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A CONVENIENT METHOD FOR DESIGN OF REINFORCED CONCRETE MEMBERS

BY

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ABSTRACT

A simplified procedure for design of reinforced concrete members along with realistic moment coefficients for two way slab design for local conditions are presented. Modern way of designing reinforced concrete structures at ultimate loads is used indirectly in the derivation and the resulting expressions are developed in the already familiar format using the actual loads. This design procedure is proposed to be employed for general construction in Pakistan where quality control and construction facilities are not meeting American or British standards. The amount of steel required to resist moment calculated from the proposed formulae comes out to be 7% to 24% lesser than the conventional working strength design, which is considerably conservative.

INTRODUCTION

Reinforced concrete design in Pakistan is gradually changing from the working stress method of the British Standard Specifications (BSS) Code (CP114, 1957) to the ultimate strength design of the American Concrete Institute (ACI, 1995) Code. The provisions of the BSS Code are based on a concrete strength determined by 6 in. cube specimens, whereas, the ACI Code recommendations are derived for a concrete strength obtained by using 6 in. diameter and 12 in. high cylinders. The ultimate strength design becomes economical as the degree of indeterminacy of the structure increases. This method uses the reserve capacities of the materials like strain hardening in steel, durability, and redistribution of stresses, etc. This method is based on the ultimate or collapse mode of the structure, therefore, it becomes more reliable as well as economical. The design formulae of the old BSS Code (CP114, 1957) are more familiar as these are convenient and easy to use and hence some engineers prefer these even now-a-days for design of structures due to their simplicity and adequacy for design of such structures.

The ultimate design approach requires relatively more effort to find out the steel reinforcement. Because of the economic nature of the modern design techniques, one has to be very careful in quality control, construction methods, and estimation of expected loads. These factors can not easily be controlled for general construction in Pakistan due to availability of lesser funds for design process, poor formwork, local site practices and almost no engineering management. As a result, the concrete compressive strength seldom reaches the minimum level of 3000 psi cube strength (2250 psi cylinder strength). Whereas, the engineers in the developed countries are employing high strength concrete for reinforced concrete construction. The recent literature does not show any evidence of use of concrete lesser than 3000 psi cylinder strength (4000 psi cube

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strength). It follows that a considerable improvement in our general construction industry is required before one can safely apply the provisions of ACI or BS8110 as such.

For design of two way continuous slabs the coefficient method of design is discontinued from the ACI Code and only direct design method, equivalent frame method, or a complete detailed analysis is recommended. However, it still allows the coefficient method in a broader sense if satisfying the compatibility and equilibrium requirements. The coefficients are reproduced in relevant books and are separately given for negative moments, positive moments due to dead loads, and positive moments due to live loads. It requires some work to calculate all these moments and combine them to get the critical design moment. Further, for some cases, the coefficients for positive moments lead to considerably small values which are even lesser than that given by the yield line analysis (Nilson and Winter, 1991). A considerable amount of torsional strength and redistribution of stresses is utilized in the derivation of these coefficients. Authors think that this refined behaviour of the structure is not justified for ordinary construction in Pakistan due to poor quality control. On the other hand the coefficients given in CP114, CP110 and BS8110 can be safely employed (Reynolds and Steedman, 1988). These lead to a conservative design which is particularly true in case of CP114. There is a need to provide a design technique based on ultimate strength in a form which is simple to apply and is applicable for practical concrete strengths according to local conditions. Simplified design formulations are presented in this paper to be employed for general construction in Pakistan, where quality control and construction facilities are not satisfying the limits of BSS, ASTM, and ACI. These formulations will provide safe, easy, and economical designs for a wide variety of ordinary construction. Economy and accurate prediction of possible failure modes is taken from the latest ACI ultimate strength procedure whereas the convenience of the earlier BSS design code is maintained.

THE FORMULATION AND COEFFICIENTS

The proposed expressions are derived for concrete cylinder strengths (f_c') of 2 ksi for ordinary quality control construction, steel yield strengths (f_y) of 40 ksi and 60 ksi, a combined load factor of 1.53 for dead and live loads, and capacity reduction factor of 0.85 for shear and torsion and 0.90 for moment. Ultimate strength formulae of ACI-95 Code are applied for the derivation but service load moments (M_s) are used with the above mentioned combined load factor of 1.53.

