Paper:128

ECONOMICAL DESIGN

OF

MASJID DOME

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ABSTRACT

A MASJID-DOME of spherical shape, with the semicentral angle greater than 90 degrees, has been completely analysed with such profilings and patterns as will make it adoring and befitting a worshipping place _____ THE MASJID.

1.0 *INTRODUCTION*

The use of a dome-like structure to cover a space goes back to very ancient times, However, over the last thousand years or more, the dome in conjuction with the tower called the minaret has comprised the basic architecture of the Muslim place of worship_____ the Mosque or Masjid. The dome has, in fact, become a kind of symbol of the Muslim world because of the expertise that was acquired in the past by the builders of domes. Apart from the primary function of forming part of a mosque, dome construction has also traditionally been employed for covering the rotunda of a mausoleum.

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Shallow domes are used in large areas of the Middle East, whereas the ogival or pointed dome is common in Iran. In Pakistan and in India, domes of spherical, parabolic, elliptical and other meridional shapes can be observed as part of historical structures in cities like Lahore and Multan.

Although many domes are constructed in Pakistan by masons even today the methods and materials used are traditional and handed down from builder to builder. Reinforced concrete domes are more or less non-existent and the use of shell theory for the evaluation of the stress resultants is quite unknown.

The present paper deals with the structural design of a modest sized mosque dome in reinforced concrete. The shell is deeper than a hemisphere, and has a thickness of 4 in. It is hoped that the calculations will be of interest to engineers in Pakistan to whom literature on shell design and construction is hardly available.

2.0 DEVELOPMENT OF THE DESIGN-THEORY

2.1 Definitions

2.1.1 Stress Resultants

The Forces and Moments acting per unit length of a shell cross-section are called stress-resultants and have the units kip.per foot (k/ft).

2.1.2 Edge-Disturbance

The reactive forces which are present at the edge of shell which is connected to some rigid support, such as a ring-beam, set up stresses into the shell body. The upward penetration and distribution of these forces follows the pattern of damping waves i.e., the intensity or wave amplitude is greatest at the shell edge and vanishes in its propogation as damped waves. This distribution (of hoop

stresses, meridional stresses and bending moments) is termed edge disturbance.

2.1.3 Membrane Stresses

If the shell is idealized as a membrane incapable of resisting bending moments the stresses thus computed are called membrane stresses. The boundry condition of the shell edge for such a state of stress is shown in Fig. (a)

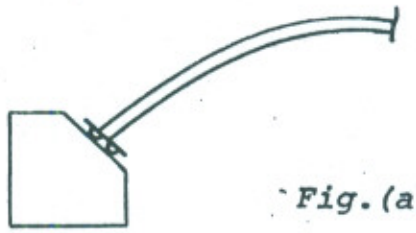


Fig.(a)

2.1.4 Bending Stresses

The stress-resultants, computed under the assumption that shell body is capable of developing resisting moments and hence the transverse shear, are classed bending stresses.

2.2 Conventions

Bending moments producing tension at the inner surface of the shell are positive.

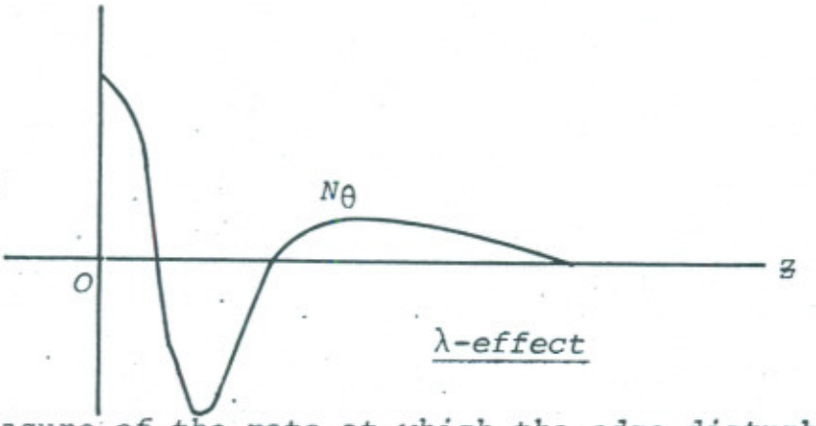
Clockwise rotations of the shell meridian are positive.

Horizontal translations of the shell and the ring-beam that tend to maintain continuity between the shell and supporting ring-beam are positive.

The angular movements of the shell and the ring-beam that tend to maintain angular continuity are positive.

