ALIGNMENT CHARTS.

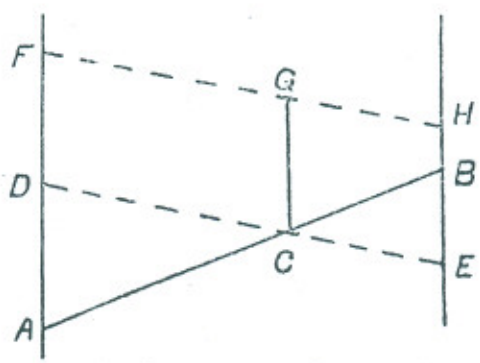
By A. C. Padday, Executive Engineer,
Western Jumna Canal, Delhi.

Introduction—Chart for distributary discharges—Chart for Kennedy's critical velocity ratios—Nomogram for weir discharges—Chart for retaining walls—Nomograms for discharges of orifices.

Introduction.

Alignment charts or nomograms afford a ready means of diagramatically expressing many of the formulæ connected with engineering problems. Their advantage lies in clearness of reading and ease of construction as compared with charts plotted from rectangular co-ordinates, and it is here proposed to show, without going into their mathematical theory beyond indicating the general principle, how some of the more simple can be constructed.

If any base as $A\ B$ is taken and parallel lines are drawn through each end, then a straight line, such as $D\ E$, passing



through a point C in the base, cuts the parallel straight lines in such a way that

$$\frac{A\ D}{B\ E} = \frac{A\ C}{C\ B}$$

and, obviously, if scales with the same unit are plotted along the lines $A\ D$ and $B\ E$, with zeros at A and B, and the point C is selected to give a required ratio m, then

$$A\ D = m\ B\ E.$$

The base $A\ B$ can be scaled for various values of m, and the simplest form of Z chart is thus obtained, connecting three variables $A\ D$, m, and $B\ E$ by means of a straight line, usually known as a transversal, across the three scales,

If different scale units are used for $A\,D$ and $B\,E$ the position of C is affected accordingly. The introduction of constants in the equation has a similar effect to a change in the scale units.

Again, if through C a line $C\,G$ is drawn parallel to $A\,D$ and through G a line $F\,H$ parallel to $D\,E$, then

$$\begin{aligned} C\,G &= A\,F - A\,D \\ &= A\,F - m\,B\,E \\ &= A\,F - m\,(E\,H - B\,H) \end{aligned}$$

but $E\,H = C\,G$

therefore $C\,G\,(1 + m) = A\,F + m\,B\,H$.

If m is a constant, then the scales for $A\,F$, $C\,G$ and $B\,H$ represent three variables. In the simplest case, where $m = 1$

$$2\,C\,G = A\,F + B\,H,$$

and for direct reading the scale unit for $C\,G$ would have to be half that adopted for $A\,F$ and $B\,H$.

The scale units obviously affect each other and the relative spacing of the parallel lines. In practice they are selected to suit the range of readings required, and their relative values and the spacing of the lines can be determined graphically.

The two diagrams for the equations,

$$Q = k\,d^{\frac{5}{3}}$$

$$V = V/V_{\circ} \times \cdot 84\,d^{\cdot 64}$$

illustrate the Z form of chart.

Chart for Distributary Discharges.

The equation

$$Q = k\,d^{\frac{5}{3}}$$

will give the discharge of a canal distributary in cusecs within the working range as usually required for a discharge table, provided that the channel is of the normal type in earth, with side slopes of about $\frac{1}{2}$ to 1, as can be checked easily from discharge diagrams such as those of Kennedy. The value of k is obtained from a single, or averaged from several observations. On the diagram, the diagonal connects the zeros of the two parallel side scales, for which inches and tenths have been used in plotting. The scale for d has been obtained from the intersection of lines connecting a selected value of k and corresponding calculated values of Q for the depth desired, thus for three feet depth, connect $k = 10$ with $Q = 62{\cdot}4$.

From the equation it is obvious that the scale readings of both Q and k can be multiplied by the same factor. This property may be taken advantage of to avoid acute intersections of the transversal with the scale for d, and to obtain values which would otherwise be off the side scales.

