EXPERIMENTS ON BROAD CRESTED WEIRS.

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Circumstances leading to the experiments—Method of conducting the experiments—Demonstration that a broad crested weir under certain conditions creates no afflux—Explanation and formulæ deduced—Standing waves—Appendices.

Circumstances leading to the Experiments.

Some years ago, when the writer was calculating the afflux over the Abazai weir, Mr. W. B. Harvey suggested to him that in a big flood the weir would probably cause no afflux. At the time the writer did not attach very much importance to the suggestion, but a year later he was provided with a visual corroboration of Mr. Harvey's theory, when, during the construction of the weir, it was necessary to make a ring bund round the first half, strong enough to resist the highest recorded winter freshet. The river at Abazai is very narrow, and the building of a ring bund round half the weir caused great constriction in the river, the effect being much the same as that of a weir, only that the constriction in area was at the side instead of in the bed. The bed of the river consisted of boulders and shingle which could not be scoured out except with very high velocities. With the ordinary low winter supply in the river there was considerable afflux; *i. e.*, the water against the upstream part of the ring bund was considerably higher than that downstream, but when large freshets occurred, it was found that there was no appreciable afflux, the difference in the water levels against the upstream and downstream sides of the bund being approximately that caused by the natural slope in the river. Eventually a flood about twice as big as the maximum recorded for the winter season was safely passed, though calculations based on any of the accepted formulæ shewed that the afflux should have been so great as to have overtopped the bund. On numerous occasions since then, the writer has noticed, in common no doubt with most hydraulic engineers, how a smooth obstruction in a stream (whether in the bed or in the sides) causes a smooth depression in the water surface at the obstruction; the water surface downstream of the obstruction rising smoothly till it reaches the same level (neglecting the friction slope) as it had upstream.

About Christmas 1917 the writer was told by Mr. Ora that it was suggested to increase the height of the weir in t design of the headworks for the new left bank Indus canal Kalabagh, but that in his opinion the increased afflux level caus thereby would flood out Kalabagh City, and he expressed gr surprise when both Mr. Harvey and the writer stated that in th opinion there would probably be no afflux at all in a big flo when the depth of water was great. Mr. Oram was so interes that he asked the writer to make some rough experiments prove the correctness or otherwise of his opinion that br crested weirs cause no afflux, beyond the slight loss of head due extra friction, when the depth of water was greater than a cer value. The writer accordingly carried out some experime which have confirmed his previous opinion, while incident he was able to show that, with relatively small depths of wa where an afflux occurred and there was a standing wave do stream of the weir, variations in the downstream level had effect on conditions at the weir crest or upstream of it, until water level downstream of the standing wave nearly approac the level of the water surface upstream of the weir, or, in ot words, that the weir behaved as if it had a free fall until downstream level nearly approached the level of the w surface upstream of the weir. He also found that it was poss to calculate with considerable nicety the water surface cu down the glacis and the position of the standing wave, w conditions were such as to produce one.

Method of conducting the Experiments.

Close to Mardan, a minor, having a well fall about twenty feet from the exit from the head culvert, takes out of a d butary with a considerable working head. The earthen cha from the head to the fall was converted into a rectangular flume two-and-a-half feet wide, and a six inch high weir was in the bed with the upstream edge of its crest about seven below the culvert. This small weir was made the full of the flume, and was proportioned so as to be a model o usual type of river weir, being given an upstream slo 1 in 5, a crest width of one foot, and a downstream sl 1 in15. At the lip of the well fall, which was about eighte below the downstream edge of the weir crest, grooves provided for the insertion of small karris, by means of the depth of water downstream of the weir could be regu The arrangement is shewn on the plan attached. The one side wall was cement plastered and given a sharp

edge, the level of which was recorded at one foot intervals. The water surface level was measured, with a decimally divided vulcanite scale, from the top edge of this side wall. The advantage of the great working head into the minor was that small fluctuations in its water level above the weir would have a nearly negligible effect on the discharge for a given gate opening.

Below the fall the minor was cut to a regular shape for a length of 120 feet, and a bund put in at the end of this length. A small quantity of water was then admitted into the tank thus formed, so as to give a perfectly level floor, and measurements were made of the tank. The head regulator gate of the minor was then opened suddenly, and the depth of water at the downstream edge of the crest measured at regular intervals. The time taken to fill the tank to various depths was observed, and the discharge in cusecs was thus obtained.

It was found that for all depths measured at the downstream edge of the crest of the weir, the discharge of the weir could be represented by

$$q = y^{\frac{3}{2}} \sqrt{g}$$

where q was the discharge in cusecs per foot run of the weir and y the depth at the downstream edge of the crest. The differences between the observed discharges and those calculated from this formula were only about one or two per cent., which is as near as a gauge can be read.

Demonstration that when the depth of water exceeds a certain value, a broad crested weir causes no afflux, and that when the depth of water is less than this value, changes in the downstream water level, within limits, have no effect on conditions at the weir crest.

A fixed discharge was admitted into the flume and allowed to flow away freely, karris were then inserted in the grooves at the lower end of the flume, and the downstream water level was thus raised so as to cause the formation of a standing wave below the glacis of the weir. Karris were thereafter gradually inserted, causing a gradual increase in the downstream water level, which corresponded to decreasing gradually the surface slope of the miniature river, and thus increasing the depth of water for a fixed discharge. By means of the approximately level courses in the masonry side walls of the flume it became at once apparent visually, without making any measurements, that two facts were at least approximately, if not absolutely, true. In the first place it was seen that the level of the water below the

standing wave had no effect on conditions at the weir crest, upstream of it, until the downstream level was nearly the same the upstream. As the downstream level was raised, the standi wave gradually crept up the glacis, yet the depth of water the weir crest, and the level of the water upstream of the wei remained the same as when the weir was discharging free. I the second place it was seen that when the raising of th downstream level caused the standing wave to reach the cres of the weir, the wave disappeared, giving place to smooth undulations, and that the upstream and downstream water levels were then the same, or, in other words, that the weir caused no afflux when the depth of water in the stream reached a certain value. As the depth was still further increased, though the constant discharge was maintained, the undulations disappeared, leaving a single smooth depression in the water surface over the crest of the weir, while the upstream and downstream water levels still remained the same.

Fig. (1) shews the water surfaces with a constant discharge of 3·54 cusecs per foot width of weir, (*a*) with the water flowing away freely below the glacis, (*b*) with the downstream depth increased so as to cause a standing wave just clear of the glacis, and (*c*) with the downstream level at a height above the weir crest of nearly seven-eighths of that of the upstream level. The small circles shew points actually observed. It will be seen from this figure that, within these limits, the downstream level of the water has nothing to do with the conditions at the weir crest, and that even the water surface curve down the glacis is only affected in the near vicinity of the standing wave. (The dotted curves refer to another subject and will be explained later).

Fig. (2) shews three water surfaces for a discharge of 4·62 cusecs per foot run of weir, with three different depths of water downstream (representing the natural depth in the stream when unobstructed by the weir, while the difference between the downstream and upstream water levels gives the afflux caused by the weir in each case). In the case of the lowest depth there is a standing wave and a difference of about 0·05 foot between the upstream and downstream levels. With the middle depth, the writer was able to get the exact point where the standing wave changed to smooth undulations, and, it will be noticed that water levels are the same above and below the weir, or, in other words, that there is no afflux. The top curve shews

the water surface when the depth was still further increased. Here the undulations have disappeared, leaving a smooth depression over the weir crest, while again there is no afflux.

Fig. (3) shews, for a discharge of 2·60 cusecs per foot run of weir, the observed water levels for increases in the downstream depth and the stage above which there is no afflux. It will be noticed that with a depth downstream of 0·88 ft. above the weir crest (or a total depth of 1·38 ft.) the upstream and downstream water levels are the same, while the smooth depression in the water surface over the crest will again be noticed in this case.

Fig. (4) shews two conditions for the very small discharge of 0·49 cusec per foot run of weir. In the lower curve the weir is discharging as with a free fall (the peculiar double wave downstream will be referred to later), and in the upper curve the water surface is shewn when the natural (*i.e.* the downstream) depth in the stream is increased to the value at which, and above which, no afflux occurs.

Numerous other experiments were made, but those given are sufficient.

An essential point to notice in the experiments is, that so long as the standing wave is well clear of the downstream edge of the crest, the depth of water at the latter point is always exactly

$$\frac{2}{3}\left(H+\frac{u^2}{2g}\right)$$

where H is the difference in levels between the weir crest and the upstream water level, and u is the velocity of approach. For instance, in Fig. (1) the discharge per foot width is 3·54 cusecs, and the total upstream depth is 1·52 ft., therefore the velocity of approach is $\frac{3\cdot54}{1\cdot52}$ or 2·31 ft. per sec. H is 1·02, therefore $\frac{2}{3}\left(H+\frac{u^2}{2g}\right)$ is 0·73 ft. which is seen to be the depth at the downstream edge of the crest.

Unwin refers to this, and the proof of this "two-thirds" value is given in Appendix I. As shewn in the same appendix, it follows that the velocity is $\sqrt{gy}$ at the downstream edge of the crest of the weir, where y is the depth, and that, if q be the discharge per foot run, $q=y^{\frac{3}{2}}\sqrt{g}$, as found when calibrating the weir by means of the measuring tank.

Explanation and Formulæ Deduced.

The experiments shew that all the energy generated in increasing the velocity of a stream over a weir (or through a contracted area) is not entirely destroyed, but that a great part and in some cases all of it, is recovered. They also shew that where there is an occurrence of foam, as in the ordinary abrupt standing wave, there will be a considerable loss of energy, but that where the water, downstream of the obstruction, rises in a smooth curve without forming any foam, there will be no appreciable loss of energy (beyond that due to increased friction owing to the higher velocity past the obstruction) and the water surface downstream will then be the same (neglecting friction head) as that upstream; or, in other words, there will be no afflux.

A very important point, which may here be mentioned in parenthesis, is that if the exit from siphons, bridges, culverts, etc., as well as from weirs, be made gently diverging, almost any degree of recovery of head can be obtained. On the Kabul River Canal a reinforced concrete siphon with a fairly blunt bell-mouthed entrance and a carefully designed diverging exit was constructed a year ago. Now the accepted formulæ give the "entrance and velocity" head as varying from $1{\cdot}08\frac{v^2-u^2}{2g}$ for bell-mouthed, to $1{\cdot}5\ \frac{v^2-u^2}{2g}$ for sharp entrances (the formulæ say nothing about exits), where v is the velocity through the siphon and u the velocity of approach. The head through the siphon was calculated accordingly for $1{\cdot}15\frac{v^2-u^2}{2g}$, and in consequence the estimate had to provide many thousands of rupees for raising the canal banks for five miles upstream of the siphon. Since construction, and with a supply slightly in excess of that designed for, it has been found, by measuring the total head through the siphon and allowing the smallest possible value for the friction head given by any formula, and deducting this from the total head, that the "entry and velocity" head is only about $0{\cdot}08\ \frac{v^2-u^2}{2g}$ instead of $1{\cdot}08\frac{v^2-u^2}{2g}$. At the exit from the siphon there is scarcely a ripple, and as the velocity decreases between the gently diverging wing walls, the curious spectacle can be seen, along the courses of the masonry, of the water surface rising gradually downstream as the velocity decreases till it is equal to the ordinary canal velocity.

