./lb)
106	0.125, 0.25, 0.375	2E-05,3.9E-05,5.8E-05	-
132	0.125, 0.25, 0.375	3E-05,6E-05,9E-05	4E-05,8E-05,0.00012
150	0.125, 0.25, 0.375	4E-05,7.9E-05,0.000108	5.25E-05,8E-05,0.00012
179	0.125, 0.25, 0.375	5.1E-05,0.00011,0.000165	8E-05,0.00015,0.00023

Appendix A

Theoretical Southwell Plots for four Steel Columns with Initial Crookedness

Modulus of Elasticity = 29 x 10^6 psi

X-section of Columns = 2.5" x 0.375"

Calculations

KL/r	b (in.)	h (in.)	A (in^2)	Imin (in^4)	r	L (in.)	Pcr (lbs.)	Pcr (Kg)
106	2.5	0.375	0.9375	0.010986328	0.108253175	11.4748366	23881.26287	10835.41873
132	2.5	0.375	0.9375	0.010986328	0.108253175	14.28941916	15400.01548	6987.302848
150	2.5	0.375	0.9375	0.010986328	0.108253175	16.23797632	11925.77198	5410.967325
179	2.5	0.375	0.9375	0.010986328	0.108253175	19.37731841	8374.578498	3799.718012

Column#1: KL/r=106, a0=1/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.002673081	5.34616E-06
1000	0.005462985	5.46299E-06
1500	0.008377543	5.58503E-06
2000	0.0114253	5.71265E-06
2500	0.014615601	5.84624E-06
3000	0.017958684	5.98623E-06
3500	0.021465794	6.13308E-06
4000	0.025149308	6.28733E-06
4500	0.029022877	6.44953E-06
5000	0.033101599	6.62032E-06
5500	0.037402218	6.8004E-06
6000	0.041943346	6.99056E-06
6500	0.04674574	7.19165E-06
7000	0.051832615	7.40466E-06
7500	0.05723002	7.63067E-06
8000	0.062967285	7.87091E-06
8500	0.069077553	8.12677E-06

Column#1: KL/r=106, a0=2/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.005346161	1.06923E-05
1000	0.01092597	1.0926E-05
1500	0.016755087	1.11701E-05
2000	0.022850601	1.14253E-05
2500	0.029231201	1.16925E-05
3000	0.035917368	1.19725E-05
3500	0.042931589	1.22662E-05
4000	0.050298616	1.25747E-05
4500	0.058045753	1.28991E-05
5000	0.066203199	1.32406E-05
5500	0.074804436	1.36008E-05
6000	0.083886692	1.39811E-05
6500	0.093491481	1.43833E-05
7000	0.10366523	1.48093E-05
7500	0.11446004	1.52613E-05
8000	0.125934569	1.57418E-05
8500	0.138155106	1.62535E-05

Column#1: KL/r=106, a0=3/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.008019242	1.60385E-05
1000	0.016388956	1.6389E-05
1500	0.02513263	1.67551E-05
2000	0.034275901	1.7138E-05
2500	0.043846802	1.75387E-05
3000	0.053876052	1.79587E-05
3500	0.064397383	1.83993E-05
4000	0.075447923	1.8862E-05
4500	0.08706863	1.93486E-05
5000	0.099304798	1.9861E-05
5500	0.112206654	2.04012E-05
6000	0.125830039	2.09717E-05
6500	0.140237221	2.1575E-05
7000	0.155497845	2.2214E-05
7500	0.17169006	2.2892E-05
8000	0.188901854	2.36127E-05
8500	0.207232659	2.43803E-05

9000	0.075598423	8.39982E-06	9000	0.151196845	1.67996E-05	9000	0.226795268	2.51995E-05
9500	0.082572721	8.69187E-06	9500	0.165145441	1.73837E-05	9500	0.247718162	2.60756E-05
10000	0.090049444	9.00494E-06	10000	0.180098887	1.80099E-05	10000	0.270148331	2.70148E-05
10500	0.098084913	9.34142E-06	10500	0.196169825	1.86828E-05	10500	0.294254738	2.80243E-05
11000	0.106744192	9.70402E-06	11000	0.213488384	1.9408E-05	11000	0.320232577	2.91121E-05
11500	0.116102858	1.00959E-05	11500	0.232205715	2.01918E-05	11500	0.348308573	3.02877E-05
12000	0.126249206	1.05208E-05	12000	0.252498411	2.10415E-05	12000	0.378747617	3.15623E-05
12500	0.13728705	1.0983E-05	12500	0.274574099	2.19659E-05	12500	0.411861149	3.29489E-05
13000	0.149339283	1.14876E-05	13000	0.298678567	2.29753E-05	13000	0.44801785	3.44629E-05
13500	0.162552478	1.20409E-05	13500	0.325104955	2.40818E-05	13500	0.487657433	3.61228E-05
14000	0.177102869	1.26502E-05	14000	0.354205737	2.53004E-05	14000	0.531308606	3.79506E-05
14500	0.193204265	1.33244E-05	14500	0.38640853	2.66489E-05	14500	0.579612796	3.99733E-05
15000	0.211118624	1.40746E-05	15000	0.422237249	2.81491E-05	15000	0.633355873	4.22237E-05
15500	0.231170413	1.49142E-05	15500	0.462340826	2.98284E-05	15500	0.693511239	4.47427E-05
16000	0.253766437	1.58604E-05	16000	0.507532874	3.17208E-05	16000	0.761299312	4.75812E-05
16500	0.279423729	1.69348E-05	16500	0.558847459	3.38695E-05	16500	0.838271188	5.08043E-05
17000	0.308809595	1.81653E-05	17000	0.61761919	3.63305E-05	17000	0.926428784	5.44958E-05
17500	0.342800484	1.95886E-05	17500	0.685600967	3.91772E-05	17500	1.028401451	5.87658E-05
18000	0.382570895	2.12539E-05	18000	0.765141789	4.25079E-05	18000	1.147712684	6.37618E-05