The advantages of this diagram will be very obvious to those who are constantly preparing and checking discharge tables of distributaries.

Chart for Kennedy's Critical Velocity Ratios

The second nomogram gives corresponding values of Kennedy's critical velocity ratio, the mean velocity and the depth of flow in a channel. The units for the side scales have been selected, keeping in view that the scales should be open, that the usual range $\frac{V}{V_o}$ is from 0·6 to 1·4, and that the velocity will rarely be over four feet per second. Outside values can be obtained by using a suitable factor for the side scale readings. The scale for d is divided as explained in the previous case, the values of V being calculated for $\frac{V}{V_o} = 1$ and depths required.

Nomogram for Weir Discharges

The formula used for this chart

$$Q = \tfrac{2}{3}\, c \left(d + \frac{e}{2} \right) L \sqrt{2\, g\, (d - e)}$$

contains five variables, and in the first instance, in order to reduce these to three, the constant C has been taken as 0·7, and the length of the crest L as one foot.

To put the equation in the form suitable for parallel scales logarithims are taken, giving

$$\log Q = \log \left[\tfrac{2}{3}\, c \sqrt{2g} \left(d + \frac{e}{2} \right) \right] + \tfrac{1}{2} \log (d - e).$$

Logarithmic scales for the two side variables $d + \frac{e}{2}$ and $d - e$ are first plotted, the units being selected to give the required range of values. For the central scale, for Q, two pairs of values of $d + \frac{e}{2}$ and $d -$ are calculated for the same discharge, say ten cusecs. The point of intersection of the transversals of the above pairs of values gives both the position of the scale for Q relative to the side scales, and the scale point for the discharge adopted. The scale point for a second discharge, say

ty cusecs, is similarly found. The positions of two scale readings being known, the remainder are obtained by drawing a logarithmic scale in proportion.

Any change in the constants in the equation alters the positions of the scales relatively to each other. This property is used to provide a supplementary scale to correct the discharge per foot run to suit co-efficients other than 0·7. The procedure is similar to a calculation with a slide rule, the difference between the scale length for 0·7 and the value of *c* adopted being added to or deducted from the discharge reading as first found. These differences could equally well be measured off the scale on which they are to be used, the scale units being necessarily the same.

The discharge per foot run being known, two more scales are required to give the total discharge and length of crest. The procedure is as with the first three scales, the central scale being obtained with the aid of the two dependent on it, which are first completed.

Logarithmic scales have two advantages in that the percentage of error in reading is constant, and that the scale repeats itself along its own line, so that extension, if required, is easily made by measurement.

Chart for Retaining Walls.

As in a nomogram three variables are dealt with at a time, the complete solution of a problem may have to be divided into steps. Thus, using Rankine's method to obtain the dimensions of a retaining wall, those for a vertical wall are first found, and then modified to suit the required top or bottom width. In the diagram herewith, to avoid additional scales, the face batter has not been dealt with.

For a wall with vertical sides Rankine's equation reduces to

$$\left(\frac{t}{h}\right)^2 r = \tan^2\left(45 - \frac{a}{2}\right)$$

and $\frac{t}{h}$ being found, the relation to other sections with back batter is given by the equation

$$r\left(t^2 - t_1^2\right) = t_2^2 - t^2$$

the notation being as given on the chart.

These equations contain both products and sums of variables, and the Z and parallel scale charts have to be combined.

The Z type is first used to obtain $\frac{t}{h}$ for a vertical wall. Parallel scales are drawn for $\tan^2\left(45-\frac{a}{2}\right)$ and $\left(\frac{t}{h}\right)^2$, the divisions being numbered to give the corresponding values of the angle or $\frac{t}{h}$. As in the previous examples, the diagonal scale points may be found by connecting corresponding values of $\frac{t}{h}$ and a.

The diagram will now give the ratios of width to height for vertical walls, and it remains to find equivalent top and bottom widths for a back batter.