2.3 Shell Constants (D, λ)

"D" represents the flexural rigidity of the shell and is $=Et^3/(1-\nu^2)12$. "λ" Lambda is another shell constant and it's value is $\sqrt[4]{3(1-\nu^2)(r/t)^2}$



λ-effect

It is a measure of the rate at which the edge disturbance alternates as shown above.

3.0 ASSUMPTIONS

The changes in the curvature of the shell surface are small.

The shell thickness is such that the ratio of the shell thickness to the shell radius should be equal to or less than 1

4.0 INFLUENCE-COEFFICIENTS OR FLEXIBILITIES

The magnitude of horizontal translation or angular deformation caused by a unit load is called influence-coefficient or flexibility of the structural system.

5.0 SHELL EQUATIONS

The set of equations which will be used in analysis are taken from the book "THE THIN SHELL CONCRETE STRUCTURES" by David P.Billington and, after the necessary modifications for the semi central angle greater than 90 degrees, they are listed below for ready reference.

Equations for stress resultants (N_ϕ, N_θ) and displacements for the spherical dome, under membrane state considering the dead load only.

The maginitude of the meridional force at any section of the shell at an angle ϕ is given by

$$N_\phi^m = -wr/(1+\cos\phi)$$

The magnitude of the hoop force at any section at the parallel circle at an angle ϕ is given by

$$N_{\theta}^{m} = +\omega r\left(\frac{1}{1+\cos\phi} - \cos\phi\right)$$

The horizontal displacement of the shell edge at the parallel circle ϕ = 120 degree is given by

$$\Delta_1^s = \frac{\omega r^2}{Et}\left(\frac{1+\nu}{1-\sin\beta} + \sin\beta\right)\cos\beta$$

And the angular deformation there is given by

$$\Delta_2^s = \frac{\omega r}{Et}(2+\nu)\cos\beta$$

Equations for the membrane displacement of the ring-beam considering the dead load only.

$$\Delta_1^R = \frac{\omega r \sin\beta\ r_o)^2}{(1-\sin\beta)\ EA_R}$$

Equations for stress resultants and flexibilities, under bending state for the dead load only.

Substituting for the angle-change (and starting from the edge).

$$\phi = \phi_c - \Psi = (90+\beta) - \Psi$$

The magnitude of the meridional force at any angle Ψ, starting from the edge, is given by

$$N_{\phi}^{b} = \frac{2\lambda}{r}\tan(\beta-\Psi)\, e^{-\lambda\Psi}\sin\lambda\Psi \cdot x_2 + \sqrt{2}\tan(\beta-\Psi)\, e^{-\lambda\Psi}\cos\beta\sin(\lambda\Psi-45)x_1$$

The magnitude of the hoop at any angle Ψ starting from the edge at the parallel circle is given by

$$N_{\theta}^{b} = \frac{2\sqrt{2}}{r}\lambda^2\, e^{-\lambda\Psi}\sin(\lambda\Psi-45)x_2 - 2\lambda\cos\beta e^{-\lambda\Psi}\sin(\lambda\Psi-90)x_1$$

The magnitude of the meridional bending moment at the same angle Ψ, starting from the edge, is given by

$$M_{\phi}^{b} = \sqrt{2}\, e^{-\lambda\Psi}\sin(\lambda\Psi+45)x_2 + \frac{r}{\lambda}\, e^{-\lambda\Psi}\cos\beta\sin(\lambda\Psi)x_1$$

The equations for flexibilities at the parallel circle ϕ =120 degrees i.e. Ψ= 0 are given as under considering the dead load only and under bending state of stress.

$$f^{s}_{11} = 2r\lambda\cos^2\beta\ /Et \quad , \quad f^{s}_{22} = 4\lambda^3\ /Etr$$

$$f^{s}_{21} = 2\lambda^2\cos\beta\ /Et = f^{s}_{12}$$

Equations for the ring-beam displacements under the effect of the dead load of the shell assuming the shell edge to be rigidly connected to the ring-beam.

$$f^{R}_{11} = 4r_o^2\ /EA_R \quad , \quad f^{R}_{22} = 3r_o^2\ /EA_R\ (h/2)^2$$

$$f^{R}_{21} = 3r_o^2\ /EA_R\ h/2 = f^{R}_{12}$$

6.0 ANALYSIS AND DESIGN OF A "MASJID(MOSQUE)DOME"

It is intended to present a complete analysis and design of a reinforced-Concrete Masjid Dome Shell which is spherical in shape having the semi-central angle equal to 120 degrees.