A_s	=	area of steel reinforcement, $in.^2$
A_{s1}	=	$A_{s\,max}$ for singly reinforced section, $in.^2$
A_{s2}	=	additional steel area required for compression, $in.^2$
A_{st} & A_{sc}	=	total tension and compression steel area, respectively, $in.^2$
b	=	width of rectangular section, in.
d	=	effective depth, depth of steel centroid from the compression face, in.
d'	=	depth of compression steel centroid from the compression face, in.
M_s	=	unfactored load moment, k-in.
M_{s1}	=	max. unfactored moment which singly reinforced section can resist, k-in.
M_{s2}	=	$M_s - M_{s1}$

Design for Moment

i) $f_c' = 2$ksi and $f_y = 40$ ksi

Case 1. Singly Reinforced Section:

$$A_{s\,max} = 0.0186\,bd \tag{1}$$

$$A_{s\,min}\ for\ beams = 0.005\,bd \quad \& \quad A_{s\,min}\ for\ slabs = 0.002\,bd \tag{2}$$

$$d_{min} = \sqrt{\frac{M_s}{0.341\,b}} \qquad A_s = A_{s\,max}$$

and for $d \geq d_{min}$ $\qquad A_s = \frac{M_s}{18.4\,d}$ (3)

$$d \geq \sqrt{\frac{M_s}{0.219\,b}} \qquad A_s = \frac{M_s}{20.6\,d} \tag{4}$$

$$d \geq \sqrt{\frac{M_s}{0.18\,b}} \qquad A_s = \frac{M_s}{21.2\,d} \tag{5}$$

$$d \geq \sqrt{\frac{M_s}{0.111\,b}} \qquad A_s = A_{s\,min} \text{ for beams}$$

$$d \geq \sqrt{\frac{M_s}{0.046\,b}} \qquad A_s = A_{s\,min} \text{ for slabs} \tag{6}$$

Case 2. Doubly Reinforced Section: If d is lesser than d_{min}, a doubly reinforced section is required. The moments along with tension and compression steel are given below:

$$M_{s1} = 0.341bd^2 \tag{7}$$

$$A_{s1} = 0.0186bd \tag{8}$$

$$M_{s2} = M_s - M_{s1} \tag{9}$$

$$A_{sc} = A_{s2} = \frac{M_{s2}}{23.5\,(d - d')} \quad \text{compression steel is assumed to be yielding} \tag{10}$$

$$A_{st} = A_{s1} + A_{s2} \tag{11}$$

Compression steel is yielding if $\quad d' \leq 0.278d$ (12)

ii) $f_c' = 3$ ksi and $f_y = 40$ ksi

Case 1. Singly Reinforced Section:

$$A_{s\,max} = 0.0278\,bd \tag{13}$$

$$A_{s\,min}\ for\ beams = 0.005\,bd \quad \& \quad A_{s\,min}\ for\ slabs = 0.002\,bd \tag{14}$$

$$d_{min} = \sqrt{\frac{M_s}{0.511\,b}} \qquad A_s = A_{s\,max}$$

and for $d \geq d_{min}$ $\qquad A_s = \dfrac{M_s}{18.4\,d}$ (15)

$$d \geq \sqrt{\frac{M_s}{0.328\,b}} \qquad A_s = \frac{M_s}{20.6\,d} \tag{16}$$

$$d \geq \sqrt{\frac{M_s}{0.27\,b}} \qquad A_s = \frac{M_s}{21.2\,d} \tag{17}$$

$$d \geq \sqrt{\frac{M_s}{0.113\,b}} \qquad A_s = A_{s\,min} \text{ for beams}$$

$$d \geq \sqrt{\frac{M_s}{0.046\,b}} \qquad A_s = A_{s\,min} \text{ for slabs} \tag{18}$$