At equal distances from and parallel to the scale for $\frac{t}{h}$, are drawn the scales for $\frac{t_1}{h}$ and the line A. The latter if scale would give $\left(\frac{t}{h}\right)^2-\left(\frac{t_1}{h}\right)^2$ and the unit for both it and scale $\frac{t_1}{h}$ is twice that adopted for scale $\frac{t}{h}$. To multiply the values of $\left(\frac{t}{h}\right)^2-\left(\frac{t_1}{h}\right)^2$ on scale A by r the Z chart $A\,B$ is drawn. The units for scales A and B being the same, the division for $r=1$ is at the middle point of the diagonal and the remaining scale points are readily found by calculation from temporary scales on A and B. The scale reading on B would give $\left(\frac{t_2}{h}\right)^2-\left(\frac{t_1}{h}\right)^2$ as this $=r\left\{\left(\frac{t}{h}\right)^2-\left(\frac{t_1}{h}\right)^2\right\}$.

The value $\left(\frac{t_2}{h}\right)^2-\left(\frac{t_1}{h}\right)^2$ is transferred on to the A line by a line through $r=1{\cdot}0$. The object of this is to use scales already drawn instead of adding new ones. A transversal from he point thus found on the A line to $\frac{t_1}{h}$ as already selected gives he value of $\frac{t_2}{h}$ on the $\frac{t}{h}$ scale.

The saving of labour afforded by the diagram compared with the usual graphical and other methods of calculation for retaining walls, is obvious, while the restrictions of tables made out to sui certain given conditions of angle of repose and weights of

materials are avoided. The effect of face batter is such that the stability of a wall designed for a vertical face by using the nomogram, will be improved by that batter up to the point where the set back of the top of the wall from the vertical is not greater than half the base width, provided of course that the back batter is not so great that no top width is left to the wall. Any normal face batter being thus on the side of safety, and in view of the uncertainty of the actual angle of repose, it is questionable whether there is much advantage in complicating the diagram with additional scales for an exact solution with face batter.

Nomograms for Discharges of Orifices.

The fundamental formula for the discharge of canal outlets is

$$Q = 5\ A\sqrt{h}$$

This contains three variables and could be expressed by either the Z or parallel scale type of chart. The latter in this case has certain advantages and has been adopted, the equation being put into the form

$$\log Q = \log 5\ A + \tfrac{1}{2} \log h.$$

The three main scales for area, head, and discharge, are first constructed in the manner explained for the chart for weir discharges. The scale of area is also marked, to give the positions of standard areas of outlets and circular pipes.

As thus made, the nomogram gives solutions of the equation corresponding to a co-efficient of 0·623 only. To modify the discharge or area to suit other co-efficients, a scale for the latter has been added, together with the parallel lines M and N, usually called supports, on which the transversal for the co-efficients must intersect. The procedure for different co-efficients is explained on the chart. The nomogram now gives solutions of the equations which can be corrected to suit a selected co-efficient.

For irrigation purposes, where outlets vary within limited ranges of size and length, it is more convenient to obtain the discharge or area required by a direct reading, rather than by two steps which may involve trial and error. This result is obtained by adding supplementary scales for each standard outlet. These scales are divided to correspond to the co-efficients for the lengths for which they are marked, and the correction for the position of the outlet on the main scale corresponds to the difference, between the scale length for the co-efficient for the length of outlet adopted and 0·623, for which the main scale

of areas has been drawn. Conversely, if the outlet position on the standard scale of area is projected vertically on to its supplementary scale, it will give the length of that outlet having a co-efficient of 0·623. The unit for the scale for co-efficients used for the supplementary scales is that used for the main scale of areas.

