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ABSTRACT

A relatively simple solution to the problem of computation of infinite plate with a crack lying on elastic base dynamic loading is presented. The solution to the problem involves analogue of Rakhmatulin's model with a difference that a long axis is directed deep into base. The solution is reformed by application of Laplace and Hanchels integral transformations.

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DETAILS

The problem of dynamic computation of infinite plate lying on elastic base is resolved this work [1], where elastic isotropic semispael is taken as base. Bat this solution is quite cumbrous and for gelting more simple results [2], the problem of bent of infinite plate on base described by dynamic model of H.A.Rakhmatulin [3].

The base represents infinite istropic medium in this model. However, grounds' investigations indicate that they have different elastic moduls in vertical and horizontal directions. Besides, as known in several cases the base is considered as a lyer of final corpulence. The cylinder base of final radius of infinite or final corpulence. That's why in the beginning formulas for bases transference of mentioned models will be obtained in the beginning of the work, then the bent of unlimited plates lying on the elastic base described by suggested models will be investigated.

Let's consider analogue of Rakhmatulin's model with a difference that a long axis dirrected deep into base z $(0 \le z \le \infty)$, elastic modul E_n differes from elastic modul in horizontal plane E_r $(0 < z < \infty)$. In this case vibrations of elastic surroundings is descrbed with equation

$$a^2 \cdot \left(\frac{d^2 v}{dr^2} + \frac{1}{r} \cdot \frac{dv}{dr} \right) + b^2 \cdot \frac{d^2 v}{dz^2} = \frac{d^2 v}{dt^2} \ , \qquad (1)$$

where **a** and **в** are velocity of horizontal and yertical waves.

Having applied integral transformations of Laplas and Hanchel, for this equation resolvement will find functions of base bent as follows.

$$\upsilon(r,z,t) = \frac{1}{2 \cdot \pi \cdot i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{pt} \int_0^{\infty} \lambda \cdot A(\lambda, p) \cdot e^{-\frac{z}{b}\sqrt{\lambda^2 a^2 + p^2}} \cdot J_0(\lambda, r) d\lambda \cdot dz \ , \qquad (2)$$

where $\mu = b/a$, $A(\lambda,p)$-determined function from limited condition on free surface base at $z = 0$.

In operation on the basic of concentrated power P which is applicoted at r=0:

$$A(\lambda, p) = -\frac{P \cdot b}{2 \cdot \pi \cdot E_n \cdot \sqrt{\lambda^2 \cdot a^2 + p^2}} \qquad (3)$$

in operation of loading q, which is distinguished by the circle of radius R

$$A(\lambda,p)=-\frac{b\cdot q\cdot R\cdot J_1(\lambda,R)}{\lambda\cdot E_B\cdot\sqrt{\lambda^2\cdot a^2+p^2}} \tag{4}$$

In case, if the load modes a settled vibration with the freguenty ω, the formules of explonotion of flexibility of base should be simplicoted and should take a form

$$\upsilon(r,\omega,z)=-\frac{P\cdot b}{2\cdot\pi\cdot E_B}\int_0^{\infty}\frac{\lambda\cdot J_0(\lambda,r)\cdot e^{\frac{z}{b}\sqrt{\lambda^2a^2-\omega^2}}}{\sqrt{\lambda^2\cdot a^2-\omega^2}}\cdot d\lambda \tag{5}$$

$$\upsilon(r,\omega,z)=-\frac{q\cdot b\cdot R}{E_B}\int_0^{\infty}\frac{J_1(\lambda,r)\cdot J_0(\lambda,r)\cdot e^{\frac{z}{b}\sqrt{\lambda^2a^2-\omega^2}}}{\sqrt{\lambda^2\cdot a^2-\omega^2}}\cdot d\lambda \tag{6}$$

If we may imagine the model of base as loyer of ending corprulence h, spreoding under incompressing base shen the function of base should be such as this

$$\upsilon(r,z,t)=\frac{1}{2\cdot\pi\cdot i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{pt}\int_0^{\infty}\lambda\cdot A(\lambda,p)\cdot\left[e^{\frac{z}{b}\sqrt{\lambda^2a^2+p^2}}-e^{\frac{2h}{b}\sqrt{\lambda^2a^2+p^2}}\cdot e^{\frac{z}{b}\sqrt{\lambda^2a^2+p^2}}\right]\cdot J_0(\lambda,r)d\lambda\cdot dp \tag{7}$$

Function A(λ,p), depending on application to the base of loading has such form

$$A(\lambda,p)=\frac{b\cdot Q(\lambda)}{E_B\cdot\sqrt{\lambda^2\cdot a^2+p^2}\cdot\left[1+e^{\frac{2h}{b}\sqrt{\lambda^2a^2+p^2}}\right]} \tag{8}$$

here Q(λ) - the reform of Hanchels load, applicoting to the free surface base at z=0. Let's make a significance of function of flexibility base which can consentrated the power $p\cdot e^{i\omega t}$ to the function of flexibility base in operation and in settled vibration

$$\upsilon(r,z,t)=\frac{P\cdot b}{E_B\cdot 2\cdot\pi}\cdot\int_0^{\infty}\frac{\left[e^{\frac{z}{b}\sqrt{\lambda^2a^2-\omega^2}}-e^{\left(\frac{z}{b}-2\frac{h}{b}\right)\sqrt{\lambda^2a^2-\omega^2}}\right]}{\sqrt{\lambda^2\cdot a^2-\omega^2}\cdot\left[1+e^{-2\frac{h}{b}\sqrt{\lambda^2a^2-\omega^2}}\right]}\cdot\lambda\cdot J_0(,\lambda,r)d\lambda \tag{9}$$

Now, let's make out a bend of infinite plate, lying on the elastic base which was described above, to the surface which was applicated a dinamic load q(r,ω,t). Let's write down an equation of this plate in given coordinates:

$$\nabla\nabla w(r,\omega,t)+w(r,\omega,t)=q(r,\omega,t)-p(r,\omega,t), \tag{10}$$

where is a new immeasurable cordinate $r_1 = kr$, $k = (m/D)^{1/4}$, m- massa of quadratic metre of a plate, D - a hard of plate.