Column#2: KL/r=132, a0=1/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.004194627	8.38925E-06
1000	0.008680546	8.68055E-06
1500	0.013489194	8.9928E-06
2000	0.018656695	9.32835E-06
2500	0.024224777	9.68991E-06
3000	0.030241898	1.00806E-05
3500	0.036764658	1.05042E-05
4000	0.04385959	1.09649E-05
4500	0.051605431	1.14679E-05
5000	0.060096064	1.20192E-05

Column#2: KL/r=132, a0=2/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.008389253	1.67785E-05
1000	0.017361092	1.73611E-05
1500	0.026978387	1.79856E-05
2000	0.03731339	1.86567E-05
2500	0.048449554	1.93798E-05
3000	0.060483795	2.01613E-05
3500	0.073529316	2.10084E-05
4000	0.087719179	2.19298E-05
4500	0.103210863	2.29357E-05
5000	0.120192129	2.40384E-05

Column#2: KL/r=132, a0=3/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.01258388	2.51678E-05
1000	0.026041639	2.60416E-05
1500	0.040467581	2.69784E-05
2000	0.055970085	2.7985E-05
2500	0.072674331	2.90697E-05
3000	0.090725693	3.02419E-05
3500	0.110293974	3.15126E-05
4000	0.131578769	3.28947E-05
4500	0.154816294	3.44036E-05
5000	0.180288193	3.60576E-05

5500	0.069444336	1.26262E-05	5500	0.138888672	2.52525E-05	5500	0.208333008	3.78787E-05
6000	0.079787103	1.32979E-05	6000	0.159574205	2.65957E-05	6000	0.239361308	3.98936E-05
6500	0.091291976	1.40449E-05	6500	0.182583952	2.80898E-05	6500	0.273875928	4.21348E-05
7000	0.104166475	1.48809E-05	7000	0.20833295	2.97618E-05	7000	0.312499424	4.46428E-05
7500	0.118670654	1.58228E-05	7500	0.237341307	3.16455E-05	7500	0.356011961	4.74683E-05
8000	0.135134853	1.68919E-05	8000	0.270269705	3.37837E-05	8000	0.405404558	5.06756E-05
8500	0.153985162	1.81159E-05	8500	0.307970324	3.62318E-05	8500	0.461955486	5.43477E-05
9000	0.175780825	1.95312E-05	9000	0.35156165	3.90624E-05	9000	0.527342475	5.85936E-05
9500	0.201270658	2.11864E-05	9500	0.402541317	4.23728E-05	9500	0.603811975	6.35592E-05
10000	0.231480818	2.31481E-05	10000	0.462961636	4.62962E-05	10000	0.694442454	6.94442E-05
10500	0.267856297	2.55101E-05	10500	0.535712594	5.10202E-05	10500	0.803568891	7.65304E-05
11000	0.312498901	2.8409E-05	11000	0.624997802	5.6818E-05	11000	0.937496703	8.5227E-05
11500	0.368588281	3.20512E-05	11500	0.737176562	6.41023E-05	11500	1.105764843	9.61535E-05
12000	0.441174462	3.67645E-05	12000	0.882348925	7.35291E-05	12000	1.323523387	0.000110294
12500	0.538790228	4.31032E-05	12500	1.077580456	8.62064E-05	12500	1.616370684	0.00012931
13000	0.677078967	5.2083E-05	13000	1.354157935	0.000104166	13000	2.031236902	0.000156249

Column#3: KL/r=150, a0=1/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.00547009	1.09402E-05
1000	0.011440839	1.14408E-05
1500	0.01798428	1.19895E-05
2000	0.025186958	1.25935E-05
2500	0.033153783	1.32615E-05
3000	0.042013173	1.40044E-05
3500	0.051924026	1.48354E-05
4000	0.063085337	1.57713E-05
4500	0.0757497	1.68333E-05
5000	0.090242648	1.80485E-05
5500	0.106991036	1.94529E-05
6000	0.126565788	2.10943E-05