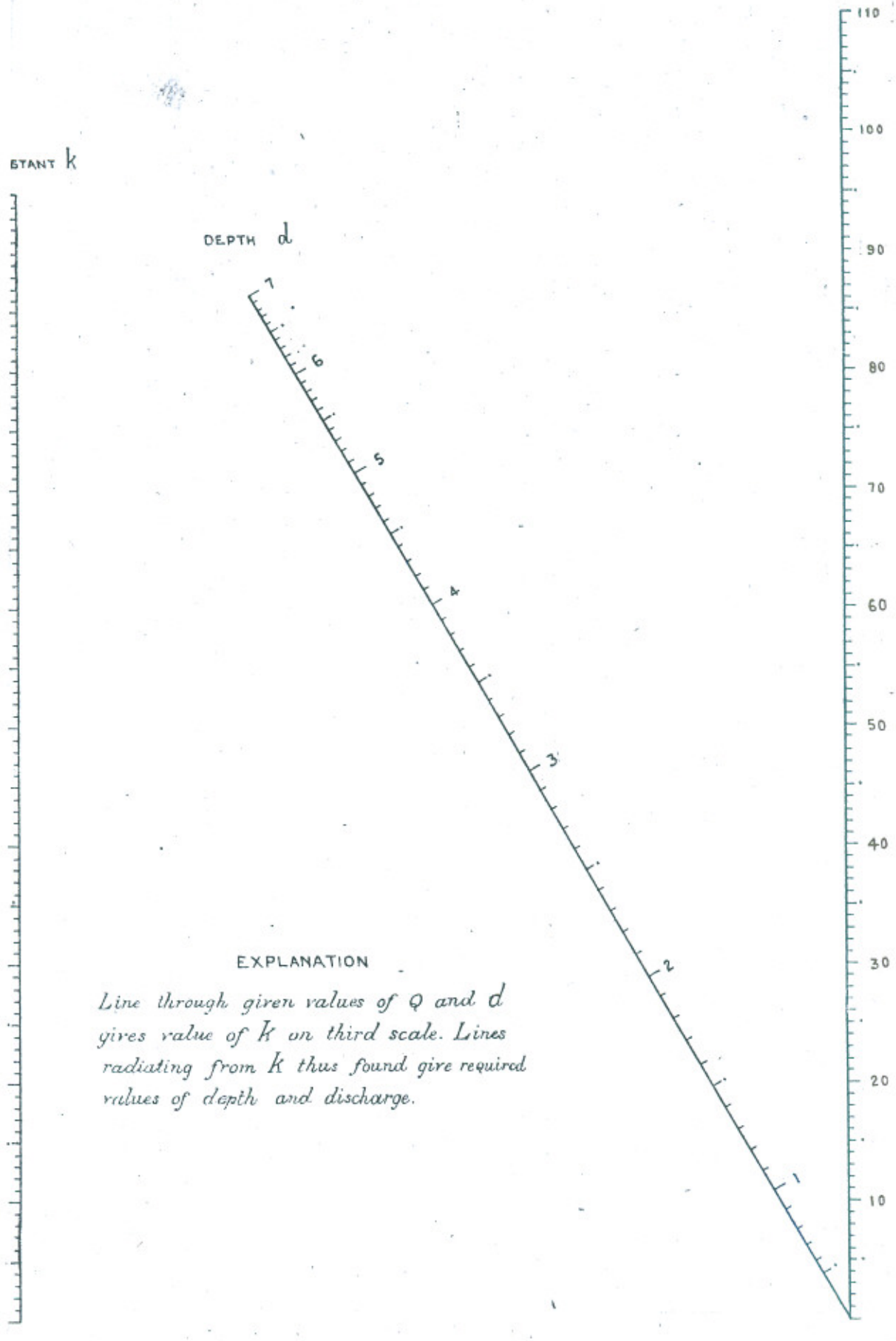
As there is some uncertainty as to co-efficients, a second set for iron pipes have been calculated. Here, instead of using parallel supplementary scales for projection, the corrected positions, to suit certain lengths of pipe, have been indicated directly on the main scale of areas by dotted lines.

The chart, which was specially constructed for canal outlets, using such data as were available, shows the amount of information which can be given by this type of diagram.