The following three laws can be deduced from these experiments:—

(1) In the case of a broad crested rectangular weir, so long as the standing wave is well clear of the vertical plane of the downstream edge of the crest, the depth at the latter point will be $\frac{2}{3}(H+\frac{u^2}{2g})$; which is, (*vide* Appendix I,) the depth giving the maximum discharge over the weir for a given water level upstream.

(2) After depression over the weir crest, the water surface can nearly, though not quite, rise in a smooth curve to the same level as that upstream when the depth at the downstream edge of the crest is $\frac{2}{3}(H+\frac{u^2}{2g})$.

(3) If calculations shew that a stream, with a water surface level the same as it had in its natural condition before the construction of a weir, can pass its discharge over the weir with a depth at the downstream edge of the crest slightly in excess of the value $\frac{2}{3}(H+\frac{u^2}{2g})$, which is the depth giving the maximum discharge for the given water level in the stream, then the weir will cause no afflux.

According to the writer's experiments, the critical depth y at the downstream edge of the crest, when the condition of no afflux begins, is given by the equation

$$y = c \times \frac{2}{3}\left(H+\frac{u^2}{2g}\right) \quad \ldots\ldots\ldots \text{Equation (1)}$$

and, except for very low velocities, c appears to be a constant having a value of 1·06. With very low velocities, as in Fig. (4), c appeared to have a higher value, approaching 1·20 for a crest velocity of 1½ feet per second.

In order to observe the effect of piers, the writer put in a wooden pier ten inches wide at the crest of the weir ; that is to say, the width of the pier was equal to the width of the opening on each side. Here the afflux practically disappeared, *vide* Fig. (5), when c was 1·19. In this case, owing to the increased friction, there was always a very slight difference between upstream and downstream water levels, which must not however

be confused with afflux due to increasing velocity. This was a very exaggerated case, such as would never occur in practice, and it may be taken that $c = 1{\cdot}1$ would cover all practical cases. From the equation, $y = c \times \frac{2}{3}\left(H + \frac{u^2}{2g}\right)$, for the depth of water at the downstream edge of the crest when the stage of no afflux begins, a simple equation is obtained (*vide* Appendix II) for the maximum height of weir which will cause no afflux for a given depth and velocity in a stream.

If x = the maximum height of weir above bed which will give no afflux,

D = the natural depth of water in the river for the given discharge Q, prior to the construction of the weir, measured from the same point in the bed from which x is measured,

u = the velocity of approach at the same point,

q_1 = the discharge per foot run over the weir, and

k = a coefficient,

then this equation is

$$x = D + \frac{u^2}{2g} - kq_1^{\frac{2}{3}} \quad \ldots\ldots\ldots \text{Equation (2)}$$

In this equation

$$k = \frac{1}{\sqrt[3]{\frac{8g}{9} \cdot c^2 \left(1 - \frac{2}{3}c\right)}} = \frac{0.3269}{\sqrt[3]{c^2 \left(1 - \frac{2}{3}c\right)}}$$

If c be given the value, observed by the writer, of $1{\cdot}06$, then k becomes $\cdot 473$. For $c = 1{\cdot}1$, $k = \cdot 476$. For $c = 1{\cdot}2$, $k = \cdot 495$.

It is shewn in Appendix III that it is immaterial (in cases where the bed slope and surface slope of the river are not parallel) where x is measured from, so long as D and u are measured at the same place.

A bar such as that which is always found at the point where a river debouches from a rocky gorge forms a natural weir. With small discharges, when the height of the bar, relatively to the depth of water, is greater than the value for x in equation (2), there will be a rapid on the downstream glacis of the bar and a considerable difference between the upstream and downstream water levels. With a big flood, however, x will probably have a value less than that given by equation (2), the rapid will disappear, and it will usually be possible to increase the height of the bar by the construction of a masonry weir thereon without causing any afflux.

If the height of a weir is greater than the value for x in equation (2) then afflux will occur, and the height of the afflux level above the weir crest is given, *vide* Appendix IV, by the equation

$$H^3 + H^2\left\{2x - \frac{3}{2}\cdot\frac{q^2}{g^{\frac{1}{3}}}\right\} + Hx\left\{x - 3\frac{q^2}{g^{\frac{1}{3}}}\right\} - \frac{2}{3}x^2\cdot\frac{q^2}{g^{\frac{1}{3}}} + \frac{q^2}{2g} = 0 \quad \ldots\ldots(3)$$

where x is the height of the weir and q_1 and q are the discharges per foot run over the weir and in the river upstream respectively. This equation is of the form

$$H^3 + A\,H^2 + B\,H + C = 0$$

of which the three roots are

$$-\frac{A}{3} + \frac{2}{3}(+\sqrt{A^2 - 3B})\begin{cases}\cos a \\ \cos(120^\circ - a) \\ \cos(120^\circ + a)\end{cases}$$

where $a = \frac{\beta}{3}$, and $\cos\beta = \dfrac{9AB - 2A^3 - 27\,C}{2(+\sqrt{A^2 - 3B})^3}$

The ordinary weir is complicated somewhat by the existence of undersluices. To find if there will be any afflux, it is necessary to calculate, with the ordinary gradient of the river, the discharge through the undersluice flume channel (remembering that there will be no appreciable loss of head through the undersluice openings if the piers are well shaped—especially at the downstream end—or if the width of the pier is small compared with that of the openings, to deduct this discharge from the discharge of the river (thus obtaining q_1), and then to find whether x is greater or less than the value given in equation (2). If x be greater than this value, there will be afflux, and both the weir and the undersluice openings will discharge as broad crested weirs with free overfalls; the depths of water at the downstream ends of the undersluice openings and at the downstream edge of the crest of the weir will have their respective "two-thirds" values, and the afflux level will begiven, *vide* Appendix V, by the equation

$$l_1\left(H - \frac{u^2}{2g}\right)^{\frac{3}{2}} + l_2\left(H + x + \frac{u^2}{2g}\right)^{\frac{3}{2}} = \frac{1.837\,Q.}{g} \quad \ldots\ldots(4)$$

where Q is the discharge of the river, l_1 and l_2 are the widths of the weir and the undersluice openings respectively, x is the height of the weir, H is the height of the afflux level

above the weir crest, and u is the velocity of approach. *Vide* Fig. (12).

Standing Waves.

If the velocity of a stream be greater than $\sqrt{gd}$ in some upstream part of its course (as on the glacis of a weir below the downstream edge of the crest, where the velocity is exactly $\sqrt{gd}$), and less than $\sqrt{gd}$ in some downstream part of its course (as in the natural stream below the weir), then a standing wave will occur at some intermediate point, and the depth at this point can be calculated with considerable accuracy. In the case of a horizontal bed, this is given by a simple equation, but, as explained later, when the standing wave occurs on a glacis, the equation for the depth becomes a cubic, and it is necessary to plot both the water surface curve down the glacis and also the graph of the depth, when the point where the two curves intersect will give the position of the standing wave. Unwin's formula will give the position of a standing wave correctly if it occurs below the glacis, but, when comparing observed positions for standing waves on a glacis, the writer found that Unwin's formula would not work as it stood, and he is indebted to Mr. W. B. Harvey for the suggestion of the method for altering the formula to make it suit other conditions.

When the floor is horizontal, the force destroying momentum is correctly, as taken by Unwin, the difference in pressures on a vertical plane downstream and on a vertical plane upstream of the standing wave. When the wave occurs on a glacis, however, the force dissipating momentum is less than the difference of the pressure on these two planes of water, as the pressure on the lower part of the downstream plane is resisted by the portion of the masonry glacis below the bottom point of the upstream plane. In Fig (13) the force destroying momentum is not the difference between the pressures on the planes CE and AB, because the pressure on DE is resisted by the portion BF of the glacis.

As shewn in Appendix VI, the equation for H_o is

$$H_o^3 - H_o \left\{ (H_1 - a)^2 + \frac{2q^2}{gH} \right\} + \frac{2q^2}{g} = 0 \quad \ldots\ldots(5)$$

where H_1 is the depth downstream of the standing wave, H_o is the depth upstream at the point where the standing wave will occur, q is the discharge per foot width, and a is the height

from the floor where H_1 is measured to the point on the glacis where H_0 is measured.

By taking, say, three or four values of a, a graph of H_0 can be plotted, and the point at which this graph cuts the water surface curve on the glacis is that where the standing wave will occur. This standing wave is not a vertical wall, its shape being somewhat as shewn in Fig. (7); and the calculated position of the wave will be the crest of the wave where the water surface becomes nearly horizontal.

Calculated positions for a standing wave are shewn in Figs. (1) and (3), and these can be compared with the observed positions. In these diagrams the horizontal scale is one-tenth of the vertical while each large square measures two feet horizontally and 0·2 foot vertically.

To find the position of the standing wave it is first necessary to plot the water surface curve down the glacis. The easiest method of doing this is probably to start with the known depth $y = \sqrt[3]{\frac{q^2}{g}}$, and known velocity $v = \sqrt{gy}$, at the downstream edge of the crest, to take a depth (with its corresponding velocity) slightly less than this value, and then find at what distance from the crest this second depth occurs. Having found this second point, the process is repeated to find at what distance from it another slightly less depth occurs. As shewn in Appendix VII, if l be the distance apart of any two points where the depths and velocities are y_0 v_0 and y_1 v_1; S the slope of the glacis; v and R the velocity and hydraulic radius for the mean depth (*i. e.* $\frac{y_0 + y_1}{2}$); and if s be the surface slope necessary to overcome friction (*Vide* Fig. 14), then

$$l = \frac{\frac{v_1^2 - v_0^2}{2g} - (y_0 - y_1)}{S - s}$$

The writer has found, in the case of his experiments, that by using Manning's formula for S, a water surface curve can be plotted which will be practically indistinguishable from the

observed curve. Examples are worked out in Appendix VII, and the results are shewn on Figs. (1) and (3).

It sometimes happens that instead of an ordinary standing wave, a smooth wave, of height $\frac{u^2 - v^2}{2g}$ above the water surface immediately upstream, will occur, and this smooth wave will be followed by a standing wave. This phenomenon is shewn in Figs. (4) and (6). The conditions necessary for its occurrence are very difficult to obtain, and the writer has not been able to satisfy himself conclusively as to its cause. He has reasons for suggesting that it may occur when the graph of H_0 (mentioned above) is identical, or nearly so, with the water surface curve for some little distance, or when the former cuts the latter at two points, thus giving more than one solution for the position of the standing wave. With his experimental weir the writer obtained this phenomenon when the depth of water at the downstream edge of the crest was 0·20 foot and the depth downstream of the glacis varied from 0·60 to 0·665 foot. The crest of this smooth wave occurred from 3·4 to 4·4 feet below the downstream edge of the crest.