Column#3: KL/r=150, a0=2/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.01094018	2.18804E-05
1000	0.022881678	2.28817E-05
1500	0.03596856	2.3979E-05
2000	0.050373916	2.5187E-05
2500	0.066307566	2.6523E-05
3000	0.084026345	2.80088E-05
3500	0.103848051	2.96709E-05
4000	0.126170675	3.15427E-05
4500	0.1514994	3.36665E-05
5000	0.180485295	3.60971E-05
5500	0.213982071	3.89058E-05
6000	0.253131576	4.21886E-05

Column#3: KL/r=150, a0=3/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.01641027	3.28205E-05
1000	0.034322517	3.43225E-05
1500	0.053952839	3.59686E-05
2000	0.075560873	3.77804E-05
2500	0.099461349	3.97845E-05
3000	0.126039518	4.20132E-05
3500	0.155772077	4.45063E-05
4000	0.189256012	4.7314E-05
4500	0.2272491	5.04998E-05
5000	0.270727943	5.41456E-05
5500	0.320973107	5.83587E-05
6000	0.379697364	6.32829E-05

6500	0.149748276	2.30382E-05	6500	0.299496552	4.60764E-05	6500	0.449244828	6.91146E-05
7000	0.17763713	2.53767E-05	7000	0.355274261	5.07535E-05	7000	0.532911391	7.61302E-05
7500	0.211827451	2.82437E-05	7500	0.423654903	5.64873E-05	7500	0.635482354	8.4731E-05
8000	0.254726969	3.18409E-05	8000	0.509453939	6.36817E-05	8000	0.764180908	9.55226E-05
8500	0.310149071	3.64881E-05	8500	0.620298143	7.29763E-05	8500	0.930447214	0.000109464
9000	0.384513901	4.27238E-05	9000	0.769027803	8.54475E-05	9000	1.153541704	0.000128171
9500	0.489534881	5.153E-05	9500	0.979069762	0.00010306	9500	1.468604643	0.00015459
10000	0.649090344	6.4909E-05	10000	1.298180688	0.000129818	10000	1.947271032	0.000194727

Column#4: KL/r=179, a0=1/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.007936933	1.58739E-05
1000	0.016950121	1.69501E-05
1500	0.027274399	1.81829E-05
2000	0.039218279	1.96091E-05
2500	0.053195306	2.12781E-05
3000	0.069772913	2.32576E-05
3500	0.08975135	2.56432E-05
4000	0.114296726	2.85742E-05
4500	0.145177082	3.22616E-05
5000	0.185208316	3.70417E-05
5500	0.239165499	4.34846E-05
6000	0.315845528	5.26409E-05
6500	0.433430769	6.66817E-05
7000	0.636558771	9.0937E-05
7500	1.071944945	0.000142926
8000	2.669667387	0.000333708

Column#4: KL/r=179, a0=2/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.015873866	3.17477E-05
1000	0.033900243	3.39002E-05
1500	0.054548799	3.63659E-05
2000	0.078436559	3.92183E-05
2500	0.106390612	4.25562E-05
3000	0.139545827	4.65153E-05
3500	0.1795027	5.12865E-05
4000	0.228593452	5.71484E-05
4500	0.290354164	6.45231E-05
5000	0.370416632	7.40833E-05
5500	0.478330997	8.69693E-05
6000	0.631691056	0.000105282
6500	0.866861538	0.000133363
7000	1.273117543	0.000181874
7500	2.14388989	0.000285852
8000	5.339334774	0.000667417

Column#4: KL/r=179, a0=3/8"

P (lbs.)	Def. (in.)	Def./P (in./lb.)
500	0.023810798	4.76216E-05
1000	0.050850364	5.08504E-05
1500	0.081823198	5.45488E-05
2000	0.117654838	5.88274E-05
2500	0.159585918	6.38344E-05
3000	0.20931874	6.97729E-05
3500	0.269254049	7.69297E-05
4000	0.342890178	8.57225E-05
4500	0.435531246	9.67847E-05
5000	0.555624947	0.000111125
5500	0.717496496	0.000130454
6000	0.947536585	0.000157923
6500	1.300292307	0.000200045
7000	1.909676314	0.000272811
7500	3.215834835	0.000428778
8000	8.009002161	0.001001125

Appendix B

Experimental Southwell Plots for Steel Columns with Initial Crookedness

Column #	a0 (mm)	a0 (in)	E (psi)	Length(mm)	Length(in)	Width(mm)	Width(in)	Thickness(mm)	Thickness(in)	Area (in^2)
1	4.1	0.161417323	29462288.46	306	12.0472441	63.5	2.5	10	0.393700787	0.984251969
2	4.9	0.192913386	29036021.71	362	14.2519685	63.75	2.50984252	9.5	0.374015748	0.938720627
3	4	0.157480315	28199317.99	420	16.5354331	63.5	2.5	9.7	0.381889764	0.954724409
4	3.4	0.133858268	29242908.39	485.14	19.1	64	2.519685039	9.4	0.37007874	0.932481865
			28985134.14							