Case 2. Doubly Reinforced Section: If d is lesser than d_{min}, a doubly reinforced section is required. The moments along with tension and compression steel are given below:

$$M_{s1} = 0.511\,bd^2 \tag{19}$$

$$A_{s1} = 0.0278\,bd \tag{20}$$

$$M_{s2} = M_s - M_{s1} \tag{21}$$

$$A_{sc} = A_{s2} = \frac{M_{s2}}{23.5\,(d - d')} \quad \text{compression steel is assumed to be yielding} \tag{22}$$

$$A_{st} = A_{s1} + A_{s2} \tag{23}$$

Compression steel is yielding if $\qquad d' \leq 0.277\,d$ (24)

iii) $f_c' = 2$ ksi and $f_y = 60$ ksi

Case 1. Singly Reinforced Section:

$$A_{s\,max} = 0.0107\,bd \tag{25}$$

$$A_{s\,min}\ for\ beams = 0.0033\,bd \quad \& \quad A_{s\,min}\ for\ slabs = 0.0018\,bd \tag{26}$$

$$d_{min} = \sqrt{\frac{M_s}{0.306\,b}} \qquad A_s = A_{s\,max}$$

and for $d \geq d_{min}$ $\qquad A_s = \dfrac{M_s}{28.6\,d}$ (27)

$$d \geq \sqrt{\frac{M_s}{0.219\,b}} \qquad A_s = \frac{M_s}{30.9\,d} \tag{28}$$

$$d \geq \sqrt{\frac{M_s}{0.18\,b}} \qquad A_s = \frac{M_s}{31.7\,d} \tag{29}$$

$$d \geq \sqrt{\frac{M_s}{0.147\,b}} \qquad A_s = \frac{M_s}{32.5\,d} \tag{30}$$

$$d \geq \sqrt{\frac{M_s}{0.111\,b}} \qquad A_s = A_{s\,min} \text{ for beams}$$

$$d \geq \sqrt{\frac{M_s}{0.061\,b}} \qquad A_s = A_{s\,min} \text{ for slabs} \tag{31}$$

Case 2. Doubly Reinforced Section: If d is lesser than d_{min}, a doubly reinforced section is required. The moments along with tension and compression steel are given below:

$$M_{s1} = 0.306\,bd^2 \tag{32}$$

$$A_{s1} = 0.0107\,bd \tag{33}$$

$$M_{s2} = M_s - M_{s1} \tag{34}$$

$$A_{sc} = A_{s2} = \frac{M_{s2}}{35.3\,(d - d')} \quad \text{compression steel is assumed to be yielding} \tag{35}$$

$$A_{st} = A_{s1} + A_{s2} \tag{36}$$

Compression steel is yielding if $\quad d' \leq 0.138d$ (37)

iv) $f_c' = 3$ ksi and $f_y = 60$ ksi

Case 1. Singly Reinforced Section:

$$A_{s\,max} = 0.016035\,bd \tag{38}$$

$$A_{s\,min}\ for\ beams = 0.0033\,bd \quad \& \quad A_{s\,min}\ for\ slabs = 0.0018\,bd \tag{39}$$

$$d_{min} = \sqrt{\frac{M_s}{0.459\,b}} \qquad A_s = A_{s\,max}$$

and for $d \geq d_{min}$ $\qquad A_s = \dfrac{M_s}{28.6\,d}$ (40)

$$d \geq \sqrt{\frac{M_s}{0.328\,b}} \qquad A_s = \frac{M_s}{30.9\,d} \tag{41}$$

$$d \geq \sqrt{\frac{M_s}{0.27\,b}} \qquad A_s = \frac{M_s}{31.7\,d} \tag{42}$$

$$d \geq \sqrt{\frac{M_s}{0.22\,b}} \qquad A_s = \frac{M_s}{32.5\,d} \tag{43}$$

$$d \geq \sqrt{\frac{M_s}{0.113\,b}} \qquad A_s = A_{s\,min} \text{ for beams}$$

$$d \geq \sqrt{\frac{M_s}{0.062\,b}} \qquad A_s = A_{s\,min} \text{ for slabs} \tag{44}$$