A beautiful example of this phenomenon was observed on the Abazai weir when a small depth of water was flowing over it, but unfortunately no observations could be made of it on the spur of the moment. Mr. Colyer has told the writer that he saw a similar phenomenon at the Merala weir when there was a flood in the river.

APPENDIX I.

Proof that the depth at the downstream edge of the crest of a weir (or most contracted section) is two-thirds of the 'still-pond' depth upstream measured above the crest of the weir.

The assumption is usually made that for any given surface level upstream, nature will adopt a depth at the contracted section which will give the maximum discharge.

It is better, perhaps, to assume that for a given discharge in a channel the depth at the determining section will be such that $H + \frac{u^2}{2g}$ in the channel upstream will be minimum, or that the height K, (which is equal to $H + \frac{u^2}{2g}$), above weir crest to water surface in a still pond will be the least possible.

In Fig. (7) let H be the height of the water surface in the approach channel above the crest of the weir and let u be the velocity of approach. Let $K = H + \frac{u^2}{2g}$, *i. e.*, let K be the height above the weir crest of the water surface in an imaginary still pond from which the constant discharge Q is flowing through the channel, friction being considered absent. Let y and v be the depth and velocity at the downstream edge of the crest, and lets and q_1 be the discharges per foot width of the river and the weir respectively

Then

$$\frac{v^2 - u^2}{2g} = H - y \qquad \therefore \; v^2 = 2g\left\{H - y + \frac{u^2}{2g}\right\}$$

$$\therefore \; v = \sqrt{2g\left\{H + \frac{u^2}{2g} - y\right\}}$$

now $q_1 = vy$

$$\therefore \; q_1 = y\sqrt{2g\left\{H + \frac{u^2}{2g} - y\right\}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If we adopt the first assumption we get

$$\frac{dq}{dy}^1 = y \times \tfrac{1}{2} \times \frac{-2g}{\sqrt{2g\left(H + \frac{u^2}{2g} - y\right)}} + \sqrt{2g\left(H + \frac{u^2}{2g} - y\right)}$$

and q_1 is maximum when $\frac{dq_1}{dy} = 0$, or when

$$gy = 2g\left(H + \frac{u^2}{2g} - y\right) \text{ , or when}$$

$$y = \frac{2}{3}\left(H + \frac{u^2}{2g}\right)$$

If we adopt the second assumption, we get from equation (1), putting $H+\frac{u^2}{2g}=K$,

$$y^3-Ky^2+\frac{q_1^2}{2g}=0$$

$$\therefore K=y+\frac{q_1^2}{2gy^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\therefore \frac{dK}{dy}=1-\frac{q_1^2}{gy^3}$$

K is minimum when $\frac{dK}{dy}=0$, or when

$$y=\sqrt[3]{\frac{q_1^2}{g}}$$

bstituting in equation (2)

$$K=\sqrt[3]{\frac{q_1^2}{g}}+\frac{q_1^2}{2g\left(\frac{q_1^2}{g}\right)^{2/3}}$$

$$\therefore K=\frac{3}{2}\sqrt[3]{\frac{q_1^2}{g}}$$

$$\therefore K=\frac{3}{2}y \qquad \therefore y=\frac{2}{3}K, \text{ or}$$

$$y=\frac{2}{3}\left(H+\frac{u^2}{2g}\right)$$

NOTE.—The velocity at the downstream edge of the crest is given by

$$v=\sqrt{2g\left(H+\frac{u^2}{2g}-y\right)}$$

But $H+\frac{u^2}{2g}=\frac{3}{2}y \qquad \therefore v=\sqrt{gy}$

And the discharge per foot width of the weir, q_1, is given by

$$q_1=y^{3/2}\sqrt{g}$$

APPENDIX II.

Maximum height of weir which will give no afflux.

In Fig. (8), let y be the depth over the downstream edge of the crest of the weir; let D and u be the depth and velocity in the stream above the weir; let x be the height of the weir above river bed; let q and q_1 be the discharges per foot width across the river and the weir respectively; let H be the height above the weir crest of the upstream surface level; let c be the coefficient which, when multiplied by $\frac{2}{3}\left(H+\frac{u^2}{2g}\right)$ will give a depth for y when the stage of no afflux begins; and let v be the velocity at the downstream edge of the weir crest.

Then $q_1 = vy = v \times c \times \frac{2}{3}\left(H+\frac{u^2}{g^2}\right)$

Now $\frac{v^2 - u^2}{2g} = H - y = H - c \times \frac{2}{3}\left(H+\frac{u^2}{2g}\right)$

$\therefore \quad \frac{v^2}{2g} = H + \frac{u^2}{2g} - \frac{2}{3}c\left(H+\frac{u^2}{2g}\right) = \left(1-\frac{2}{3}c\right)\left(H+\frac{u^2}{2g}\right)$

$\therefore \quad v = \sqrt{2g\left(1-\frac{2}{3}c\right)\left(H+\frac{u^2}{2g}\right)}$

$\therefore \quad q_1 = \left(H+\frac{u^2}{2g}\right)^{\frac{3}{2}} \times \frac{2}{3}\, c\sqrt{2g\,(1-\tfrac{2}{3}c)}$

$\therefore \quad \left(H+\frac{u^2}{2g}\right)^{\frac{3}{2}} = \frac{2}{3}\,\frac{q_1}{c\sqrt{2g\,(1-\frac{2}{3}c)}}$

$\therefore \quad H = \sqrt[3]{\frac{q_1^2}{\frac{8g}{9}c^2\left(1-\frac{2}{3}c\right)}} - \frac{u^2}{2g}$ But $H = D - x$

$\therefore \quad x = D + \frac{u^2}{2g} - \sqrt[3]{\frac{q_1^{\frac{2}{3}}}{\frac{8g}{9}c^2\left(1-c\right)}} = D + \frac{u^2}{2g} - kq_1^{\frac{2}{3}}$

Where $k = \frac{1}{\sqrt{\frac{8g}{9}c^2\left(1-\frac{2}{3}c\right)}} = \frac{\cdot 3269}{\sqrt[3]{c^2\left(1-\frac{2}{3}c\right)}}$

This equation will also give the maximum amount of contraction which will give no afflux, as in undersluice openings. Here x may be put equal to o, when the maximum value of q_1 which will cause no afflux is got from:—

$$q_1 = \left(\frac{D+\frac{u^2}{2g}}{k}\right)^{\frac{3}{2}}$$

APPENDIX III.

It is immaterial where x, in Appendix II, is measured from

In Fig. (9) let D_1 u_1 be the depth in and the velocity of the stre of the at the point where x_1 is measured from, and let D_2 u_2 be the depth an velocity at a point where x_2 is measured from. Then, according t Appendix II.

$$x_1 = D_1 + \frac{u_1^2}{2g} - k\, q_1^{\frac{2}{3}} \quad \ldots\ldots(1)$$

$$\& \quad x_2 = D_2 + \frac{u_2^2}{2g} - k\, q_1^{\frac{2}{3}} \quad \ldots\ldots(2)$$

If x_1 and x_2 give the same level of weir crest then

$$x_2 = x_1 + D_2 - \left(D_1 + \frac{u_1^2 - u_2^2}{2g} \right)$$

therefore equation (2) becomes

$$x_1 + D_2 - D_1 - \frac{u_1^2 - u_2^2}{2g} = D_2 + \frac{u_2^2}{2g} - k\, q_1^{\frac{2}{3}}$$

Or $$x_1 = D_1 + \frac{u_1^2}{2g} - k\, q^{\frac{2}{3}}$$

which is the same as equation (1).

APPENDIX IV.

Afflux caused by a weir when there is a standing wave, and the latter is clear of the downstream edge of the crest.

Let q and q_1 be the discharges per foot run across the river and the weir respectively in Fig. (11). In this case the depth on the downstream edge of the crest is $\frac{2}{3}\ (H + \frac{u^2}{2g})$, and, *vide* note of Appendix I,

$$q_1 = \left\{ \tfrac{2}{3} (H + \frac{u^2}{2g}) \right\}^{\frac{3}{2}} \sqrt{g}$$

$$\therefore H + \frac{u^2}{2g} = \frac{3}{2} \cdot \frac{q^{\frac{2}{3}}}{g^{\frac{1}{3}}} \quad \therefore H = \frac{3}{2} \cdot \frac{q_1^{\frac{2}{3}}}{g^{\frac{1}{3}}} - \frac{u^2}{2g}$$

Now

$$u = \frac{q}{H+x} \quad \therefore u^2 = \frac{q^2}{(H+x)^2}$$

$$\therefore H = \frac{3}{2} \cdot \frac{q_1^{\frac{2}{3}}}{g^{\frac{1}{3}}} - \frac{q^2}{2g\,(H+x)^2}$$

$$\therefore H\,(H+x)^2 - \frac{3}{2} \cdot \frac{q_1^{\frac{2}{3}}}{g^{\frac{1}{3}}} (H+x)^2 + \frac{q^2}{2g} = 0.$$

This is a cubic of the form $H^3 + AH^2 + BH + C = 0$ of which the three roots are

$$H = -\frac{A}{3} + \frac{2}{3}\ (+\sqrt{A^2 - 3B}) \cos \left\{ \tfrac{1}{3} \cos^{-1} \frac{9AB - 2A^3 - 27C}{2(+\sqrt{A^2 - 3B})^3} \right\}$$

The plus square root, as indicated, must be taken.

$$\text{If } \cos b = \frac{9\,A\,B - 2\,A^3 - 27\,C}{2\left(+\sqrt{A^2 - 3\,B}\right)^3} \text{ and if } a = \frac{b}{3}$$

then the three roots are :—

$$H = -\frac{A}{3} + \frac{2}{3} \left(+\sqrt{A^2 - 3B}\right) \cos \begin{cases} a \\ 120^\circ - a \\ 120^\circ + a \end{cases}$$

NOTE.—The writer is indebted for this complete solution of a cubic equation to Paper No. 1355 of the American Society of Civil Engineers, page 344. It will be found particularly useful in reinforced concrete calculations, in which cubic equations frequently occur.

APPENDIX V.

Afflux above a weir with undersluices, when the height of the weir is greater than the height which will give no afflux.