Column #	Imin (in^4)	r (in)	KL/r	Pcr (Kg)	Pcr(lb)	y-intercept
1	0.01271	0.113651628	106.0015094	11375.44372	25071.478	6.43829E-06
2	0.01094	0.107969046	132.0005037	6996.346944	15419.9487	1.25106E-05
3	0.0116	0.110242079	149.9920287	5510.967135	12146.1716	1.29654E-05
4	0.01064	0.10683253	178.7844955	3788.496068	8349.84533	1.60312E-05

Column #2

Load (Kg)	Load(lbs)	Defl.x0.01mm	Net Def. (mm)	Net Def.(in.)	Def./Load
0	0	45	0	0	#DIV/0!
100	220.4	54	0.09	0.003543307	1.6077E-05
200	440.8	63	0.18	0.007086614	1.6077E-05
300	661.2	75	0.3	0.011811024	1.7863E-05
400	881.6	85	0.4	0.015748031	1.7863E-05
500	1102	96	0.51	0.02007874	1.822E-05
600	1322.4	104	0.59	0.023228346	1.7565E-05
700	1542.8	110	0.65	0.025590551	1.6587E-05
800	1763.2	117	0.72	0.028346457	1.6077E-05
900	1983.6	125	0.8	0.031496063	1.5878E-05
1000	2204	133	0.88	0.034645669	1.5719E-05
1100	2424.4	143	0.98	0.038582677	1.5914E-05
1200	2644.8	153	1.08	0.042519685	1.6077E-05
1300	2865.2	161	1.16	0.045669291	1.5939E-05

Regression Output:		
Constant	1.24951E-05	
Std Err of Y Est	2.04036E-07	
R Squared	0.995483305	
No. of Observations	22	
Degrees of Freedom	20	
X Coefficient(s)	6.67555E-05	
Std Err of Coef.	1.00546E-06	

y=mx+c
1.24951E-05
1.27317E-05
1.29682E-05
1.32836E-05
1.35464E-05
1.38355E-05
1.40457E-05
1.42034E-05
1.43874E-05
1.45977E-05
1.48079E-05
1.50707E-05
1.53335E-05
1.55438E-05

1400	3085.6	171	1.26	0.049606299	1.6077E-05				1.58066E-05
1500	3306	181	1.36	0.053543307	1.6196E-05				1.60694E-05
1600	3526.4	193	1.48	0.058267717	1.6523E-05				1.63848E-05
1700	3746.8	206	1.61	0.063385827	1.6917E-05				1.67265E-05
1800	3967.2	216	1.71	0.067322835	1.697E-05				1.69893E-05
1900	4187.6	228	1.83	0.072047244	1.7205E-05				1.73047E-05
2000	4408	241	1.96	0.077165354	1.7506E-05				1.76463E-05
2100	4628.4	255	2.1	0.082677165	1.7863E-05				1.80143E-05
2200	4848.8	267	2.22	0.087401575	1.8025E-05				1.83297E-05
2300	5069.2	286	2.41	0.09488189	1.8717E-05				1.8829E-05
2400	5289.6	300	2.55	0.100393701	1.8979E-05				1.91969E-05
2500	5510	318	2.73	0.107480315	1.9506E-05				1.967E-05
2600	5730.4	336	2.91	0.114566929	1.9993E-05				2.01431E-05
2700	5950.8	356	3.11	0.122440945	2.0576E-05				2.06687E-05
2800	6171.2	374	3.29	0.129527559	2.0989E-05				2.11418E-05
2900	6391.6	397	3.52	0.138582677	2.1682E-05				2.17463E-05
3000	6612	418	3.73	0.146850394	2.221E-05				2.22982E-05
3100	6832.4	444	3.99	0.157086614	2.2991E-05				2.29815E-05
3200	7052.8	468	4.23	0.166535433	2.3613E-05				2.36123E-05
3300	7273.2	500	4.55	0.179133858	2.4629E-05				2.44533E-05
3400	7493.6	538	4.93	0.194094488	2.5901E-05				2.5452E-05

Column #3

Load (Kg)	Load(lb.)	Defl.x0.01mm	Net Def. (mm)	Net Def. (in.)	Def./Load				y=mx+c
0	0	0	0	0	#DIV/0!		Regression Output:		1.33336E-05
200	440.8	26	0.26	0.01023622	2.3222E-05	Constant		1.33336E-05	1.44365E-05
300	661.2	34	0.34	0.013385827	2.0245E-05	Std Err of Y Est		1.18556E-06	1.47758E-05
400	881.6	44	0.44	0.017322835	1.9649E-05	R Squared		0.975616886	1.52E-05
500	1102	55	0.55	0.021653543	1.9649E-05	No. of Observations		20	1.56667E-05
600	1322.4	65	0.65	0.025590551	1.9352E-05	Degrees of Freedom		18	1.60909E-05
700	1542.8	79	0.79	0.031102362	2.016E-05				1.66847E-05
900	1983.6	89	0.89	0.03503937	1.7665E-05	X Coefficient(s)	0.000107747		1.71089E-05