Besides taking into consideration that on the surface base

$$w(r,\omega,t)=v(r,\omega,t) \tag{11}$$

and a contact tention under a plate may be determined from correlotion

$$p(r,\omega,t) = -E_B \frac{dv(r,\omega,t)}{dz}\Big|_{z=0} \qquad (12)$$

Using to the equation (10) - (12) of Hanchel's and Laplas's reform.We'll get a expression for the flexibility of plate. If the plate is on the infinite massive with the different moduls of elasticity in vertical & horisontal directions then

$$w_{\omega}(r,t) = \frac{1}{2\cdot\pi\cdot i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\int_0^{\infty}\frac{\lambda\cdot Q(\lambda,p)\cdot J_0(\lambda,r)}{\left[\lambda^4 - p^2 + E_B\cdot\frac{\sqrt{\lambda^2\cdot a^2+p^2}}{b}\right]}d\lambda dp \qquad (13)$$

here Q(λ,p) - the Hanchel's reform applicoting to the plate of load.

An applicating load to the plate by the circle of area ρ, doing the settled vibiotion with the frequenty ω, then

$$w_{\omega}(r,t) = q\cdot\rho\int_0^{\infty}\frac{J_1(\lambda,\rho)\cdot J_0(\lambda,r)}{\left[\lambda^4 + \omega^2 + E_B\cdot\frac{\sqrt{\lambda^2\cdot a^2-\omega^2}}{b}\right]}d\lambda \qquad (14)$$

if the plate is on layer with capacity h, the curre of plate:

$$w(r,t) = \frac{1}{2\cdot\pi\cdot i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt}\int_0^{\infty}\frac{Q(\lambda,\rho)\cdot\left[1-\frac{2\cdot e^{\frac{-h}{b}\sqrt{\lambda^2 a^2+p^2}}}{1+e^{\frac{-2h}{b}\sqrt{\lambda^2 a^2+p^2}}}\right]\cdot\lambda\cdot J_0(\lambda,r)}{\left[\lambda^4 - p^2 + E_B\cdot\frac{\sqrt{\lambda^2\cdot a^2+p^2}}{b}\right]}d\lambda dp \qquad (14')$$

In some cases circuit hinge or even circuit crack on radius $r = \rho_i$ can be formed in plate under loading. As it was shown in work [4] the solution of such sums can be reformed by using of Laplas and Hanchel's integral reforms to equation (10), but in this case in the right part of equatin (10), in spite of the given loading function $q_0(r,t)$ loading $q_i(r,t)$ should be enclosed.

$$q_i(r) = A_i\cdot L_i(r)\cdot\delta(r-\rho) \qquad (i=1,2), \qquad (15)$$

where $L_i(r)$ - an operator of edge conditions according lines $r=\rho$.

Having done overmentioned operations, we'll write exression for plate curve with through crack on line $r=\rho_1$ when any dynamic loading have operation on plate:

$$w(r,t)=w_{\infty}(r,t)+\frac{1}{2\cdot\pi\cdot i}\cdot\int_{-i\infty}^{i\infty}e^{pt}\left\{A_1\cdot\left[\int_0^{\infty}\frac{\lambda^3 J_0(\lambda,\rho)\cdot J_0(\lambda,r)d\lambda}{\lambda^1-p^2+E_B\cdot\mu^{-1}\sqrt{\lambda^2+p^2/a^2}}\right.\right.$$

$$\left.-\frac{1-\nu}{\rho}\int_0^{\infty}\frac{\lambda^2\cdot J_1(\lambda,\rho)\cdot J_0(\lambda,r)d\lambda}{\lambda^1-p^2+E_B\cdot\mu^{-1}\sqrt{\lambda^2+p^2/a^2}}\right]-A_2\cdot\left[\frac{1}{2\cdot\rho}h(r-\rho)+\int_0^{\infty}\frac{\left(p^2-E_B\cdot\mu^{-1}\cdot\sqrt{\lambda^2+p^2/a^2}\right)}{\lambda^1-p^2+E_B\cdot\mu^{-1}\cdot\sqrt{\lambda^2+p^2/a^2}}\right. \qquad (16)$$

$$\left.\left.\cdot\frac{J_1(\lambda,\rho)\cdot J_0(\lambda,r)}{}d\lambda\right]\right\}dp$$

here h(r-ρ) - function Heviside:

$$h(r-\rho)=\begin{cases}1 & r<\rho\\ 0{,}5 & r=\rho\\ 0 & r>\rho\end{cases} \qquad (17)$$

Having differenced (16), we'll defermine functions of curving moments and crossed forces.

Coefficients A_i is determined according edge conditions on lines r=ρ;

$$M_r(\rho)=0 \qquad N_r(\rho)=0 \qquad (18)$$

If fluxible hinge forms by lines r=ρ, than in (16) $A_2=0$ should be applied.

CONCLUSIONS

The paper provides an appropriate mathematical treatment of the problem of dynamic computation of infinite plate with a crack lying on elastic base by dynamic loading. The equatons provided here may serve as a usefull tool for interested engineers.

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