Case 2. Doubly Reinforced Section: If d is lesser than d_{min}, a doubly reinforced section is required. The moments along with tension and compression steel are given below:

$$M_{s1} = 0.459\,bd^2 \tag{45}$$

$$A_{s1} = 0.016035\,bd \tag{46}$$

$$M_{s2} = M_s - M_{s1} \tag{47}$$

$$A_{sc} = A_{s2} = \frac{M_{s2}}{35.3\,(d - d')} \quad \text{compression steel is assumed to be yielding} \tag{48}$$

$$A_{st} = A_{s1} + A_{s2} \tag{49}$$

Compression steel is yielding if $\quad d' \leq 0.138\,d$ (50)

Out of the above equations, the formula having maximum denominator coefficient is to be used for determination of steel reinforcement provided the effective depth criterion is satisfied.

Design of T and L Beams

The following symbols are used for the derivations:

h_f	=	depth of flange, in.
b	=	effective width of slab acting as flange of the T-beam, in.
b_w	=	width of stem, in.
S	=	centre-to-centre spacing of beams, in.
S_c	=	clear spacing of beams, in.
A_{sf}	=	area of steel required to balance compression in the flange, in.^{2}
ρ_w	=	steel ratio in T-beams = $A_s/(b_w d)$
A_{sw}	=	area of steel reinforcement required for rectangular section, in.^{2}
l	=	span of the beam

Effective Flange Width: For T-beams, b should be minimum of (l/4), ($16h_f + b_w$) and S, and for L-beam, b must be minimum of (l/12 + b_w), ($6h_f + b_w$) and (S_c/2 + b_w). For the isolated T-beams, h_f should not be lesser than b_w/2 and b is not to be more than $4b_w$.

Design of T-beam with $f_c' = 2$ ksi: The T-beam is first designed as a rectangular beam of dimensions b and d. Depth of the neutral axis from the compression face, c, is calculated using Eq. 51 and if $c \le h_f$, the design is the same as that of the rectangular one. Otherwise, A_{sf} is evaluated using Eq. 52 and the remaining moment, M_r, is determined by Eq. 53. The rectangular web of dimensions b_w and d is then designed for this moment (M_r) to calculate the additional area of steel, A_{sw}. Equation 54 is employed to obtain the total area of steel required.

$$c = 27.7\frac{A_s}{b} \quad \text{Grade 40}$$
$$c = 41.5\frac{A_s}{b} \quad \text{Grade 60} \qquad (51)$$

$$A_{sf} = 0.0425\,(b - b_w)\,h_f \quad \text{Grade 40}$$
$$A_{sf} = 0.0283\,(b - b_w)\,h_f \quad \text{Grade 60} \qquad (52)$$

$$M_r = 1.1\,M_s - 26.1\,A_{sf}\left[d - {}^{h_f}\!/_{2}\right] \quad \text{Grade 40}$$
$$M_r = 1.1\,M_s - 39.2\,A_{sf}\left[d - {}^{h_f}\!/_{2}\right] \quad \text{Grade 60} \qquad (53)$$

$$\text{total steel} \quad A_s = A_{sf} + A_{sw} \qquad (54)$$

Maximum Permissible Steel Ratios for T-beams:

$$\rho_{w,max} = 0.0186 + 0.0319\frac{(b-b_w)h_f}{b_w d} \qquad \text{Grade 40}$$

$$\rho_{w,max} = 0.0107 + 0.0212\frac{(b-b_w)h_f}{b_w d} \qquad \text{Grade 60} \qquad (55)$$

Minimum Depth of Beams and One-way Slabs for Deflection Control

The values of the minimum depth of beams and one way slabs are taken from ACI 1995 Code and are reproduced in Table 1.