1000	2204	106	1.06	0.041732283	1.8935E-05	Std Err of Coef.	4.01489E-06		1.78301E-05
1100	2424.4	113	1.13	0.044488189	1.835E-05				1.8127E-05
1200	2644.8	122	1.22	0.048031496	1.8161E-05				1.85038E-05
1300	2865.2	136	1.36	0.053543307	1.8687E-05				1.91027E-05
1400	3085.6	150	1.5	0.059055118	1.9139E-05				1.96966E-05
1500	3306	165	1.65	0.06496063	1.9649E-05				2.03329E-05
1600	3526.4	183	1.83	0.072047244	2.0431E-05				2.10964E-05
1700	3746.8	202	2.02	0.079527559	2.1225E-05				2.19024E-05
1800	3967.2	229	2.29	0.09015748	2.2726E-05				2.30477E-05
1900	4187.6	253	2.53	0.099606299	2.3786E-05				2.40658E-05
2000	4408	282	2.82	0.111023622	2.5187E-05				2.5296E-05
2100	4628.4	305	3.05	0.12007874	2.5944E-05				2.62717E-05
2200	4848.8	326	3.26	0.128346457	2.647E-05				2.71625E-05
2400	5289.6	380	3.8	0.149606299	2.8283E-05				2.94532E-05
2500	5510	432	4.32	0.17007874	3.0867E-05				3.1659E-05
2600	5730.4	485	4.85	0.190944882	3.3321E-05				3.39073E-05
2700	5950.8	550	5.5	0.216535433	3.6388E-05				3.66646E-05
2760	6083.04	712	7.12	0.280314961	4.6081E-05				4.35366E-05

Column #4

Load (Kg)	Load(lb.)	Defl.x0.01mm	Net Def. (mm)	Net Def.(in.)	Def./Load				y=mx+c
0	0	0	0	0	#DIV/0!		Regression Output:		
100	220.4	10	0.1	0.003937008	1.7863E-05	Constant		1.58981E-05	1.64439E-05
200	440.8	21	0.21	0.008267717	1.8756E-05	Std Err of Y Est		9.49116E-07	1.70443E-05
300	661.2	35	0.35	0.013779528	2.084E-05	R Squared		0.993544337	1.78085E-05
400	881.6	52	0.52	0.020472441	2.3222E-05	No. of Observations		10	1.87364E-05
500	1102	66	0.66	0.025984252	2.3579E-05	Degrees of Freedom		8	1.95005E-05
600	1322.4	86	0.86	0.033858268	2.5604E-05				2.05922E-05
700	1542.8	99	0.99	0.038976378	2.5263E-05	X Coefficient(s)	0.000138641		2.13018E-05
800	1763.2	110	1.1	0.043307087	2.4562E-05	Std Err of Coef.	3.95114E-06		2.19022E-05
900	1983.6	124	1.24	0.048818898	2.4611E-05				2.26663E-05
1000	2204	161	1.61	0.063385827	2.8759E-05				2.46859E-05

1100	2424.4	180	1.8	0.070866142	2.923E-05				2.5723E-05
1200	2644.8	208	2.08	0.081889764	3.0963E-05				2.72513E-05
1400	3085.6	227	2.27	0.089370079	2.8964E-05				2.82884E-05
1500	3306	260	2.6	0.102362205	3.0963E-05				3.00896E-05
1600	3526.4	293	2.93	0.115354331	3.2712E-05				3.18909E-05
1700	3746.8	329	3.29	0.129527559	3.457E-05				3.38558E-05
1800	3967.2	374	3.74	0.147244094	3.7115E-05				3.63121E-05
1900	4187.6	414	4.14	0.162992126	3.8923E-05				3.84954E-05
2000	4408	463	4.63	0.182283465	4.1353E-05				4.117E-05
2100	4628.4	497	4.97	0.195669291	4.2276E-05				4.30258E-05
2200	4848.8	557	5.57	0.219291339	4.5226E-05				4.63007E-05
2300	5069.2	627	6.27	0.246850394	4.8696E-05				5.01216E-05
2400	5289.6	774	7.74	0.304724409	5.7608E-05				5.81452E-05
2420	5333.68	846	8.46	0.333070866	6.2447E-05				6.20752E-05
2430	5355.72	908	9.08	0.357480315	6.6747E-05				6.54594E-05

Column #1

Load (Kg)	Load(lb)	Defl x0.01mm	Net Defl.mm	Net Def.(in)	Defl./Load				y=mx+c
500	1102	66	0.06	0.002362205	2.1436E-06		Regression Output:		6.46622E-06
1000	2204	92	0.32	0.012598425	5.7162E-06	Constant		6.33726E-06	7.02505E-06
1500	3306	121	0.61	0.024015748	7.2643E-06	Std Err of Y Est		2.97744E-07	7.64835E-06
2000	4408	159	0.99	0.038976378	8.8422E-06	R Squared		0.974627024	8.4651E-06
2500	5510	190	1.3	0.051181102	9.2888E-06	No. of Observations		6	9.13139E-06
3000	6612	228	1.68	0.066141732	1.0003E-05	Degrees of Freedom		4	9.94814E-06
3500	7714	268	2.08	0.081889764	1.0616E-05				1.08079E-05
4000	8816	333	2.73	0.107480315	1.2192E-05	X Coefficient(s)	5.4593E-05		1.22049E-05