The analysis is to be carried out considering:

- The self Load of the shell,
- A temperature differential of $\pm$ 40^{o}F .

As there is no snow fall in the area where this dome is to be erected (Lahore City), no snow load, therefore, is being considered in the said analysis.

A side computation shows that the wind stresses (considering sufficiently high velocity winds) are extremely low, hence the wind stresses are neglected.

As is generally the case in regard to domes that temperature stresses, by far, exceed all other types of stresses, hence temperature is to be considered as the controlling effect.

ANALYSIS

A. Geometery of the Dome shell

As shown in Fig.1
Radius of the Shell
r = 6 ft.

Radius of the base
r_o = 5.2 ft.

Semi central angle
ϕ_c = 120 degrees.

Angle below 90 degree is indicated as β= 30 degrees.

Poisson's effect

ν = 1/6

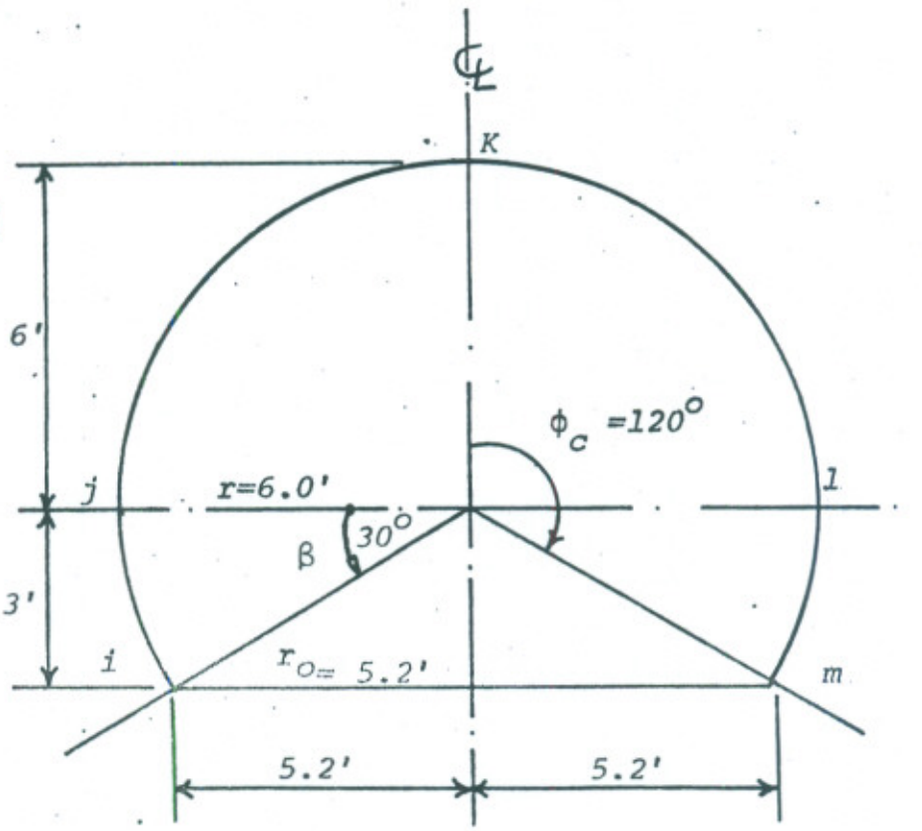


Fig.1

B, Preliminary Section

* The thickness of the shell in the upper part i.e., between jkl is to be constant and indicated as t_j=4 in.

* The thickness in the lower part i.e., in the zones j-i and l-m is to vary gradually being a maximum at the edge and is indicated as $t_i = t_m$ = 6 in.

C. Loads and allowable stresses for steel and concrete

Self load of the dome-shell (including marble tiles on the outer surface and plastering) in the upper portion jkl

is to be indicated as W_{DL}= 65 lb/ft^2 of the shell-surface area.

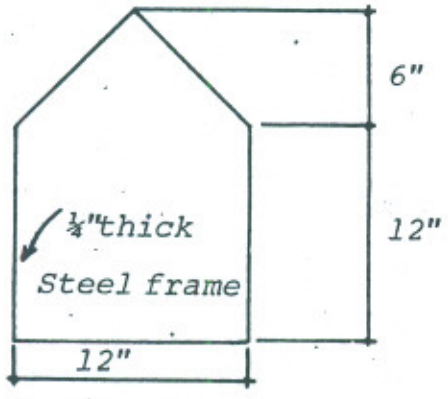
* In the lower portion ijlm the dead load of the shell is W_{DL} = 100 lb/ft^2 of the shell surface area.