In Fig. (12), let H be the height above weir crest of the afflux level, let q_1 and q_2 be the discharges per foot width across the weir and the undersluices respectively; let l_1 and, l_2 be the length of the weir and the total width of the undersluices respectively; let Q and u be the discharge and velocity of the river upstream; let Q_1 and Q_2 be the discharges of the weir and the undersluices; and let x be the height of the weir above river bed. Then, *vide* note to Appendix II.

$$q_2 = \sqrt{g}\left\{\frac{2}{3}\left(H+x+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}$$

$$\therefore Q_2 = l_2\sqrt{g}\left\{\frac{2}{3}\left(H+x+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}$$

$$\therefore Q_1 = Q - l_2\sqrt{g}\left\{\frac{2}{3}\left(H+x+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}$$

$$\therefore q_1 = \frac{Q - l_2\sqrt{g}\left\{\frac{2}{3}\left(H+x+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}}{l_1}$$

Also $q_1 = \sqrt{g}\left\{\frac{2}{3}\left(H+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}$

$$\therefore \sqrt{g}\left\{\frac{2}{3}\left(H+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}} = \frac{Q}{l_1} - \frac{l_2}{l_1}\sqrt{g}\left\{\frac{2}{3}\left(H+x+\frac{u^2}{2g}\right)\right\}^{\frac{3}{2}}$$

$$\therefore l_1\left(H+\frac{u^2}{2g}\right)^{\frac{3}{2}} + l_2\left(H+x+\frac{u^2}{2g}\right)^{\frac{3}{2}} = \frac{Q}{\left(\frac{2}{3}\right)^{\frac{3}{2}}\sqrt{g}}$$

$$\therefore l_1\left(H+\frac{u^2}{2g}\right) + l_2\left(H+x+\frac{u^2}{2g}\right)^{\frac{3}{2}} = \frac{1{\cdot}837Q}{\sqrt{g}}$$

Or $l_1\left(H+\frac{u^2}{2g}\right)^{\frac{3}{2}} + l_2\left(H+x+\frac{u^2}{2g}\right)^{\frac{3}{2}} = 3{\cdot}37Q.$

from which H can be found.

APPENDIX VI.

Standing waves on the glacis of a weir.

In Fig (13) the force destroying momentum between the planes $C\ E$ and $A\ B$ is not the difference between the pressures on these planes, because the pressure on $D\ E$ is resisted by the portion $B\ F$ of the glacis.

Let a be the height of B above E, let $C\ E = H_1$, and let $A\ B = H_\circ$.

The pressure, P_1, on $C\ D = \frac{w(H_1 - a)^2}{2}$, while that, $P_\circ$, on $A\ B = \frac{wH_\circ^2}{2}$

$\therefore$ the pressure destroying momentum is $\frac{w}{2}\left\{(H_1 - a)^2 - H_\circ^2\right\}$

In a short time t let the planes $A'\ B'$ and $C\ E$ move to $A\ B$ and $C'\ E'$. Let $v_\circ$ be the velocity at $A\ B$, and v_1 that at $C\ E$.

The change in momentum in the time is $\frac{w}{g}(A\ A'\ B'\ B\ v_\circ - C\ C'\ D'\ D\ v_1)$ but

$A\ A'\ B'\ B = H_\circ v_\circ t$, and $C\ C'\ D'\ D = H_1 v_1 t$

$\therefore$ the change in momentum is $\frac{w}{g}(H_\circ v_\circ^2 - H_1 v_1^2)\ t$.

Now the product of a force and its time of action is equal to the change of momentum in the time.

$$\therefore \frac{w}{2}\left\{(H_1 - a)^2 - H_\circ^2\right\} t = \frac{w}{g}(H_\circ v_\circ^2 - H_1 v_1^2)\ t$$

$$\therefore \frac{1}{2}\left\{(H_1 - a)^2 - H_\circ^2\right\} = \frac{H_\circ v_\circ^2 - H_1 v_1^2}{g} = \frac{q^2}{g}\left(\frac{1}{H_\circ} - \frac{1}{H_1}\right) = \frac{q^2}{g}\,\frac{(H_1 - H_\circ)}{H_\circ H_1}$$

$$\therefore (H_1 - a)^2 - H_\circ^2 = \frac{2q^2}{g} \cdot \frac{H_1 - H_\circ}{H_\circ H_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

$$H_1 - a = \sqrt{H_\circ^2 + \frac{2q^2}{g} \cdot \frac{H_1 - H_\circ}{H_\circ H_1}}$$

$$a = H_1 - \sqrt{H_\circ^2 + \frac{2q^2}{g} \cdot \frac{H_1 - H_\circ}{H_\circ H_1}}$$

For known values of H_1 and q the graph of $H_\circ$ can be plotted by taking different values of $H_\circ$ and finding the corresponding values of a.

Example (1).—In the observed upper standing wave of Fig. (1) $H_1 = 1{\cdot}37$, and $q = {\cdot}72\sqrt[3]{\ }$

$$\therefore \frac{2q^2}{g H_1^3} = \frac{2\times{\cdot}72^2\times g}{g\times 1{\cdot}37^3} = \frac{{\cdot}7465}{1{\cdot}37} = {\cdot}5449$$

$$\therefore a = 1{\cdot}37 - \sqrt{H_\circ^2 + {\cdot}5449\,\frac{1{\cdot}37 - H_\circ}{H_\circ}}$$

(1) Put $H_\circ = 0{\cdot}4$ then $a = 0{\cdot}15$

(2) „ $H_\circ = 0{\cdot}45$ „ $a = 0{\cdot}22$

(3) „ $H_\circ = 0{\cdot}50$ „ $a = 0{\cdot}275$

If this curve of $H_\circ$ be plotted, as shewn in Fig (1), it cuts the observed (or calculated) water surface curve at a point which agrees, as closely as can be observed, with the actual position of the standing wave.

Example (2).—In the observed lower standing wave of the same figure $H_1 = 1{\cdot}14$, $a = 0{\cdot}03$ about, $H_\circ = 0{\cdot}45$, and q as before $= 0{\cdot}72\sqrt[3]{g^2}$

$$\therefore \frac{2q^2}{g H_1^3} = \frac{0{\cdot}7465}{1{\cdot}14} = 0{\cdot}6548.$$

We should find therefore that

$$0{\cdot}03 = 1{\cdot}14 - \sqrt{{\cdot}45^2 \quad 6\ 548\ \frac{1{\cdot}14 - {\cdot}45}{{\cdot}45}}$$

while the right hand side of this equation is act ually $= 0{\cdot}04$, which again is as near as can be observed.

Example (3).—In the observed lowest standing wave of Fig. (3) $H_1 = 1{\cdot}19$, and $q = 0{\cdot}6\sqrt[3]{g^2}$

$$\frac{2q^2}{g H_1^3} = \frac{2\times{\cdot}6^2}{1{\cdot}19} = 0{\cdot}3630$$

$$a = 1{\cdot}19 - \sqrt{H_\circ^2 + 0{\cdot}3630\,\frac{1{\cdot}19 - H_\circ}{H_\circ}}$$

(1) Put $H_\circ = 0{\cdot}36$ then $a = 0{\cdot}17$.

(2) „ $H_\circ = 0{\cdot}38$ „ $a = 0{\cdot}23$.

(3) „ $H_\circ = 0{\cdot}42$ „ $a = 0{\cdot}27$.

Where this graph of H_o cuts the calculated water surface curve on the glacis, a position is obtained which agrees with the observed position, as shewn in Fig. (3).

Note.– Equation (1) above can be put in the form of a cubic in H_o :—

$$H_o^3 - H_o\left\{(H_1 - a)^2 + \frac{2q^2}{gH_1}\right\} + \frac{2q^2}{g} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

There are three roots of this equation. One root is negative, the second root is the same as that obtained from equation (1), and the third root will give a graph of H_o which, in three cases worked out by the writer, is approximately the actual curve of the water surface below the standing wave, from the calculated position of the latter to the normal downstream water level.

APPENDIX VII.

Water surface curve on a glacis or rapid.

In Fig. (14) let v_o y_o and v_1 y_1 be the velocities and depths at two vertical planes distant 1 feet apart.

Let H be the difference in water surface levels at these two planes. Let S be the slope of the glacis. Let s be the surface slope necessary to overcome friction with a velocity v, which is the mean between v_o and v_1. Let h_1 be the head necessary to generate the increased velocity, and let h_2 be that necessary to overcome friction.

Then $H = h_1 + h_2$,

$$h_1 = \frac{v_1^2 - v_o}{2g}, \quad \& \; h_2 = \frac{h}{1}^2 \times 1 = s\,1$$

$$\therefore \quad H = \frac{v_1^2 - v_o^2}{2g} + s\,1,$$

But also $H = y_o + S\,1 - y_1$

$$\therefore \quad \frac{v_1^2 - v_o^2}{2g} + s\,1 = S\,1 + (y_o - y_1)$$

$$1 = \frac{\frac{v_1^2 - v_o^2}{2g} - (y_o - y_1)}{S - s}$$

[*Note.*—Where S is greater than s, *i.e*, when, as in the case of a rapid or glacis, the bed slope is steeper than the slope necessary to overcome friction, the denominator is positive and therefore 1 is only positive when $\frac{v_1^2 - v_o^2}{2g}$ is greater than $y_o - y_1$. In a rectangular channel

$$\frac{v_1^2 - v_o^2}{2g} = \frac{q^2}{2g} \cdot \frac{y_o^2 - y_1^2}{y_o^2\; y_1^2}$$

therefore this equation only holds good so long as $\frac{q^2}{2g} \cdot \frac{y_o + y_1}{y_o^2\; y_1^2}$ is greater than unity (if y_o and y_1 are infinitely close this becomes $\frac{q^2}{2g} \cdot \frac{2y}{y^4}$), or when q is greater than $y^{\frac{3}{2}}\sqrt{g}$, or when v is greater than $\sqrt{gy}$, or when y is less than $\frac{v^2}{g}$, that is, *vide* note to Appendix

when y has a value less than $\frac{2}{3}\left(H-\frac{u^2}{2g}\right)$. Similarly, when the bed slope is less than the slope necessary to overcome friction the denominator of equation (1) is negative, and therefore y must have a value greater than $\frac{v^2}{g}$]

Example.—In Fig. (1) let us start with the known depth, $\frac{2}{3}\left(H-\frac{u^2}{2g}\right)$, or 0·72, at the downstream edge of the crest. The channel is rectangular of breadth 2·5 feet.

Using Manning's formula for s :—

$$v=\frac{1\cdot4858}{n}R^{\frac{2}{3}}s^{\frac{1}{2}}$$

we get s to be about $0\cdot00009\frac{v^2}{R^{\frac{4}{3}}}$

The velocity at the crest, $v_c=\sqrt{\cdot72g}$, the velocity v at any point where the depth is y is $\frac{v_c\times 72}{y}$ therefore $\log v=\cdot5399267-\log y$.

(1) Find at what distance from the crest a depth of 0·6 will occur.

$y_0=0\cdot72$, $y_1=0\cdot6$, $y=0\cdot66$

mean area = 1·65, mean wetted perimeter = 3·82, $\frac{v_1^2}{2g}=0\cdot3600$, $\frac{v^2}{2y}=0\cdot5184$.