TABLE. 1. Minimum Depth of Beams and One Way Slabs

Member	Steel Grade	Simply Supported	One End Continuous	Both Ends Continuous	Cantilever
Solid Slab	40	l/25	l/30	l/35	l/12.5
	60	l/20	l/24	l/28	l/10
Beams	40	l/20	l/23	l/26	l/10
	60	l/16	l/18.5	l/21	l/8

Minimum Depth of Two Way Slabs

h = depth of slab, in.
l_l = longer span length, in.
l_{ns} = clear shorter span length, in.
l_{nl} = clear longer span length, in.
l_s = shorter span length, in.

i) ACI 1963 Code Method:

$$h_{min} = \frac{\text{panel perimeter}}{180} \qquad \text{not less than 3.5 in.} \qquad (56)$$

for slabs supported on walls or very rigid beams

For different aspect ratios, the above formula simplifies to:

square panel:	$h_{min} = l_s / 45$
3:4 ratio panel:	$h_{min} = l_s / 38.5$
2:3 ratio panel:	$h_{min} = l_s / 36$

ii) ACI 1995 Code Method:

$$h_{min} = \frac{l_{ns}}{36\frac{l_{ns}}{l_{nl}} + 9} \qquad \text{for Grade 40} \tag{57}$$

Increase the depth h_{min} by 10% for Grade 60

This criterion gives the minimum depth of two way slabs for different aspect ratios as under:

square panel:	$h_{min} = l_{ns} / 45.0$ (Grade 40) &	$h_{min} = l_{ns} / 40.9$ (Grade 60)
3:4 ratio panel:	$h_{min} = l_s / 36.0$ (Grade 40) &	$h_{min} = l_{ns} / 32.7$ (Grade 60)
2:3 ratio panel:	$h_{min} = l_s / 33.0$ (Grade 40) &	$h_{min} = l_{ns} / 30.0$ (Grade 60)

Moment Coefficients for Two Way Continuous Slabs

The positive moment coefficients given by Nilson and Winter, reproduced from method 3 of the ACI 1963 Code for slab panels supposed by relatively stiff beams or walls, are provided separately for dead and live loads. The calculation of critical design moment is relatively more involved. The positive moment coefficients are too low for most cases due to involvement of a considerably large inelastic redistribution. Accordingly the negative moment coefficients are high for the corresponding cases. For example, in case of simply supported square slab, the ACI 1963 moment is $0.036wl_n^2$ in comparison with $0.048wl^2$ obtained from exact elastic analysis (Timoshenko and Woinowsky-Krieger, 1959), $0.042wl^2$ obtained from ordinary yield line analysis considering same properties in all directions, $0.047wl^2$ if corner effects are considered in yield line analysis, and $0.0625wl^2$ if deflection is equated for middle strips in both directions. On the other hand, the coefficients given in CP114 are too conservative. The coefficients given in BS8110 are also conservative as compared to the ACI 1963 coefficients but are more realistic in most cases.

Considering the local conditions and the fact that construction management pay less attention to the placement of negative steel, the slab moment coefficients of BS8110 are proposed to be used and are reproduced in Table 2. However, for simply supported slabs, the coefficients given by Timoshenko and Woinowsky-Krieger (1959) for elastic homogenous plates are also included in this table.

Table 2. Moment Coefficients for Two Way Slabs (BS8110).