* The yield stress for steel is F_y = 40 ksi.
* The allowable stress for steel in tension and in bending is f_s = 20 ksi.
* The yield stress for concrete f_c = 2.5 ksi.
* Bending stress in concrete f_c = 1.125 ksi.
* Concrete stress in direct compression = 0.750 ksi.

D. Shell Profile

In the external profile scheming the shell is to have Mehrab-shape cuts (fig.2) at every third feet. The purpose of these Mehrabs is to provide more light in day times and to make the shell prominent in night times with light emittance. It is also to be noted that Mehrabs are a characteristic feature of Muslim architecture.

* Dimensions of the Mehrab are taken to be 18x12 in.
* The continuity on the shell at the mehrab cuts is to be maintained by providing a 4 inch wide and ¼ inch thick steel frame made air-tight and water proof by framing in glass fitted with rubber gaskets. Exact analysis considering the stress resultants around the cut has been made and only a nominal steel section is found necessary. (As shown in the figure).



E. Shell constants

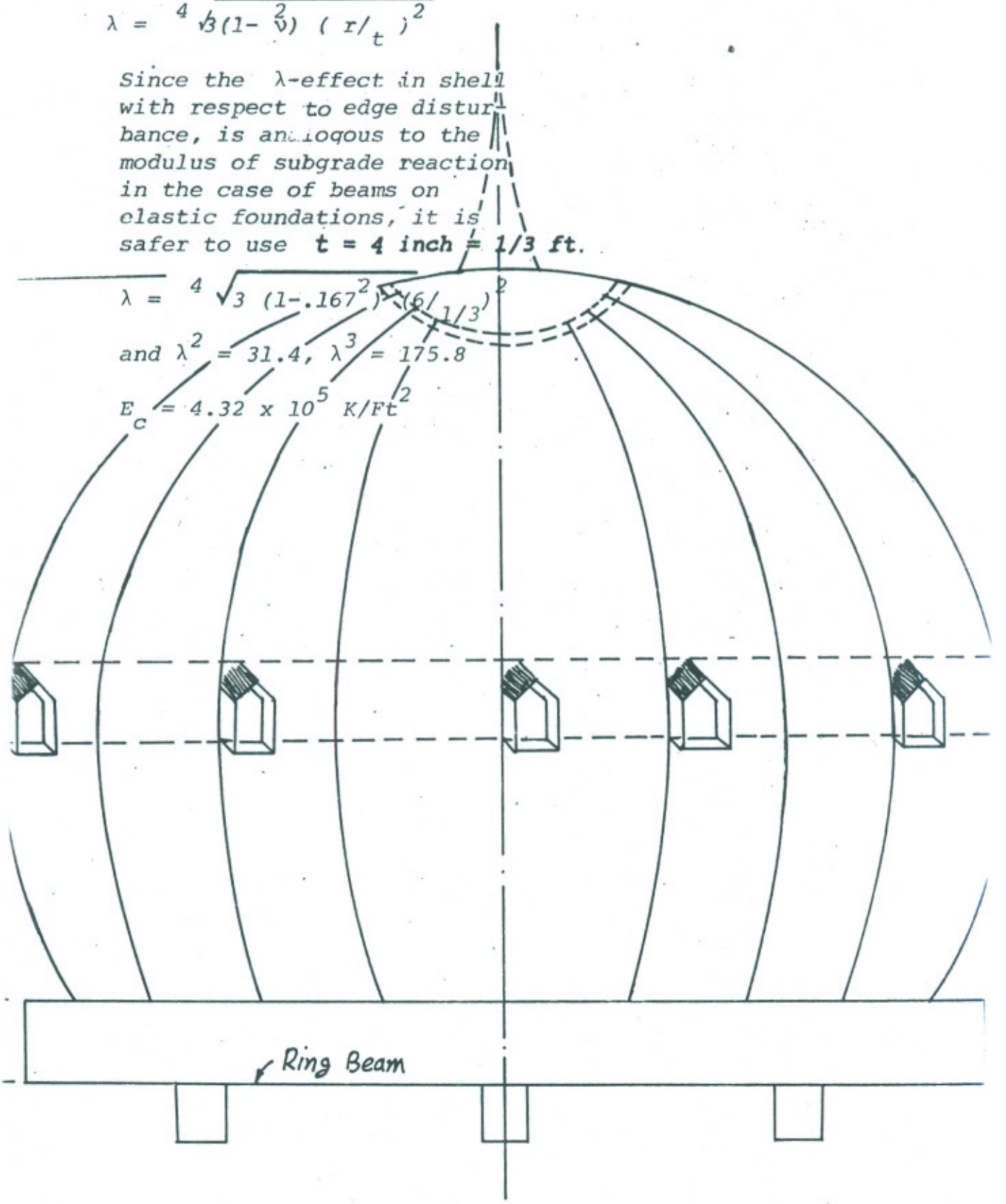
$$\lambda = \sqrt[4]{3(1-\nu^2)\,(r/t)^2}$$