	·5399267	$\log A$	= ·2174839
$\log\cdot66$	= $\bar{1}\cdot8195439$	$\log W.P.$	= ·5820634
$\log v$	= $\overline{\cdot720382}8$	$\log R$	= $\bar{1}\cdot6354205$
$\log v^2$	= $\bar{1}\cdot4407656$	$\log R^4$	= $\bar{2}\cdot5416820$
$\log\cdot00009$	= $\bar{5}\cdot9542425$	$\log R^{\frac{4}{3}}$	= $\bar{1}\cdot5138940$
	$\bar{3}\cdot3950081$		
$\log R^{\frac{4}{3}}$	= $\bar{1}\cdot513894$0		
$\log s$	= $\bar{3}\cdot8811141$		

$s=0\cdot007605$ $\therefore l=\frac{0\cdot5184-0\cdot3600-(0\cdot72-0\cdot60).}{S-s}$

$$l=\frac{0\cdot0384}{S-0\cdot0076}$$

In the first part of the glacis S is about 0·046.

$$\therefore l=\frac{\cdot0384}{\cdot0384}=1\cdot0.$$

(2) Distance from place where depth is 0·6 to place where depth is 0·5

$y_0=0\cdot6$, $y_1=0\cdot5$, $y=0\cdot55$

mean area = 1·375, mean wetted perimeter = 3·6

$$\frac{v_0}{2g} = \cdot5184, \frac{v_1^2}{2g} = 0\cdot7465$$

	·5399267	log A	= ·1383027
log 55	= $\bar{1}$·7403627	log $W.P.$	= ·5563025
log v	= ·7995640	log R	= $\bar{1}$·5820002
log v^2	= $\bar{1}$·5991280	log R^4	= $\bar{2}$·3280008
log ·00009	= $\bar{5}$·9542425	log $R^{\frac{4}{3}}$	= $\bar{1}$·4426669
	$\bar{3}$·5533705		
log $R^{\frac{4}{3}}$	= $\bar{1}$·4426669		
log s	= $\bar{2}$·1107036		

$$\therefore s = \cdot01289 \quad \therefore \quad 1 = \frac{\cdot7465 - \cdot5184 - (\cdot6 - \cdot5)}{\cdot07 - \cdot0129}$$

$$\therefore \quad 1 = 2\cdot25$$

(3) Distance from place where depth is 0·5 to place where depth is 0·44.

$y_0 = 0\cdot5$, $y_1 = 0\cdot44$, $y = 0\cdot47$

mean area = 1·175, mean wetted perimeter = 3·44

$$\frac{v_0^2}{2g} = 0\cdot7465, \frac{v_0^2}{2g} = 0\cdot0640$$

	·5399267	log A	= ·0770379
log y	= $\bar{1}$·6720979	log $W.P.$	= ·5365584
log v	= ·8678288	log R	= $\bar{1}$·5334795
log v^2	= ·7356576	log R^4	= $\bar{2}$·1339180
log ·00009	= $\bar{5}$·9542425	log $R^{\frac{4}{3}}$	= $\bar{1}$·3779727
	$\bar{3}$·6899001		
log $R^{\frac{4}{3}}$	= $\bar{1}$·3779727		
log s	= $\bar{2}$·3119274		

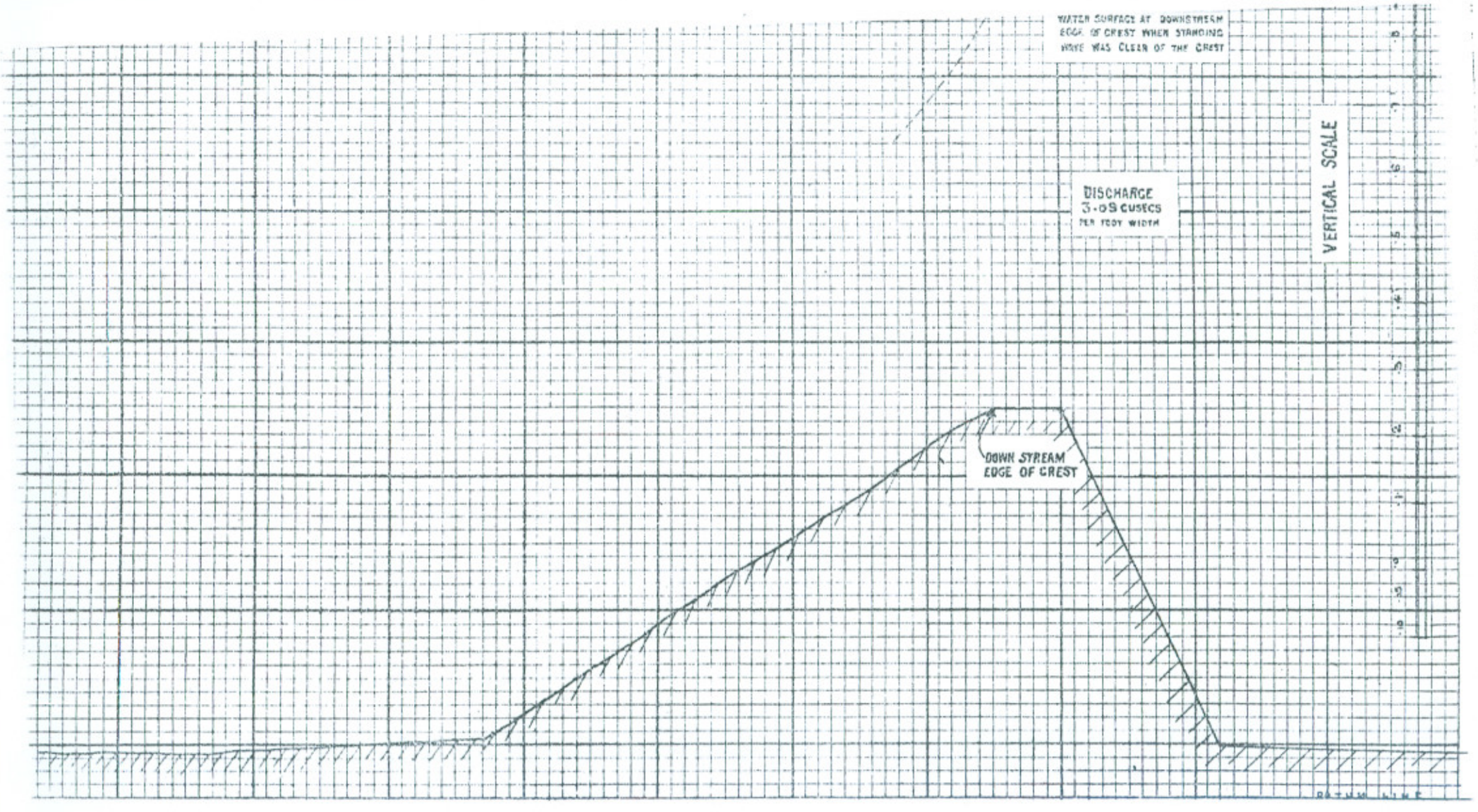
$$\therefore \quad s = 0\cdot02051 \quad \therefore 1 = \frac{\cdot9340 - \cdot7465 - (\cdot5 - \cdot44)}{\cdot065 - \cdot0205}$$

$$\therefore 1 = 3\cdot54$$

We therefore get :—

Depth	·72	·60	·50	·44
Distance from down-stream edge of crest	0	1·0	3·25	6·79

This curve is shewn dotted in fig (1), and it agrees, as closely as it possible to measure, with the observed water surface curve.


WATER SURFACE AT DOWNSTREAM EDGE OF CREST WHEN STANDING WAVE WAS CLEAR OF THE CREST
DISCHARGE 3.05 CUSECS PER FOOT WIDTH
VERTICAL SCALE
DOWN STREAM EDGE OF CREST