Ratio l_y / l_x	Coefficient C	Case 1 *	Case 1 **	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.0	C_{sx}^-	—	—	0.033	—	0.047	0.046	0.058	—	0.040	0.039
	C_{sx}^+	0.056	0.048	0.024	0.034	0.036	0.034	0.044	0.043	0.030	0.029
	C_{ty}^-	—	—	0.033	0.045	0.045	—	—	0.057	0.037	0.037
	C_{ty}^+	0.056	0.048	0.024	0.034	0.034	0.034	0.044	0.043	0.028	0.028
1.1	C_{sx}^-	—	—	0.037	—	0.056	0.050	0.065	—	0.049	0.044
	C_{sx}^+	0.065	0.056	0.028	0.046	0.042	0.038	0.048	0.053	0.036	0.033
	C_{ty}^-	—	—	0.027	0.037	0.037	—	—	0.047	0.031	0.031
	C_{ty}^+	0.047	0.041	0.020	0.028	0.028	0.028	0.036	0.036	0.023	0.024
1.2	C_{sx}^-	—	—	0.043	—	0.063	0.054	0.072	—	0.056	0.048
	C_{sx}^+	0.073	0.063	0.032	0.056	0.047	0.041	0.053	0.064	0.042	0.036
	C_{ty}^-	—	—	0.023	0.032	0.032	—	—	0.040	0.026	0.026
	C_{ty}^+	0.039	0.035	0.017	0.024	0.024	0.024	0.030	0.030	0.020	0.020
1.3	C_{sx}^-	—	—	0.047	—	0.068	0.057	0.077	—	0.063	0.052
	C_{sx}^+	0.081	0.070	0.035	0.065	0.052	0.043	0.057	0.072	0.047	0.038
	C_{ty}^-	—	—	0.019	0.027	0.027	—	—	0.034	0.022	0.023
	C_{ty}^+	0.033	0.030	0.014	0.020	0.020	0.020	0.026	0.026	0.017	0.017
1.4	C_{sx}^-	—	—	0.050	—	0.074	0.060	0.081	—	0.068	0.055
	C_{sx}^+	0.087	0.076	0.037	0.072	0.055	0.045	0.060	0.078	0.052	0.042
	C_{ty}^-	—	—	0.017	0.023	0.023	—	—	0.029	0.018	0.019
	C_{ty}^+	0.029	0.026	0.013	0.017	0.018	0.018	0.022	0.023	0.015	0.015
1.5	C_{sx}^-	—	—	0.054	—	0.077	0.062	0.085	—	0.073	0.058
	C_{sx}^+	0.093	0.082	0.040	0.078	0.058	0.047	0.064	0.084	0.055	0.043
	C_{ty}^-	—	—	0.015	0.020	0.020	—	—	0.026	0.017	0.017
	C_{ty}^+	0.026	0.022	0.011	0.015	0.015	0.015	0.019	0.019	0.013	0.013

Table 2. (Continued)

Ratio l_t / l_s	Coefficient C	Case 1		Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
		*	**								
1.6	C_s^-	—	—	0.056	—	0.082	0.064	0.087	—	0.077	0.060
	C_s^+	0.097	0.087	0.042	0.084	0.061	0.048	0.066	0.090	0.058	0.045
	C_t^-	—	—	0.013	0.018	0.017	—	—	0.023	0.015	0.015
	C_t^+	0.023	0.020	0.009	0.014	0.014	0.014	0.017	0.017	0.011	0.012
1.7	C_s^-	—	—	0.058	—	0.085	0.066	0.090	—	0.080	0.063
	C_s^+	0.102	0.091	0.044	0.089	0.064	0.050	0.068	0.094	0.061	0.046
	C_t^-	—	—	0.012	0.016	0.016	—	—	0.018	0.013	0.013
	C_t^+	0.020	0.017	0.008	0.012	0.012	0.012	0.015	0.014	0.009	0.010
1.8	C_s^-	—	—	0.060	—	0.087	0.067	0.093	—	0.084	0.065
	C_s^+	0.105	0.095	0.045	0.094	0.066	0.052	0.070	0.098	0.063	0.048
	C_t^-	—	—	0.010	0.014	0.014	—	—	0.018	0.012	0.012
	C_t^+	0.017	0.015	0.007	0.011	0.010	0.011	0.014	0.013	0.008	0.008
1.9	C_s^-	—	—	0.063	—	0.090	0.068	0.096	—	0.087	0.066
	C_s^+	0.108	0.099	0.047	0.097	0.067	0.052	0.072	0.103	0.065	0.049
	C_t^-	—	—	0.008	0.013	0.012	—	—	0.016	0.010	0.010
	C_t^+	0.016	0.013	0.006	0.010	0.008	0.010	0.012	0.012	0.008	0.008
2.0	C_s^-	—	—	0.063	—	0.093	0.070	0.097	—	0.088	0.067
	C_s^+	0.111	0.102	0.048	0.100	0.069	0.053	0.074	0.105	0.066	0.050
	C_t^-	—	—	0 008	0.012	0.011	—	—	0.015	0.009	0.010
	C_t^+	0.014	0.012	0.006	0.009	0.008	0.009	0.011	0.011	0.007	0.007

A shaded edge indicates that the slab continues across or is fixed; an unmarked edge indicates a roller support.