Since the λ-effect in shell with respect to edge disturbance, is analogous to the modulus of subgrade reaction in the case of beams on elastic foundations, it is safer to use **t = 4 inch = 1/3 ft.**

$$\lambda = \sqrt[4]{3\,(1-.167^2)\,(6/_{1/3})^2}$$

and $\lambda^2 = 31.4$, $\lambda^3 = 175.8$

$E_c = 4.32 \times 10^5$ K/Ft^2



F. *Ring-beam design*

* *An edge ring-beam is considered to be the best supporting structure for spherical shell (Fig.2) Generally the design is started by assuming a reasonable preliminary section to be ultimately checked for the reactive forces due to shell. In the present design the preliminary section is taken as 18 in. X 18 in.*

* *The vertical deflection of the ring-beam[1] supported either continuously or at a number of points, may be neglected in its analysis.*

* *Fig 3(a) shows the ring-beam and the shell arrangement, wherein it is to be noted that the meridional shell reaction passes almost through the centre of gravity of the ring-beam, hence it does not produce any torsion.*

* *Fig. 3(b) shows the edge separated from the ring-beam and the restoring forces.*

* *The explanation of fig.3(b) is that the shell edge tends to move away from ring-beam under the effects of applied loads as well as temperature. The forces that are required to keep the shell edge and the ring-beam together are used to compute design forces.*

* *These edge-disturbances are evaluated by first finding the horizontal and angular membrane displacements of the shell edge (assuming it separated from the ring-beam) that are likely to occur under a given loading system and from these the restoring forces are computed as follows:*

The preliminary Section is taken to be 18 X18.

The shell thickness is made 6" at the junction.

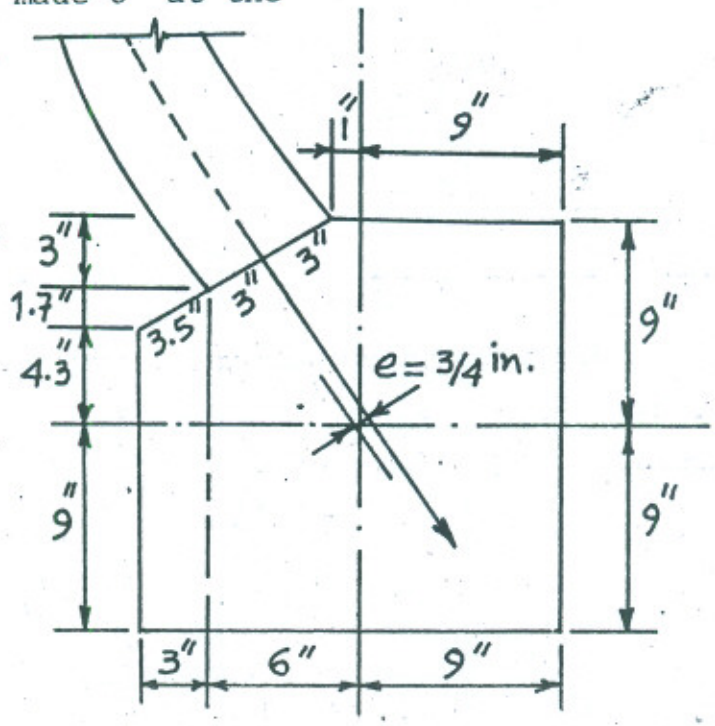


Fig. 3

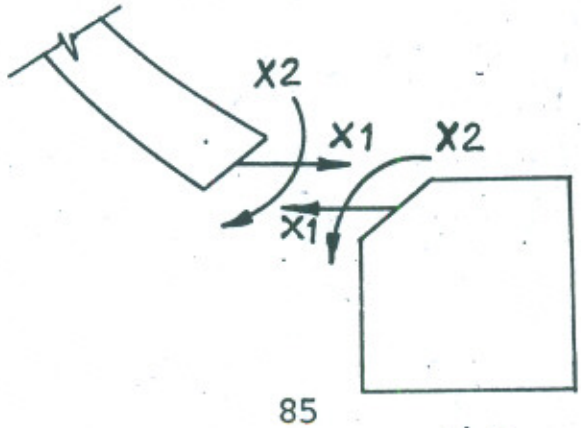
(a)