These examples may convey sufficient information for the construction of similar nomograms, but those, who wish for further details, should consult " The Construction of Graphical Charts " by J. B. Peddle, while the " Traité de Nomographie" by M. d'Ocagne deals still more fully with the subject which is one which deserves to be more familiarly known than it appears to be at present

DISTRIBUTARY DISCHARGE DIAGRAM

FOR $Q = k\ d^{\frac{5}{3}}$



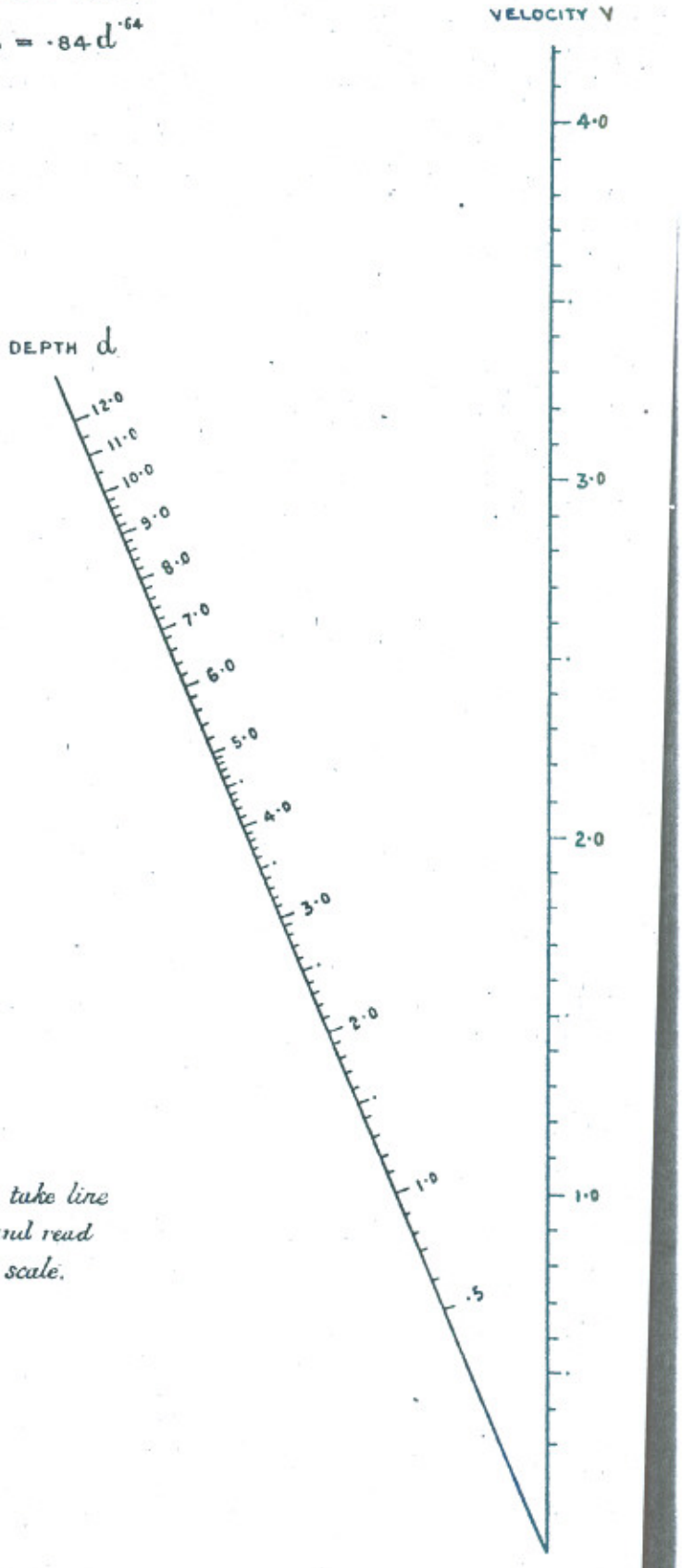
EXPLANATION

Line through given values of Q and d gives value of K on third scale. Lines radiating from K thus found give required values of depth and discharge.

DIAGRAM FOR KENNEDY'S CRITICAL VELOCITY RATIO

$V_o = .84\, d^{.64}$

$\frac{V}{V_o}$



EXPLANATION

To obtain critical velocity ratio take line through velocity and depth and read ratio at intersection on third scale.