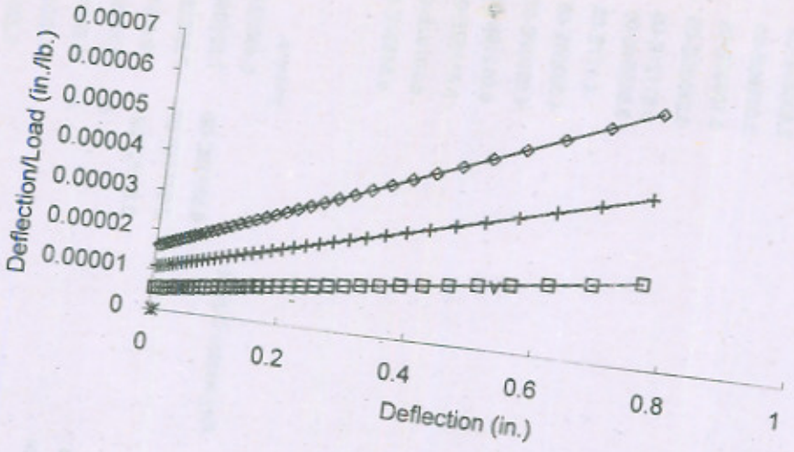


Fig.1a

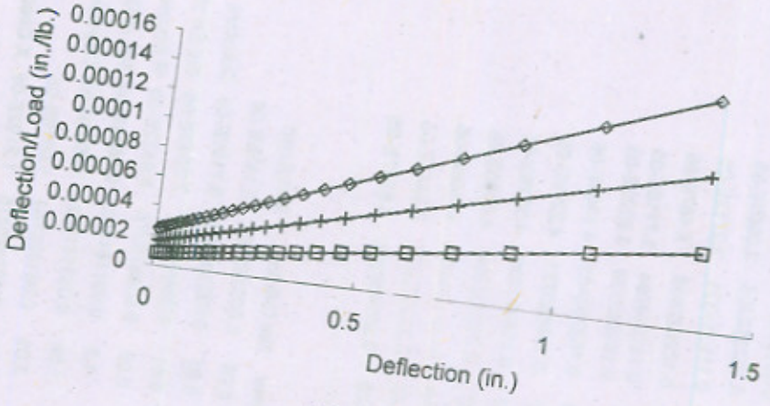


Fig.1b

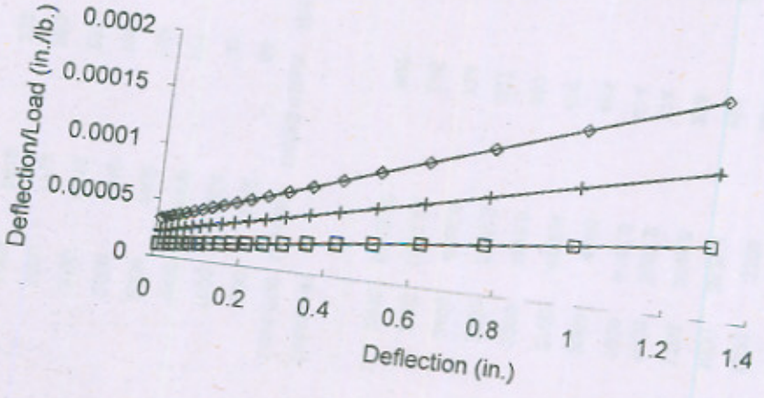


Fig.1c

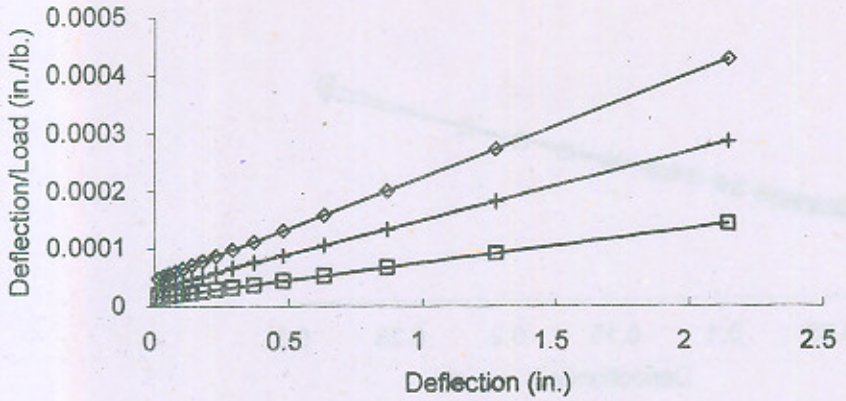


Fig.1d

FIG:1 Theoretical Smith-Southwell Plots for Slender Steel Columns itial Crookedness

(a) Kl/r=106 (b) Kl/r=132

© Kl/r=150 (d) Kl/r=179

Initial Crookedness for each column is 1/8", 2/8", 3/8". The steepest line on each graph is for a0=3/8"