r_o at the parallel circle Φ = 120 is = 5.2 F_r $\beta = 30^o$

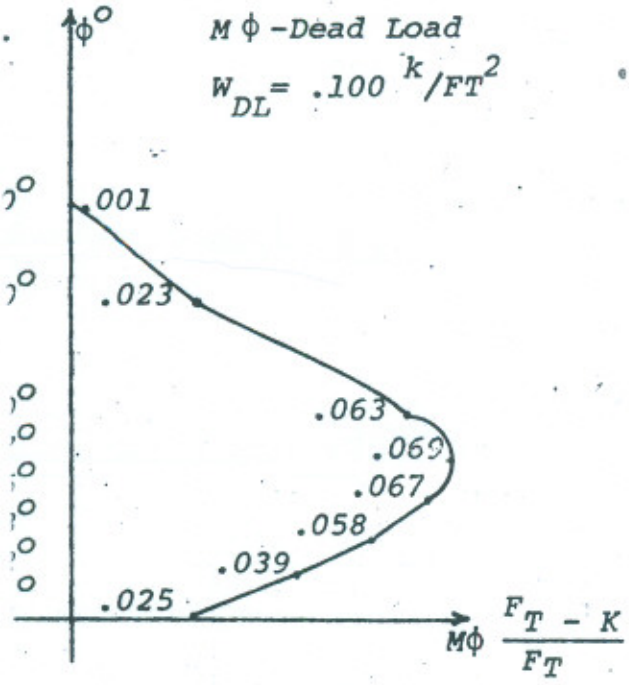
Area RING-BEAM $A_R = 2.1\ F_T^2$

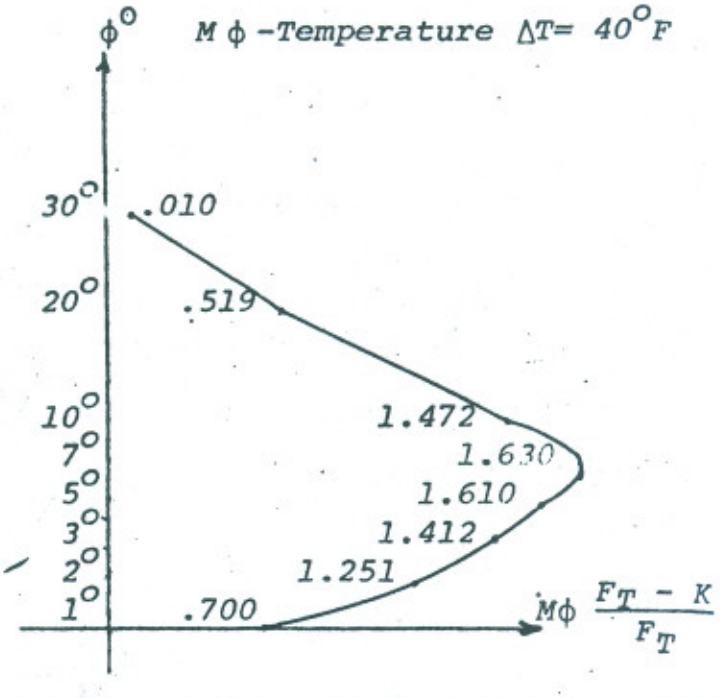
$E = 4.32 \times 10^5\ K/F_T^2$; $r_o^2 = 27.04$, $I_{xx} = .37\ F_T^4$.