WEIR DISCHARGES

$$Q = C \times 5.35\, l \left(d + \frac{e}{2}\right) \sqrt{d - e}$$

Q = discharge in cusecs
d = depth on crest
e = " drowned
f = clear fall
l = length of crest
C = coefficient

CUSECS PER FOOT RUN OF WEIR

120 110 100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4 3 2

CLEAR FALL f

10 9 8 7 6 5 4 3 2 1 ·9 ·8 ·7 ·5 ·4

LENGTH OF CREST

4 3 2 1 ·5

TOTAL DISCHARGE Q CUSECS

200 150 100 90 80 70 60 50 40 30 20 10 5 2

RRECTIONS FOR COEFFICIENT

·9 ·8 ·7 ·6 ·5

PLANATION

or deduct from
discharge per
n as coefficient
ter or less than ·7

PLANATION

connecting $d + \frac{e}{2}$
gives discharge
t run of weir

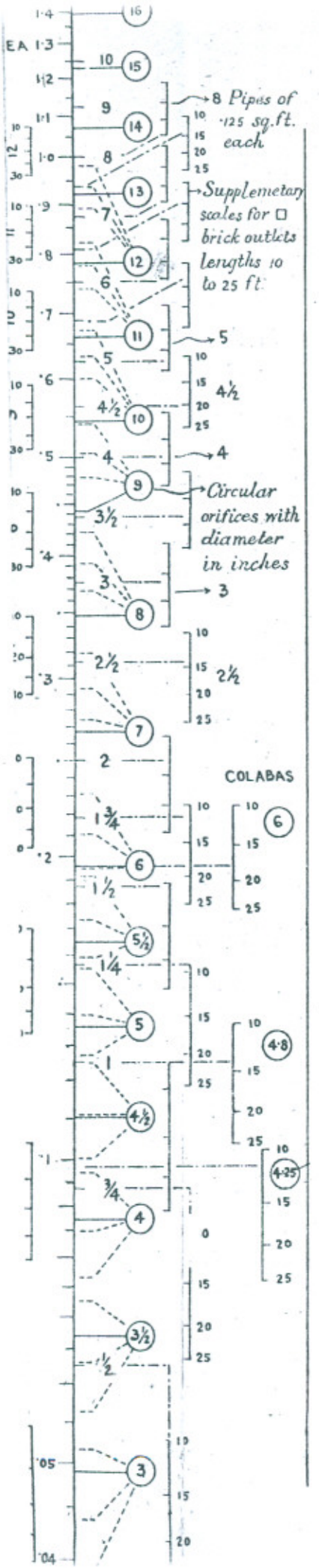
EXPLANATION

Line connecting discharge per foot run and total discharge gives length of crest.

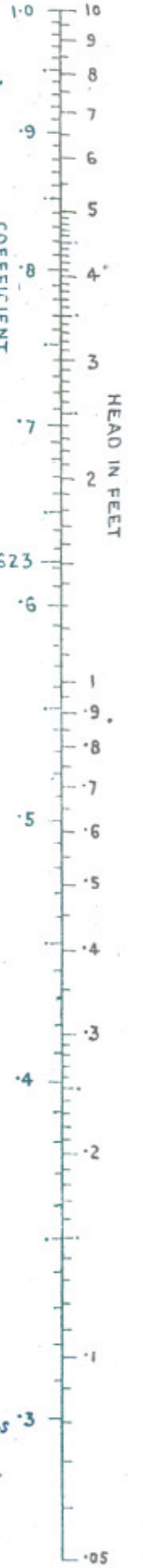
DISCHARGES OF ORIFICES

WITH COEFFICIENT OF ·623 $Q = 5A\sqrt{h}$

AND FOR OTHER COEFFICIENTS & CANAL OUTLETS







EXPLANATION

Straight line across scales of area, discharge and head gives corresponding values in equation

$Q = 5A\sqrt{h}$

Q being discharge in cusecs
A .. area in sq.ft.
h .. head in feet

To correct area obtained to suit a coefficient other than ·623, connect area to ·623 on scale C, then line through intersection with support M and selected coefficient gives area required.