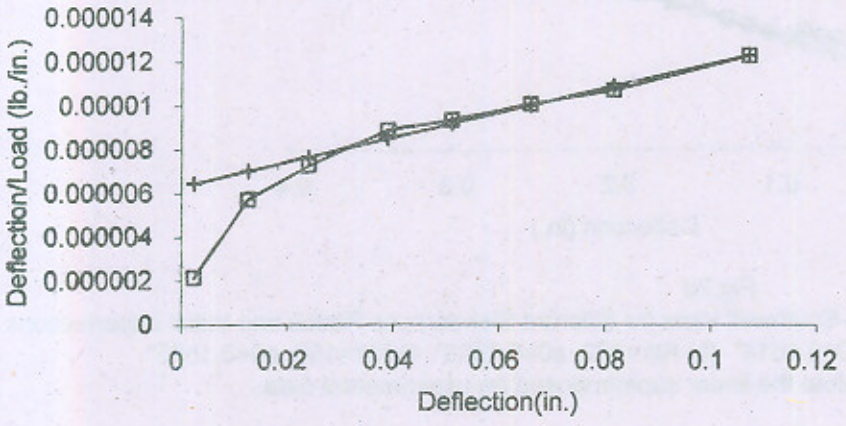


Fig.2a

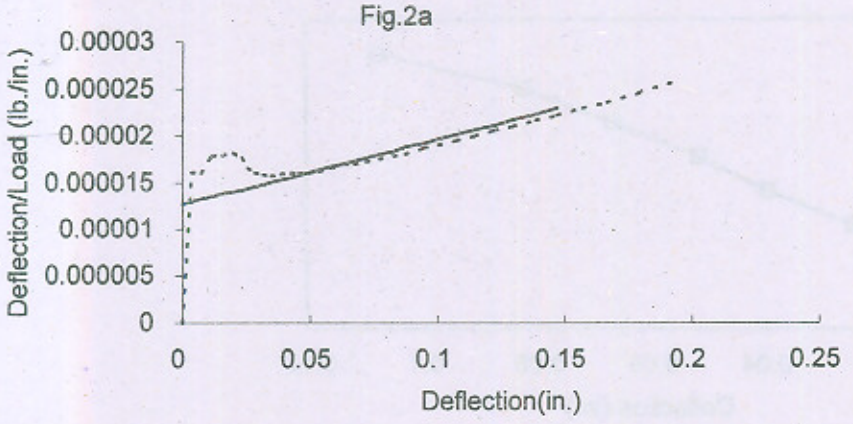


Fig.2b

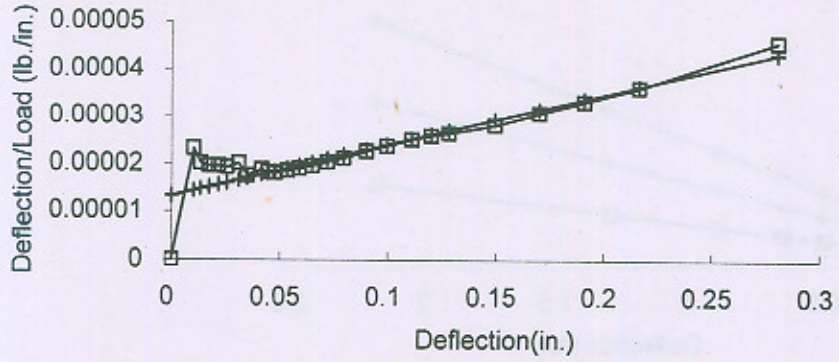


Fig.2c

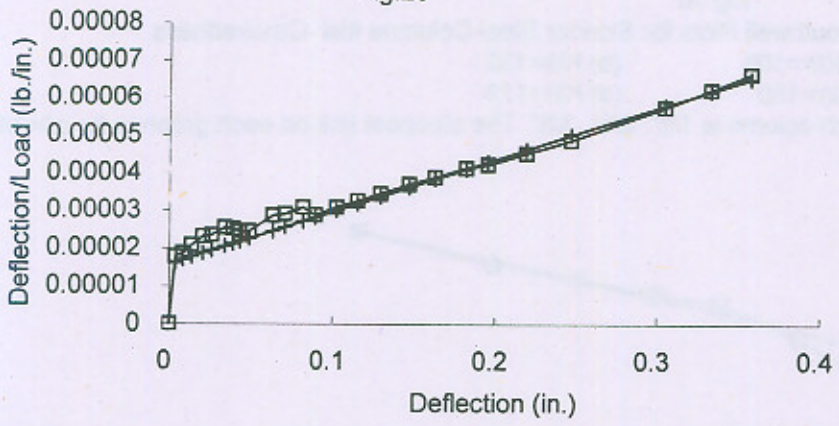


Fig.2d

FIG:2 Experimental Smith-Southwell Plots for different Slenderness Ratios and Initial Imperfections (a) Kl/r=106, a0=0.1614" (b) Kl/r=132, a0=0.1929" © Kl/r=150, a0=0.1575" (d) Kl/r=179,a0=0.1339". Note the linear superimposed on experimental data.