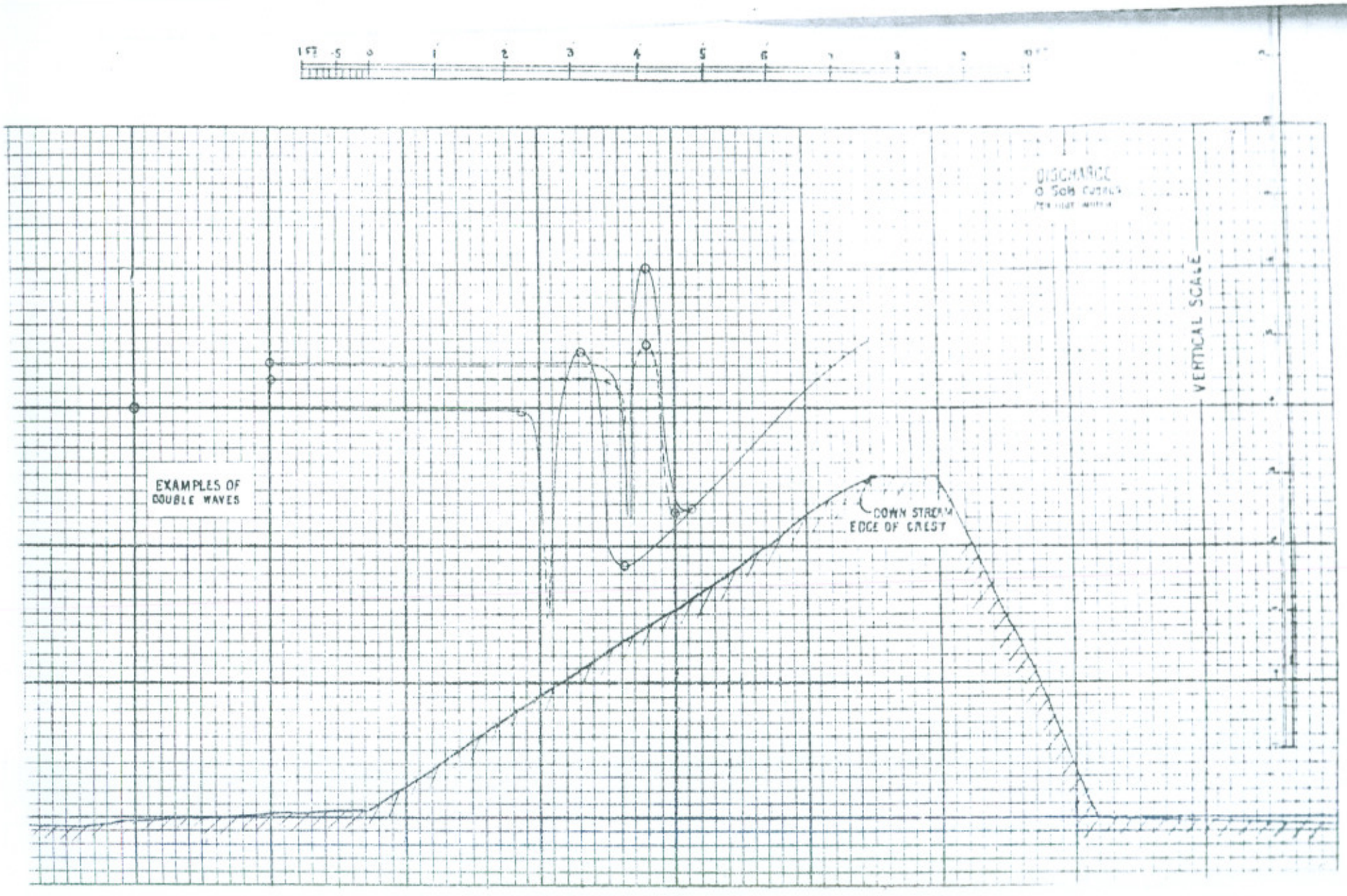

DISCHARGE
VERTICAL SCALE
EXAMPLES OF
DOUBLE WAVES
DOWN STREAM
EDGE OF CREST

FIG 7

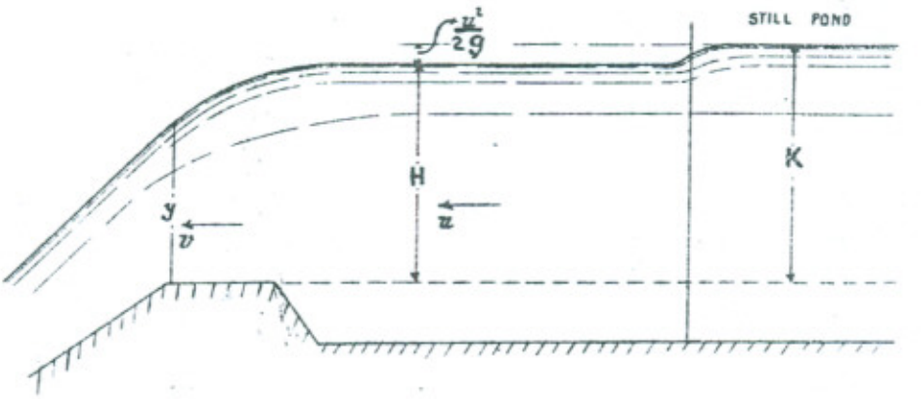


FIG 8

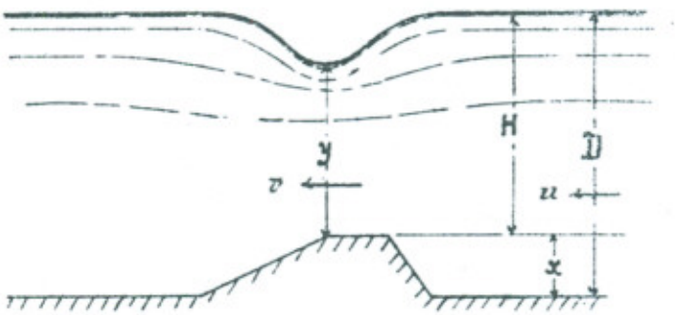


FIG 9

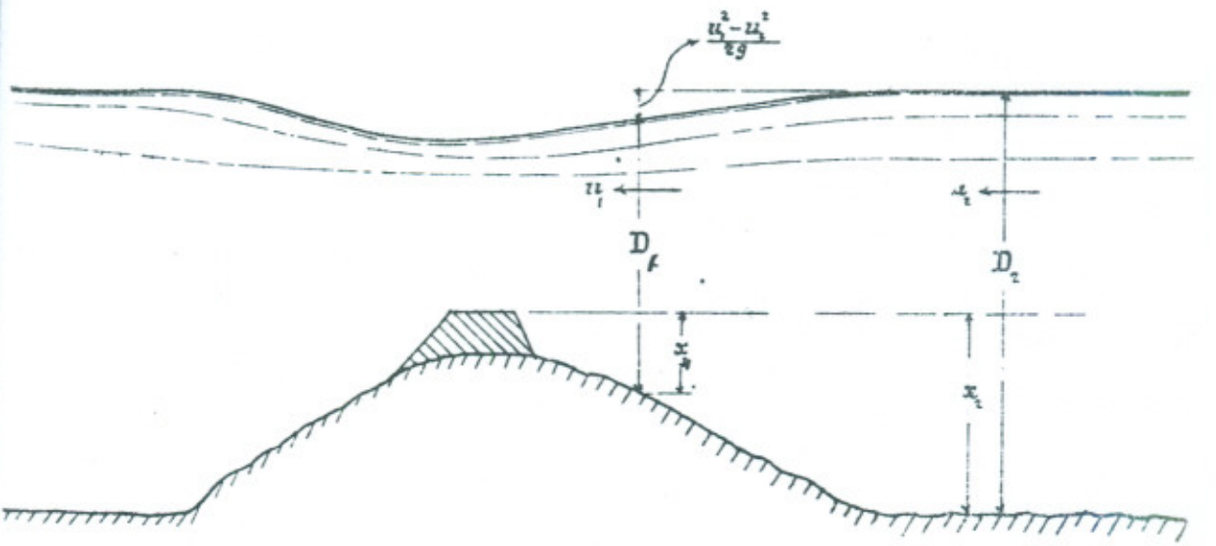
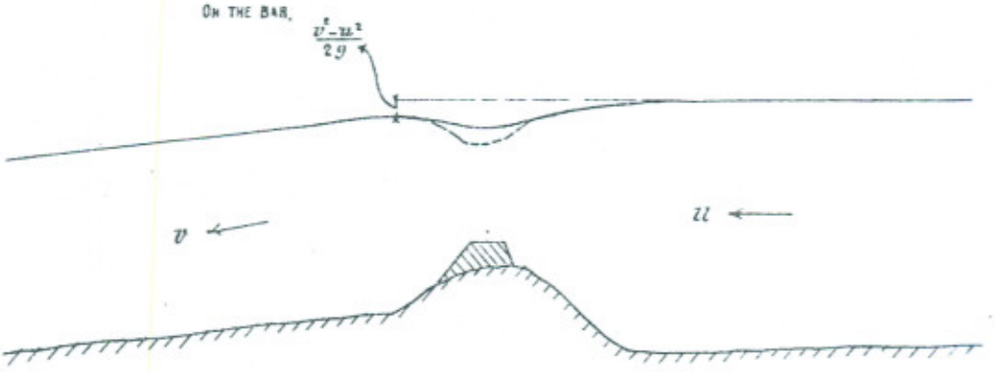


FIG 10

SHEWING EFFECT OF CHANGE IN BED SLOPE OF A STREAM AFTER A NATURAL BAR. AND SHEWING THE EFFECT OF CONSTRUCTING A MASONRY WEIR ON THE BAR.



IN THIS CASE THERE WILL ALWAYS BE A DIFFERENCE IN LEVEL BETWEEN THE UPSTREAM & DOWNSTREAM WATER LEVELS = $\frac{v^2-u^2}{2g}$.

THE ADDITION OF A MASONRY WEIR, IF THE LATTER IS NOT TOO HIGH, WILL STILL LEAVE A DIFFERENCE IN LEVEL OF $\frac{v^2-u^2}{2g}$, BUT THE DEPRESSION OF THE WATER-SURFACE OVER THE BAR WILL BE INCREASED.

FIG 11

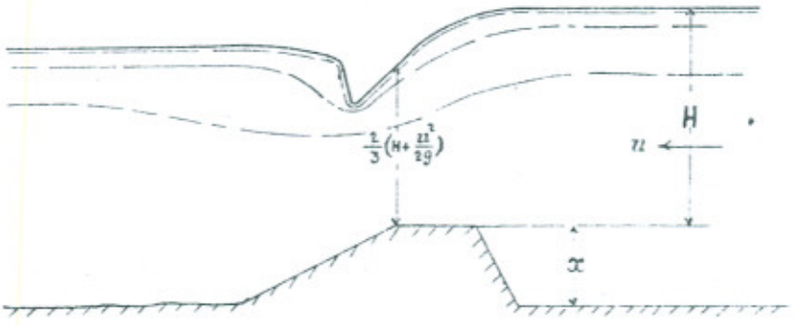
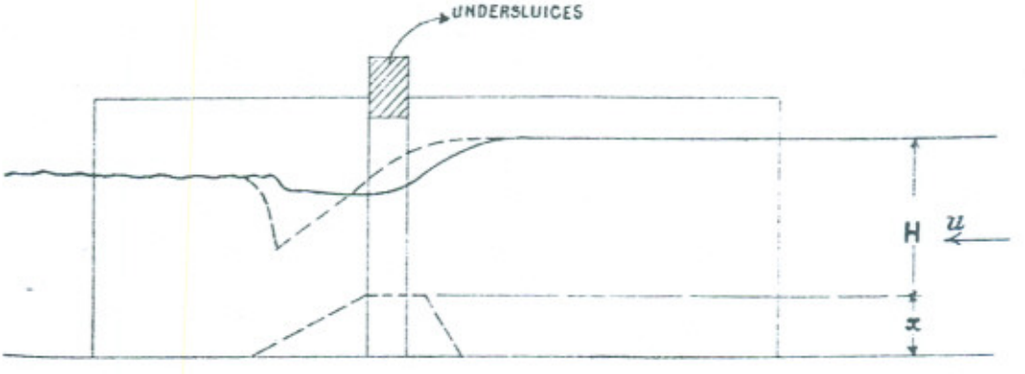


FIG 12.



WEIR AND WATER SURFACE OVER WEIR SHEWN DOTTED

FIG 13

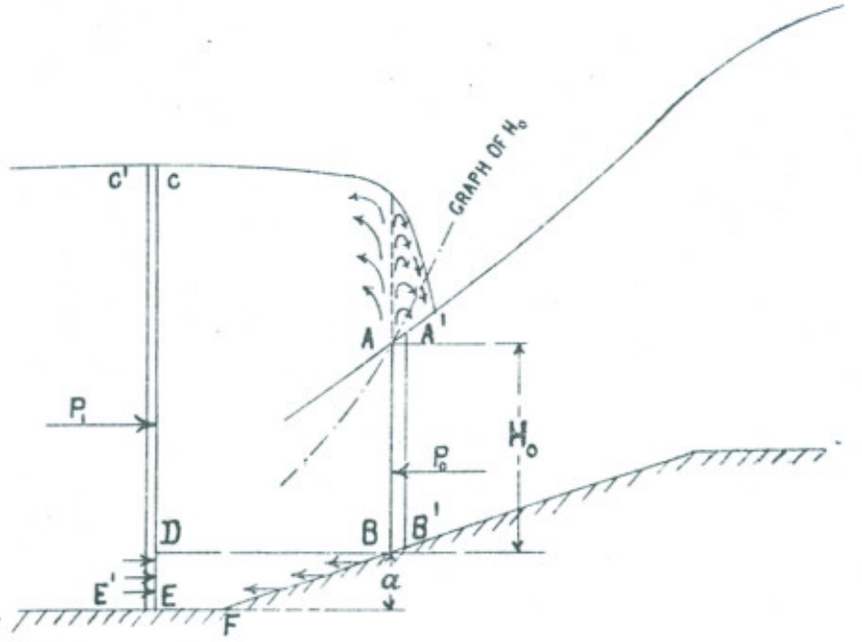
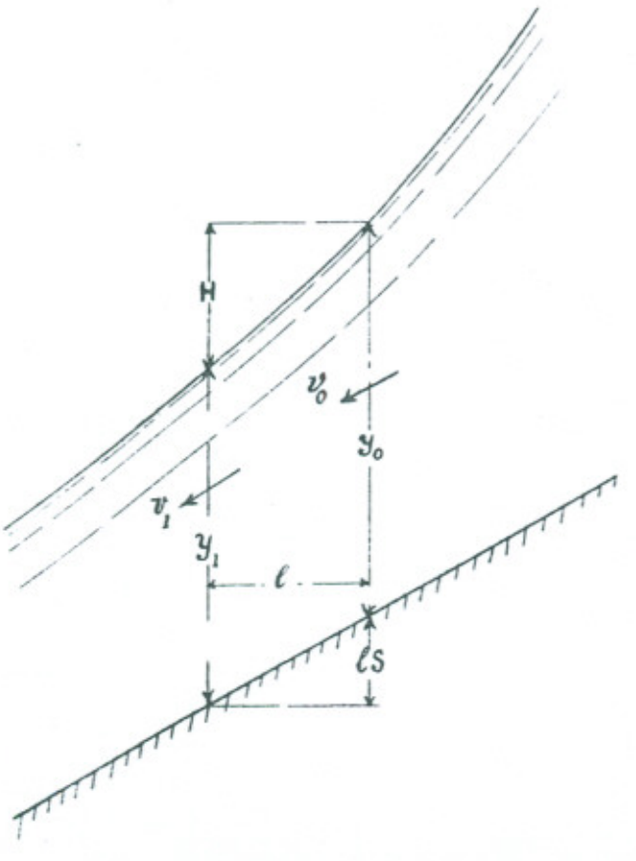


FIG 14



DISCUSSION.