To correct discharge to suit a coefficient other than ·623 connect discharge with selected coefficient on scale C and then line through intersection with support N and ·623 on scale C gives modified discharge

Dotted positions are corrected for direct reading for smooth iron pipes of 9, 18, 27 ft. length taking coefficient for entrance and 3 diameters length ·84 and remaining pipe loss from Haxen-Williams' formula with c = 130

Supplementary scales of lengths projected on main scale give corrected positions of outlets for direct reading to suit lengths with coeff? obtained from Sirhind canal experiments

DISCUSSION.

LALA WAZIR CHAND CHOPRA thought, with reference to Plate No. 3, that a separate nomogram based on the formula $Q = c\,l\,d^{\frac{3}{2}}$ should be used for free falls. The "clear fall" represented by f on the Plate was equivalent to $d—e$ in the case of a drowned weir, but for a free fall this was not correct, as its value was always greater than d, and the f line for free falls might be misleading, although he was aware that equal values taken on the f and $d + \frac{e}{2}$ scales would meet the case.

He thought the information contained on the nomogram for outlets was excessive and likely to lead to confusion. It would be better to split it up. The nomogram for Bazin's formula was given in d'Ocagne's book quoted by Mr. Padday, though this had unfortunately not been translated into English. It had however been reproduced in "Engineering News" in 1909. Kutter's formula was in more frequent use, and he would like to know whether a similar solution was possible in this case, Kennedy's and Garrett's hydraulic diagrams met the case to some extent, but they were primarily intended for canals, and it was often necessary to use Kutter in calculations for drainage diversions when a nomogram would be useful.

MR. LINDLEY said that the subject was well-worth bringing to the notice of the Congress, as many formulæ, in frequent use, could be reduced to a diagram almost as quickly as a few calculations could be made, while the diagram could be reproduced and was so simple that even a clerk could use it. The essential forms of the diagram might however have been made clearer, and the geometry more fully given, as the latter led to simple formulæ by which the diagram could often be more readily finished than by the graphic method.

One form of diagram consisted of scales marked on three parallel straight lines, and in the first part of an appendix, which he had prepared, were worked out the formulæ giving the relative spacing of the lines, the scales for the variables, and the position of the scales to make a diagram representing the equation

$$z = ax + by + c$$

instead of linear scales of x, y and z, logarithms of these variables could be plotted, when the diagram would represent the equation

$$z = c \times x^a \times y^b$$

a form which was more generally useful, but had the disadvantage that equations containing addition or subtraction terms could not be represented. The diagram for weir discharges was of this form. The formulæ shown in the appendix gave the spacing and scale of discharge by mental calculation, which was simpler than the graphic methed : $n = \frac{1}{3}$ and $S_z = \frac{2}{3}$.

The other form of diagram consisted of two parallel lines and one transversal crossing them in Z form, which was suitable for an equation of the following form

$$(x + a) = c \times (y + b)$$

Mr. Padday had used this for equations in which a and b were zero and c a regular variable, so that the principal advantage of this form over the parallel-line form with logarithmic scales was lost, and the scale on the transversal had to be marked off from a set of solutions either taken from a published table or calculated for the purpose, whereas if these diagrams had been made in the parallel-line form, with logarithmic scales, no calculations would have been needed except those for n and for S_z.

In the second part of the appendix the geometry of the Z form was given, showing that it could be used directly with linear scales to solve Bazin's formula for C in the hydraulic formula

$$V = C\sqrt{RS.}$$

Comparing this geometrical formula with the Bazin formula

$$C = \frac{157{\cdot}6}{1 + \frac{\gamma}{\sqrt{R}}}$$

it was obvious that if AB were graduated for R, AE for C and made 157·6 long, and ED for γ, a plain nomogram represented the equation, while the same diagram could also be used for the solution of the Kutter formula, though that called for the use of curves : and the form was described by various authorities, and would be found in Molesworth's Pocket Book.

MR. PADDAY in reply said with reference to the point raised by Mr. Chopra that, in actual use, a single nomogram for weir discharges under all conditions was sufficient. The nomogram for a drowned weir must include the condition when the amount of drowning was practically or actually nil, and the

user of the diagram presumably would have sufficient knowledge of the terms governing the weir discharge, to know that the maximum limit of clear fall was the depth on the crest.