* With torsional restraint at corners.

** Actual elastic analysis (Timoshenko and Woinowsky-Krieger, 1959)

$$M_s = C_s w l_s^2 \quad \& \quad M_t = C_t w l_t^2$$

Distribution of Slab Loads on Beams

The effective width of slab contributing load on unit width of beam for moment calculations is evaluated by taking the area supported by the beam using 45° lines method, dividing it with the span length of the beam and multiplying it with the factor to convert it into equivalent uniformly distributed load per unit length for moment calculations.

l_s = shorter span, ft.
l_l = longer span, ft.
R = l_s / l_l

Width of slab supported by external short beam, ft = $1/3\ l_s$
Width of slab supported by internal short beam, ft = $2/3\ l_s$ (56)

Width of slab supported by external long beam, ft = $l_s/2\ (1 - R^2/3)$
Width of slab supported by internal long beam, ft = $l_s\ (1 - R^2/3)$ (57)

Loads on Lintel

For those cases of lintels where a height of wall above the lintel equal to its span is available, the equivalent uniformly distributed load (udl) for moment calculation may be estimated as follows:

W = wall thickness, in.
l = span, ft.

$$\text{equivalent udl on lintel} = 0.00577\ Wl \text{ k/ft} \quad (58)$$

$$\text{maximum positive moment} = 0.0087\ Wl^3 \text{ k-in.} \quad (59)$$

Equivalent Horizontal Loads from Stairs

Loads on horizontal plane of stairs = 1.22 x loads on inclined plane (60)

Shear Design

For the derivation of the formulae ACI 1995 Code provisions are used and the ultimate concrete shear strength is taken as $2\sqrt{f_c'}$. Shear force at the supports should be calculated as simply supported reactions and then the shear due to difference of end moments, if any, should be added. For design, shear force at the required section should be evaluated. After the shear design, stirrups for torsion should be added.

V_s = service load shear, kips
s = spacing of stirrups, in.
A_v = cross-sectioned area of vertical stirrups, in.^{2}
b_w = width of web, in.

(i) Neglect shear reinforcement if $V_s/b_w d < 0.0248$ ksi (25 psi) for $f_c' = 2$ ksi
or $V_s/b_w d < 0.0304$ ksi (30 psi) for $f_c' = 3$ ksi (61)

(ii) If $V_s/b_w d > 0.0248$ ksi for $f_c' = 2$ ksi or $V_s/b_w d > 0.0304$ ksi for $f_c' = 3$ ksi and s evaluated from Eq. 63 or Eq. 64 is negative, provide stirrups at a spacing of d/2 having an area given by Eq. 62.

$$A_v = 0.000625\, b_w d \qquad \text{for Grade 40 steel} \tag{62}$$

(iii) For $f_c' = 2$ ksi:

$$s = \frac{A_v d}{0.045 V_s - 0.002236 b_w d} \qquad s_{max} = d/2 \qquad \text{Grade 40 steel} \tag{63}$$

For 2 - legged No.3 stirrups: $s = \dfrac{A_v d}{0.20 V_s - 0.01 b_w d}$ $s_{max} = d/2$

For 2 - legged No.2 stirrups: $s = \dfrac{A_v d}{0.46 V_s - 0.023 b_w d}$ $s_{max} = d/2$

iv) For $f_c' = 3$ ksi:

$$s = \frac{A_v d}{0.045 V_s - 0.002739 b_w d} \qquad s_{max} = d/2 \qquad \text{Grade 40 steel} \tag{64}$$

Development Length

The given expressions for development length include 30% increase for steel placed in top lesser quality concrete and are derived from the ACI Code for No.7 or higher number bars. Hence the value obtained will be conservative except in the case when the ACI Code minimum transverse stirrups are not provided. A further increase of 50% in development length is necessary in the later cases.

l_d = development length in tension, in.
d_b = diameter of bars, in.
l_{db} = development length in compression, in.