MR. BURKITT briefly introduced his paper, and explained more fully on the black board, for the information of members, his sketch illustrating the experiments about which there appeared to be some misconception.

MR. E. S. LINDLEY said that in May 1915 his superior had suggested to him a problem in standing waves ; after reading the little that had then been written on the subject, he submitted an estimate for experiments, which however only found a place in office records. Mr. Burkitt had been more fortunate in his opportunities.

The standing wave problem was of more than merely academic interest, and in asking for means to experiment, he had mentioned the following practical applications that he foresaw and wished to investigate :—

1. Two forms of gauge-outlet.
2. A " Venturi " flume.
3. An improvement for cross-drainage works.

One form of the first was now in use as the " Harvey Outlet," which was only a small Venturi flume ; the other form was the Stoddard-Harvey module, that had not yet been perfected (as far as he knew). The Venturi flume had been introduced in America in the very small and imperfect form, described in the Journal for Agricultural Research of April 23rd, 1917 ; possibly a contracted flume, built on the Lower Bari Doab Canal main line, was intended to be the same, while Mr. Harvey, in his paper before the present Congress, suggested that what he proposed was again the same thing.

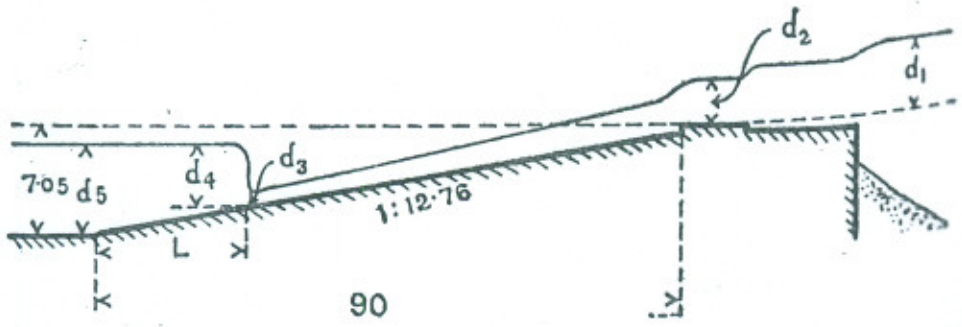
The third was what Mr. Burkitt had described on page 108, where the heading up due to a syphon was diminished by recovering the velocity head at the exit.

About 1915 an experimental flume was built at Sargodha, but no standing wave would form. It would have been more useful to build only the side-walls, and to have had a set of slabs that could be formed into weirs of different shapes and at different levels, using also a submerged orifice to give the conditions of flow obtaining near the toe of a glacis, as under a fairly wide range of conditions a wave would jump to four-fifths of the original depth. The ninety cusecs and three feet afflux available for the experiments which he wished to make, would with such a flume have given data of value compared with any recorded up to date.

The theory of standing waves had already been exhaustively discussed in Paper 1355 (1916) of the American Society of Civil Engineers, by Mr. Karl Kennison; and in Paper 4081 (1913-14) of the Institution of Civil Engineers, by Professor Gibson. These showed conclusively that the correct formula was obtained by equating momenta, as Mr. Burkitt did, and not by equating energies, as had been done in the formula long accepted as correct, and ascribed to Bidone. For waves on a sloping glacis, Professor Gibson introduced a term for the fall of the glacis, but, in testing by it some experiments of D'Arcy and Bazin, did not state how he had fixed the length in which to take the fall into account. Mr. Burkitt took this fall down to the toe of the glacis, and his experiments confirmed this, because the wave appeared always to have extended so far. In practice the wave was made to form on the glacis, and its correct theoretical position was higher up than that given by Mr. Burkitt's formula, and lower than that obtained without allowing for the slope; but the difference between the two extreme positions was not large compared with the length in which there was great turbulence, and the balance of advantage lay with the simplicity for calculation of the formula that made no allowance for slope; a greater difference would be made by an ordinary error in assuming a coefficient of rugosity.

Table I*, which he had prepared, gave the data of some observations he had made on the Rasul weir, with such accuracy as was possible, while Fig. 1 gave the details and the notation.

Rasul Weir



The weir was closed, except for about a quarter of the bay in which the observations were made, so that the

* Page 126j.

flow was practically from one still-pond to another, and not with high velocities of approach such as occurred in floods: d_1 and d_5 were measured on open gauges at the closed sides of the bay, d_2 was measured at the middle of the crest, where the surface became distinctly more horizontal at small depths: when d_2 exceeded two-thirds of d_1 d_3 was measured just above the wave, and was difficult to get accurately, in very high velocities, on hammer-dressed masonry. The standing wave had an almost vertical face with only a little curl of foam in the trough, and rose to slightly above the level of the standing water downstream, boiling all over without any marked central spout-up, so that the face of the wave was taken as giving its position. It was not possible to determine the discharge with any degree of accuracy, but the data had been used to calculate the discharge per foot of width in various ways. Q_1 was the discharge over a broad crest with a free run-off, Q_2 the same taking the actual depth and afflux, Q_3 was calculated from the data of the wave without any allowance for fall on the glacis, by a convenient adaptation of the Unwin formula (momentum), and Q_4 by Mr. Burkitt's formula.

$$Q_1 = 3{\cdot}09 \times {d_1}^{1{\cdot}5}$$

$$Q_2 = d_2 \times \sqrt{2g(d_1 - d_2)}$$

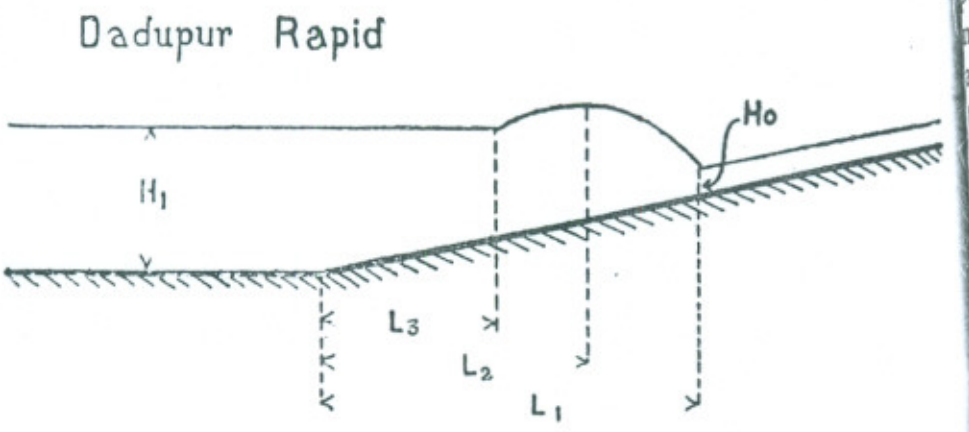
$$Q_3 = \sqrt{g/_2 \times d_3 \times d_4 \times (d_3 + d_4)}$$

$$Q_4 = \sqrt{g/_2 \times d_3 \times d_5 \, 2\frac{({d_4}^2 - d}{d_5 - d^3}}$$

In these observations $V/\sqrt{gd}$ varied from 2·3 to 2·6. The discharges and depths of flow on the glacis gave a value of $N = 0{\cdot}026$ in the Manning formula.

It will be seen that when d_1 exceeds three feet the flow would be free, and the formula Q_1 applied; with lesser depths the discharge was damped by the width of the crest, and a little grass that was found entangled in links lying on the face of the shutters forming the crest, and Q_2 applied; the fact that Q_3 agreed with these, while Q_4 was always appreciably less, showed that it was wrong to take the fall to the foo of the glacis.

Table II* gave a few data of standing waves on the Dadupur rapid, while Fig. 2 gave details and the notation.



In these observations $V/\sqrt{gd}$ was from 3·9 to 4·5, and $N = 0{\cdot}025$; the wave was much more violent, rising in ten feet from the beginning to the marked centre of spouting up, which continued for another ten feet. In an observation made by him the position of all three points was noted, but in the others made for him only the first was measured. H_0 was measured as before, just before the wave, and H_1 on a gauge downstream. Data of the crest were not given, as it was neither a broad nor a sharp crest. The total discharge passing was taken from a curve which seemed accurate, referred to a gauge almost on the crest. There was a main rapid, and a rafting bay with a crest and glacis 0·75 ft. lower, and the total discharge passing had been distributed according to the level measured in the approach channel clear of any draw-down to the weir, with an allowance for the velocity of approach there. A trial was made with velocities of approach due to the different rates of discharge, but gave results that were obviously absurd. The most doubtful point here being the position of the wave, this was calculated from the other data, both by the Unwin formula (momentum) without allowance for slope, and by Burkitt's formula, and again the former seemed to be nearer the mark. The observations showed the great increase in turbulence as compared with the Rasul wave.

Coming to the practical applications referred to above both were the securing of a constant relation between the discharge passing and the gauge-level above a work, indepen-

*Page 123.

dently of variations of level below that work. Mr. Burkitt's experiments went a good way towards showing the practical possibility of the Venturi flume, which was an open Venturi meter, with contraction sufficient to give the stage $V^2 = g \times d$ at the throat, so that only one gauge need be read since there was this constant relation. The flume had to consist of a throat long enough to allow this condition to be fully established; a bell-mouth to fill the whole throat and convert as much as possible of the head into velocity, (that was, coefficients of contraction and of velocity, both as near as possible to unity) and according to the afflux available, an expanding flume to recover the velocity head. Mr. Burkitt made the point that contraction could be effected, either by raising the bed or by narrowing the sides of a channel, and also gave data for the condition of 'no afflux,' though it would be more correct to speak of afflux that was too small to be appreciable in his small-scale tests. Further tests were wanted to show the minimum afflux required for modularity.