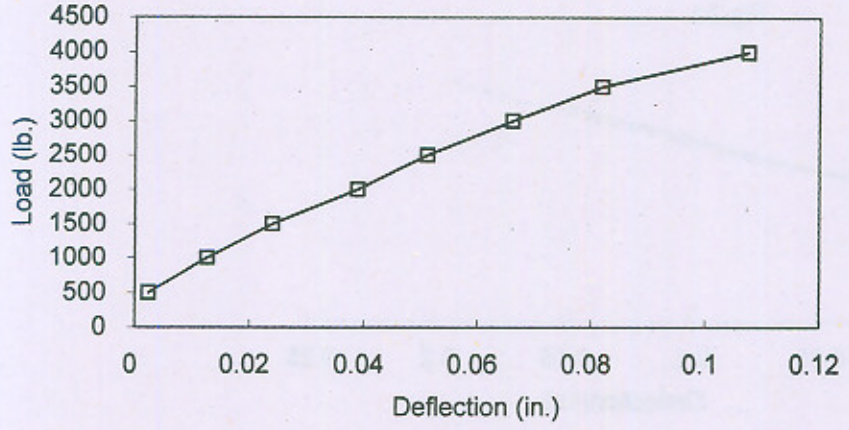


Fig.3a

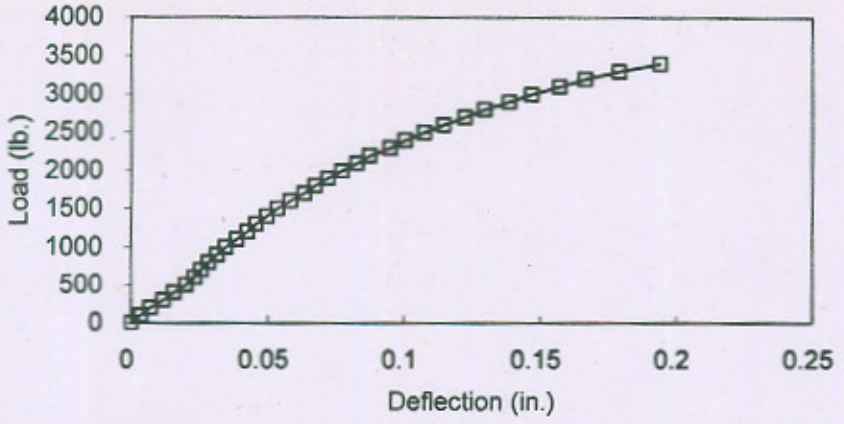


Fig.3b

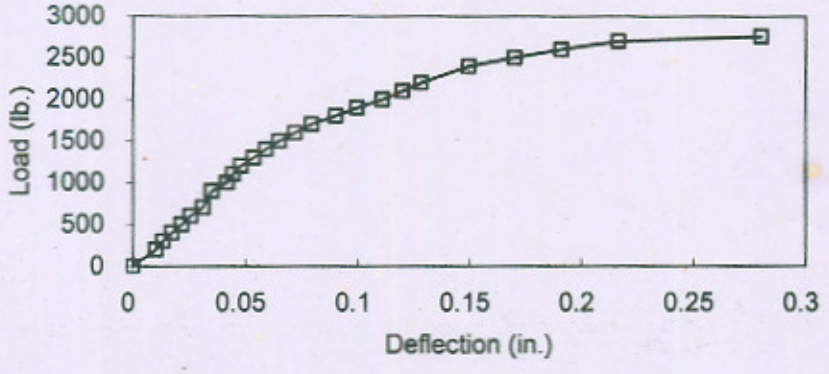


Fig.3c

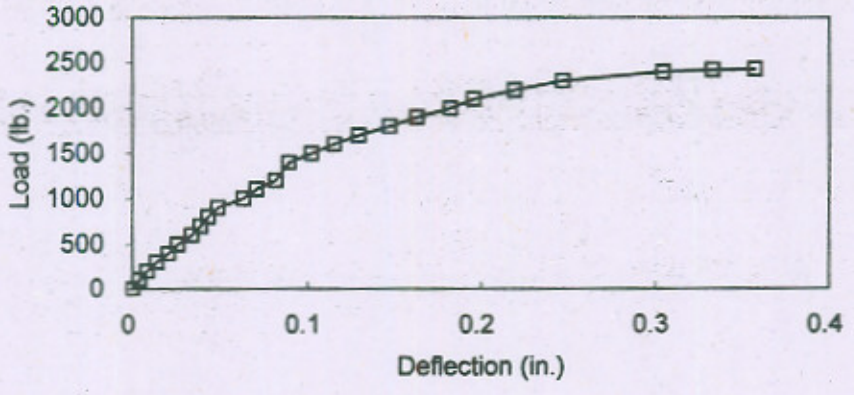


Fig.3d

FIG:3 Experimental Load Deflection Curves for Model Slender Steel Columns
(a) Kl/r=106 (b) Kl/r=132 © Kl/r=150 (d) Kl/r=179