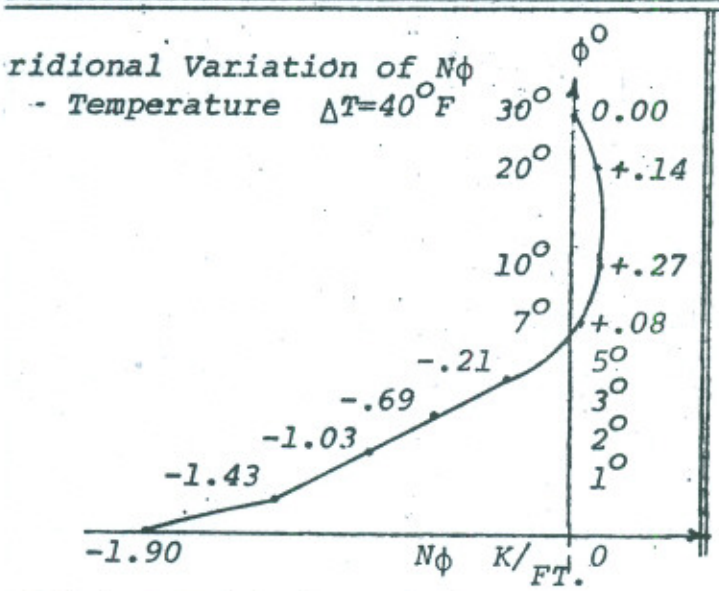
$\omega = 100$ lb/F_T^2 of shell surface in the Edge zone

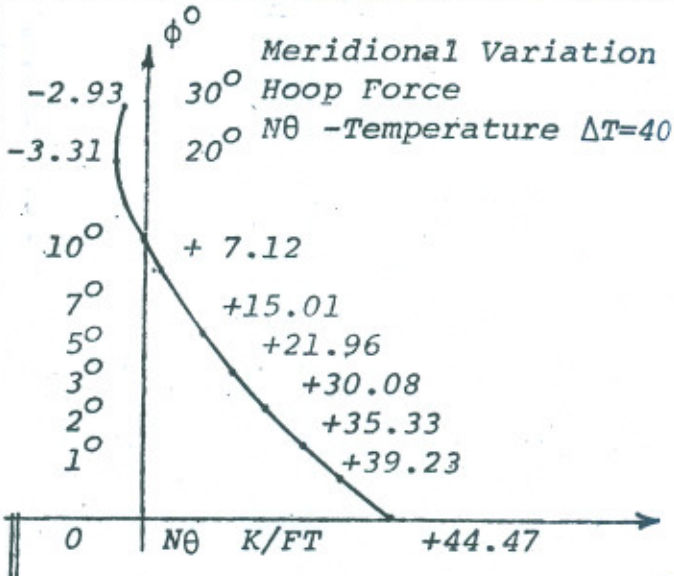


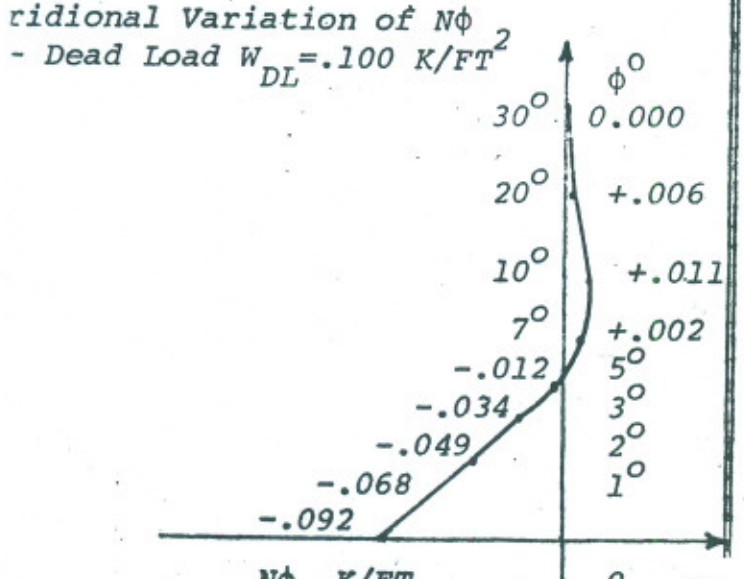
(b)