i) Development length in tension:

$$\begin{aligned} l_d &= 59\, d_b \quad \text{not lesser than 12 in.} \quad \text{Grade 40, } f_c' = 2 \text{ ksi} \\ l_d &= 88\, d_b \quad \text{not lesser than 12 in.} \quad \text{Grade 60, } f_c' = 2 \text{ ksi} \end{aligned} \tag{65}$$

ii) Development length in compression:

$$l_{db} = 18\, d_b \quad \text{not lesser than 8 in.} \quad \text{Grade 40, } f_c' = 2 \text{ ksi}$$
$$l_{db} = 27\, d_b \quad \text{not lesser than 8 in.} \quad \text{Grade 60, } f_c' = 2 \text{ ksi} \qquad (66)$$

iii) Compression lap splice length:

$$\text{lap splice length} = 27\, d_b \quad \text{not lesser than 16 in.} \quad \text{Grade 40, } f_c' = 2 \text{ ksi}$$
$$\text{lap splice length} = 40\, d_b \quad \text{not lesser than 16 in.} \quad \text{Grade 60, } f_c' = 2 \text{ ksi} \qquad (67)$$

EXAMPLES

Example 1

A simply supported rectangular beam, b=15in. and d=21.3in., with a span of 15 ft is to carry a computed dead load of 2.04 kips/ft and a service live load of 1.67 kips/ft. Steel of Grade 40 and 1:2:4 concrete without mechanical compaction are to be used. Area of steel is required to be calculated.

Total working load = 3.71 kips/ft
Total factored load = 5.70 kips/ft
Working moment = 1252 in-kips
Ultimate moment = 1924 in-kips

i) ACI ultimate strength design considering $f_c' = 2$ ksi:

$A_s = 2.80 \text{ in.}^2$

ii) The proposed procedure:

$$\sqrt{\frac{M_s}{0.219b}} = 19.5 \prec d \quad \text{ok}$$

$$A_s = \frac{1252}{(20.6)(21.3)} = 2.85 \text{ in}^2 \quad \text{only 1.8\% greater than that of Case (i)}$$

iii) CP114 working strength design:

$$A_s = \frac{1252}{(17.14)(21.3)} = 3.43 \text{ in}^2 \quad \text{22.5\% greater than that of Case (i)}$$

Example 2

A reinforced concrete slab measuring 15x18.75 ft internally is supported on 9 in. thick walls and is subjected to a total load of 0.3 ksf. Critical design bending moments are to be compared when using BS8110, ACI 1963 Code and exact elastic analysis. The corners are assumed to be held down.

$l_{ns} = 15$ ft; $l_{nl} = 18.75$ ft; $l_s = 15.75$ ft; $l_l = 19.5$ ft

$$\frac{l_{ns}}{l_{nl}} = \frac{15}{18.75} = 0.8 \quad \text{and} \quad \frac{l_l}{l_s} = \frac{19.5}{15.75} \cong 1.25$$

i) BS8110 Code:

$M_s = 0.077 \times 0.3 \times 15.75^2 \times 12 = 68.76$ in-kips
$M_l = 0.036 \times 0.3 \times 19.5^2 \times 12 = 49.28$ in-kips

ii) ACI 1963 Code:

$M_s = 0.056 \times 0.3 \times 15^2 \times 12 = 45.36$ in-kips		34.0% lesser than BS8110
$M_l = 0.023 \times 0.3 \times 18.75^2 \times 12 = 29.11$ in-kips		40.9% lesser than BS8110

iii) Exact elastic analysis (Timoshenko and Woinowsky-Krieger):

$M_s = 0.067 \times 0.3 \times 15.75^2 \times 12 = 59.83$ in-kips	13.0% lesser than BS8110
$M_l = 0.033 \times 0.3 \times 19.5^2 \times 12 = 45.17$ in-kips	8.3% lesser than BS8110

CONCLUSIONS

The presented formulae and coefficients provide safe, economical and easy method of designing reinforced concrete members. Modern way of designing reinforced concrete structures at ultimate loads is used indirectly in the derivation and the resulting expressions are presented in the already familiar form using actual unfactored loads. It is hoped that this paper will help the engineers in designing the structures with a considerable convenience. Further the formulations are derived for local conditions and material properties which will help in improving the safety as well as economy of the structures simultaneously.

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