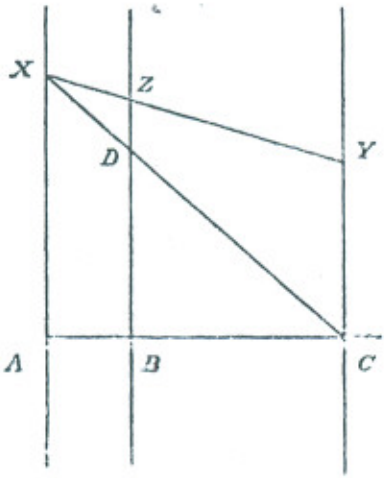
The nomogram for outlets was based on the requirements of actual practice, the constants had been obtained from such information as was available, and in view of its present rather tentative nature, it was useful for comparative purposes to have the results shown on one diagram. Where more than one type of outlet was in use, this also allowed for the selection of one or other type to meet special conditions more easily, than if each type had its own nomogram.

Messrs. Ganguillet and Kutter, when they brought out their formula, also published a nomogram for the value of the co-efficient *C* of velocity. This nomogram was subsequently extended to give the velocity, and the diagram with an explanation had been reproduced in recent editions of Molesworth's Pocket Book. Mr. Lindley had prepared the diagram for use with a scale for comparison with Bazin's constants, and would no doubt give Mr. Chopra a copy if he desired.

The further details on the calculation for and the construction of nomograms, given by Mr. Lindley, were useful, but the theory of the subject and its possible applications were beyond the limits of a Congress paper, and reference should be made to authors, like Peddle and d'Ocagne, who had dealt fully with the subject. In certain cases mental calculation was simple and a useful check on the graphical method, but where the units of scales were varied to suit the range and degree of accuracy required, the graphical method, in practice, was usually the easier, and it should certainly be applied in every case as a check on calculations, while the reverse process was not essential.

The paper was originally put up as a note on five nomograms, representing equations generally useful, and there were many other variations besides those given by Mr. Lindley. In the examples given the question of the parallel or Z form had been decided by convenience in actual use, one factor being that for departmental purposes the error of reading should be limited to the unit, for which the Z chart was usually preferable, rather than to a percentage, as given by parallel logarithmic scales.

APPENDIX.



Take three parallel straight lines AX, BZ and CY, crossed by a base line ABC, and a transversal XZY. Join XC, cutting BZ at D

Then $BZ = BD + DZ$

$$= AX\,\frac{BC}{AC} + CY\,\frac{AB}{AC}$$

and writing $\frac{AB}{AC} = n$

$$= AX\,(1-n) + CY\,n$$

Now, on AX and CY mark off equal scales of x and y, and on BZ mark a scale of z of unit length S_z times that of the scales of x and y then

$$z \times S_z = (1-n)\,x + ny$$

So, to represent the equation $S_z = ax + by$ it was necessary to make

$$\frac{n}{S_z} = b \qquad \text{and } (1-n)\,S_z =$$

$$\text{that is } n = \frac{b}{a+b} \quad \text{and } S_z = \frac{1}{a+b}$$

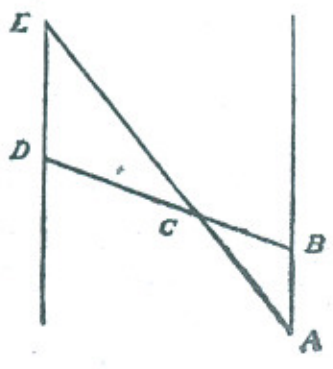
These were the fundamental formulæ giving the values of n and S_z necessary to make a nomogram to fit the above equation with co-efficients a and b

If, for any reason, it was required to use different scales for x and y, then, if the scale of y were of unit length S_y times that of x, $\frac{b}{S_y}$ would have to be substituted for b in the above equations.

To make the nomogram represent the equation

$$z = ax + by + c$$

all that was necessary was to move the scale of z till it gave a reading c on the line connecting the zero points of x and y



$$\frac{AC}{AE} = \frac{AB}{AB+DE}$$

$$AC = \frac{AE}{1+\frac{DE}{AB}}$$