In the discussion of a recent paper before the American Society of Civil Engineers by Mr. Floyd Nagler, the originator of the Venturi meter Mr. Clemens Hershell pointed out that a sharp-crested weir could not be considered a really accurate measuring device unless the conditions under which it was calibrated were accurately reproduced, and advocated a certain standard rounding of the crest, to get a standard suppression of contraction. A bettea course, however, would be to suppress entirely all contractions but the natural surface contraction, and make a broad-crest weir or Venturi flume. This had the added advantage that, instead of requiring a free fall, a small afflux was sufficient.

The Stoddard-Harvey module, involving the formation of a standing wave below a submerged orifice, was outside the subject of the present paper; but the following data were worth placing on record. On the Rasul undersluices he had formed a standing wave, with depths of 10·3 and 8·5 ft. up and down-stream with a depth of flow roughly measured as 4 ft. from a gate-opening of $5\frac{1}{2}$ ft., while theoretically a depth of flow of 0·4 of the up-stream depth would jump to 0·8 of it, calling for a working head of only a fifth, without any attempt to diminish the loss of head in the standing wave.

MR. W. C. CHOPRA instanced the case of the very broadcrested weirs in the form of super-passages on the Raya

branch of the Upper Chenab Canal, where the calculation of the discharge became very difficult, because, on the one hand, the crest was not wide enough to allow of a steady stream in the flume to admit of direct observation or of calculation by Kutter's formula, and on the other hand, the crest was so broad that the ratio of the head or the depth of water to the width of the weir became so small as to defy even the low values of coefficient given by Unwin, and he enquired what coefficient the author would recommend in such cases, there being no standing wave to speak of, on account of the small head. A coefficient of 0·577 was taken, but it was doubtful whether it applied to the case. He also instanced the case of the weir across the Bhimber diversion on the Upper Jhelum Canal, where over a length of 850 feet of crest wall, the water-level sloped from right to left, so that while there was a fall of several feet at one end, it was almost *nil* on the other, and where two standing waves were produced (the first just below the raised crest-wall, on the horizontal part of the glacis, and the other lower down, on the sloping part of the glacis), and inquired what co-efficient should be employed in such cases, and what, in the author's opinion, should, for purposes of calculation, be considered (*a*) the water-level upstream of the crest, (*b*) the water-level downstream of the weir.

He had been taking the mean values (for downstream the mean of the stream immediately below the lower standing wave), but was doubtful if that was quite correct, and would be glad to have the author's views on the matter.

MR. C. A. COLYER drew attention to Fig. 4 of Mr. Burkitt's paper, and described a double wave of the kind noted by the author, which he had seen formed on the Merala weir in a big flood.

MR. F. W. WOODS remarked on the statement that on the Swat river, large freshets passed over the Abazai weir "without appreciable afflux, the difference in the water-levels against the upstream and downstream sides of the *bund*, that obstructed half the waterway of the river, being approximately that caused by the natural slope of the river." It would have been more satisfactory if a plan, with longitudinal and cross sections of the river and the recorded water-levels of the flood, had been given, to enable them to judge of the real conditions at the site, and of the merits of the author's inferences therefrom.

The author went on to state that " eventually a flood, *about twice as big* as the maximum recorded for the winter season was *safely passed*, though *calculations based on* any of the *accepted formulae showed that* the afflux should have been so great as to have overtopped the *bund.*" The words in italics were va gue. One would have liked to know the estimated magnitude of the flood in cusecs, and how the estimate had been arrived at. Also, the recorded afflux of this flood, that is to say, the difference of the water surface levels upstream and downstream respectively of the weir, or of the *bund.* It was not enough to say that it was *safely passed*, as that might have happened even if the afflux had been very considerable. One would also have liked to see the figures of the calculations " by accepted formulæ," which Mr. Burkitt claimed to have been so erroneous in this case ; for it was, *prima facie*, improbable that formulae, which had held the field for so long, should have done so without earlier detection of their worthlessness.

Mr. Burkitt's main thesis might very possibly be correct, but he would bring conviction to their minds more readily if he treated them to precise facts and figures which they could analyze and check to their own satisfaction.

Mr. Burkitt's proposition that, when a standing wave occurred below a waterfall over a weir or flume, the discharge was the same as it would have been if the overfall were free, with the tail-water level below the level of the crest of the weir or the floor of the flume, and that neither the discharge nor the upstream water-level wasa ffected by artificial variations of the level of the tail-water surface downstream of the standing wave, was probably true. As long ago as the year 1893, Mr. Herbert, Executive Engineer of the Lower Chenab Canal at Khanki, stated that it had been found, by experience of the weirs and rapids on the Upper Bari Doab Canal, that the discharge over a weir could be estimated accurately by the " free fall " formula, so long as a standing wave existed below the weir. Colonel Ottley, Chief Engineer, and Mr. Preston, Superintending Engineer, corroborated this opinion ; though at a later date Mr. Beresford and Mr. Benton, Chief Engineers, expressed opinions to the contrary. It would simplify their calculations of the discharge of streams, and especially of streams of great magnitude in flood, if their discharge could be calculated by the simple free-fall formula

$$Q = \tfrac{2}{3} c a \sqrt{2gh}$$

in cases where a and h were known from the recorded water-

level of the flood upstream of the weir, and the known level of the crest. The difficulty of applying it to the design of particular works lay in the fact that one could not predict beforehand whether, and to what extent, standing waves would occur downstream of a designed weir, under any particular conditions of flood. It had been a matter of common observation of floods passing over various river weirs in the Punjab, that standing waves might occur below some parts of a weir and not below other parts of it ; and it could not be predicted with confidence that any given flood would produce standing waves throughout the length of the overfall.

When a flood of great magnitude careered down the sandy alluvial bed of a Punjab river, it flowed not in a single well-formed channel, but in several different channels, between shoals, and in shallow spillways over shoals. It swept away shoals in some places, or deposited them in others, and was subject to unforeseen deflections from its normal course by those shoals, and by obstructions of various kinds ; so that in its passage over a masonry weir at any point of its course, its hydraulic conditions did not always lend themselves easily to a precise estimation of the discharge, and this was the main drawback to the application, in practice, of the results of Mr. Burkitt's useful investigations.

MR. BURKITT in his reply at the Congress to the criticisms put forward by the members, said that he could not reply off hand to Mr. Lindley. It was not his formula for the position of a standing wave at a weir. He had got it from Mr. Lindley, but the formula did not agree at all until Mr. Harvey had given him his suggestions.

The two-thirds depth referred to in his paper would occur at the downstream end of a super-passage, as long as there was a free fall or a standing wave, and the downstream water-level would then not affect the discharge. He regretted that his information with regard to the construction of the Abazai weir was not more complete, but during the flood all the engineers were busily engaged in saving their *bund*. Most of the American papers had been writing a great deal about making weirs that would cause no afflux, and to avoid having an afflux, during a big flood, when the depth of water was great, the weir should be designed in such a manner that there would be no standing wave. The main thing which he wanted to prove was that a weir could be built that would not cause any afflux after the depth had reached a certain point.

Unfortunately it has been impossible to obtain any reply from Mr. Burkitt, beyond that taken down at the Congress, to the points raised by certain members, otherwise he might have pointed out that Mr. Lindley's conclusions that his formula for ascertaining the position of the standing wave was wrong, appeared to depend on the accuracy of the discharges given by certain formulæ, the applicability of which to the conditions of the case was itself open to doubt. Instancing that the formula $Q_2 = d_2 \times \sqrt{2g\left(d_1 - d_2\right)}$ gave the full theoretical discharge for a water-level of the height d_2 on the crest, without regard to frictional losses upstream of the point where the water became horizontal at this level, which under the conditions of the case were not apparently negligible, with the result that the actual discharge might well be less than that given thereby.

It is also likely that he would have qualified his reply to Mr. W. C. Chopra, that "the two-thirds depth, referred to in his paper, would occur at the downstream edge of the superpassage....." by adding "provided there was no loss due to friction in traversing the superpassage."

Data of Standing-wave Observations on Works.

TABLE I.

RASUL WEIR (Bay No. 8).

Observed.					Calculated.				
d_1	d_2	d_3	L	d_5	d_4	Q_1	Q_2	Q_3	Q_4
3·85	2·70	1 60	40	7·00	3·86	23·3	23·2	23·4	20 4
3·65	2·50	*1·25	29	6·10	3·82	21·6	21·5	19·8	18·2
3·60	2·30	†1·25	25	5·60	3·64	21·1	21·0	19·0	17·5
3·45	*2·70	1·25	29½	5·80	3·48	19·8	18·8	18·3	16·5
3·15	2·15	1·25	39	6·40	3·34	17·3	17 2	17·8	15 6
3·00	2·00	1·10	38	6·20	3·22	16·1	16·0	15·8	14·1
2·80	2·00	0·84	30	5·30	2·95	14·5	14·4	12·35	11·35
2·45	1·90	0·88	30½	5·20	2·80	11·85	11·30	12·10	11·00
2·15	1·71	0·71	32	4·70	2·19	9·75	9·10	8·55	7·64
2·15	1·72	0·75	30	4·60	2·25	9·75	9·05	9·05	8·10
2·00	1·60	0·67	29	4·55	2·27	8·75	8·12	8·50	7·74
1·65	1·42	0·57	30	4·10	1·75	6·65	5·46	6·13	5·43
1·65	1·30	0·52	30	3·95	1·60	6·55	6·17	5 35	4·71
1·35	1·15	0 40	30	3·70	1·35	4·85	4·12	3·91	3·47

$V/\sqrt{gd}$ is from 2·3 to 2 6.

TABLE II.

DADUPUR RAPID.

Observed.								Calculated.	
Date.		Q	H_0	H_1	L_1	L_2	L_3	L_0	L_B
17-7-17	...	48·0	2·09	9 21	42	†30	...	$28\frac{3}{4}$	$24\frac{1}{2}$
18-11-16	...	42·9	1·80	9·1 2	$39\frac{3}{4}$	$28\frac{3}{4}$	$18\frac{3}{4}$	$28\frac{1}{2}$	25
17-7-17	...	39·4	1·81	9·27	53	43	...	$42\frac{1}{4}$	39
18-11-16	...	34·5	1·55	9·08	50	†41	30	$43\frac{3}{4}$	$40\frac{1}{4}$
8-7-17	...	25·55	1·38	7·13	$44\frac{1}{4}$	$\dagger 37\frac{1}{4}$	...	$35\frac{1}{2}$	31
8-7-17	...	17·85	1·06	7·11	$52\frac{1}{4}$	$\dagger 47\frac{1}{4}$	...	$49\frac{1}{4}$	45

$V/\sqrt{gd}$ is from 3·9 to 4·5.

† Not observed. * Doubtful.