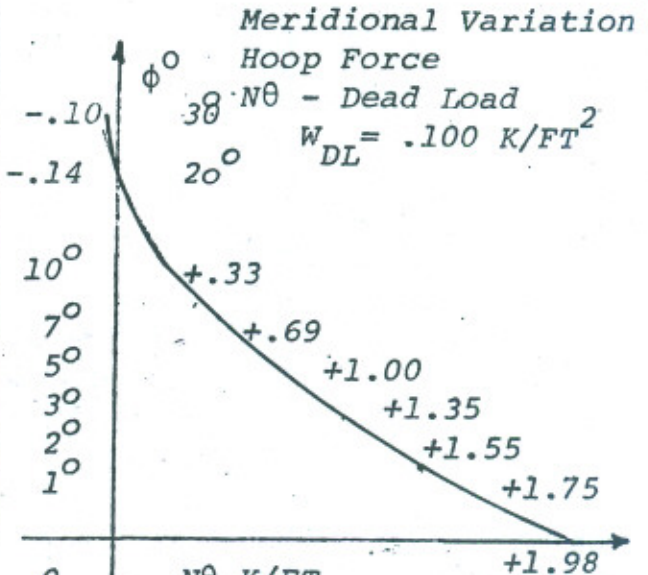

Mφ -Dead Load
W_{DL}= .100 k/FT²
φ°
.001
.023
.063
.069
.067
.058
.039
.025
Mφ $\frac{F_T - K}{F_T}$


Mφ -Temperature ΔT= 40°F
φ°
30°
.010
20°
.519
10°
1.472
7°
1.630
5°
1.610
3°
1.412
2°
1.251
1°
.700
Mφ $\frac{F_T - K}{F_T}$


ridional Variation of Nφ
- Temperature ΔT=40°F
φ°
30° 0.00
20° +.14
10° +.27
7° +.08
5° -.21
3° -.69
2° -1.03
1° -1.43
-1.90
Nφ K/FT. 0


Meridional Variation
Hoop Force
Nθ -Temperature ΔT=40
φ°
30° -2.93
20° -3.31
10° + 7.12
7° +15.01
5° +21.96
3° +30.08
2° +35.33
1° +39.23
0
Nθ K/FT
+44.47


ridional Variation of Nφ
- Dead Load W_{DL}=.100 K/FT²
φ°
30° 0.000
20° +.006
10° +.011
7° +.002
5° -.012
3° -.034
2° -.049
1° -.068
-.092
Nφ K/FT
0


Meridional Variation
Hoop Force
Nθ - Dead Load
W_{DL}= .100 K/FT²
φ°
30°
20° -.10
10° -.14
7° +.33
5° +.69
3° +1.00
2° +1.35
1° +1.55
+1.75
0
Nθ K/FT
+1.98

H. COMBINATION FOR DESIGN LOADS

TABLE OF STRESS-RESULTANTS AND STRESS-COUPLES: N_ϕ , N_θ , M_ϕ

Dead Load N_ϕ K/FT	Dead Load N_θ K/FT	Dead Load M_ϕ (FT-K)/FT	Temperature N_ϕ K/FT	Temperature N_θ K/FT	Temperature M_ϕ (FT-K)/FT	Wind N_ϕ K/FT	Wind N_θ K/FT	Design Loads N_ϕ K/FT	Design Loads N_θ K/FT	Design Loads M_ϕ (FT-K)/FT
-.195	-.195					-.000	-.000	-.195	-.195	
-.196	-.187					-.005	-.005	-.201	-.192	
-.201	-.166					-.010	-.011	-.212	-.177	
-.209	-.129					-.014	-.016	-.223	-.145	
-.221	-.078					-.017	-.023	-.238	-.101	
-.237	-.013					-.019	-.030	-.256	-.043	
-.260	+.065					-.019	-.038	-.279	+.065	
-.291	+.157					-.017	-.049	-.308	+.157	
-.332	+.265					-.011	-.062	-.343	+.265	
-.390	+.390	0.000	0.00	-2.39	+.010	-.00	-.080	-.39	+.390	+.011
+.000	-.100	+.001								
+.006	-.14	+.023	$\pm$.14	$\mp$3.31	+.519	+.018	-.105	+.164	-3.555	+.542
+.011	+.33	+.063	$\pm$.27	$\pm$7.12	+1.472	+.049	-.144	+.33	+7.450	+1.535
+.002	+.69	+.069	$\pm$.08	$\pm$15.01	+1.630			+.082	+15.700	+1.699
-.012	+1.00	+.067	$\mp$.21	$\pm$21.96	+1.610			-.222	+22.960	+1.677
-.034	+1.35	+.058	$\mp$.69	$\pm$30.08	+1.412			-.724	+31.43	+1.470
-.049	+1.55	+.050	$\mp$1.03	$\pm$35.33	+1.251			-1.079	+36.88	+1.301
-.068	+1.75	+.039	$\mp$1.43	$\pm$39.23	+1.011			-1.498	+40.98	+1.050
-.092	+1.98	+.025	$\mp$1.90	$\pm$44.47	+.700	+.104	-.208	-1.992	+46.46	+.725

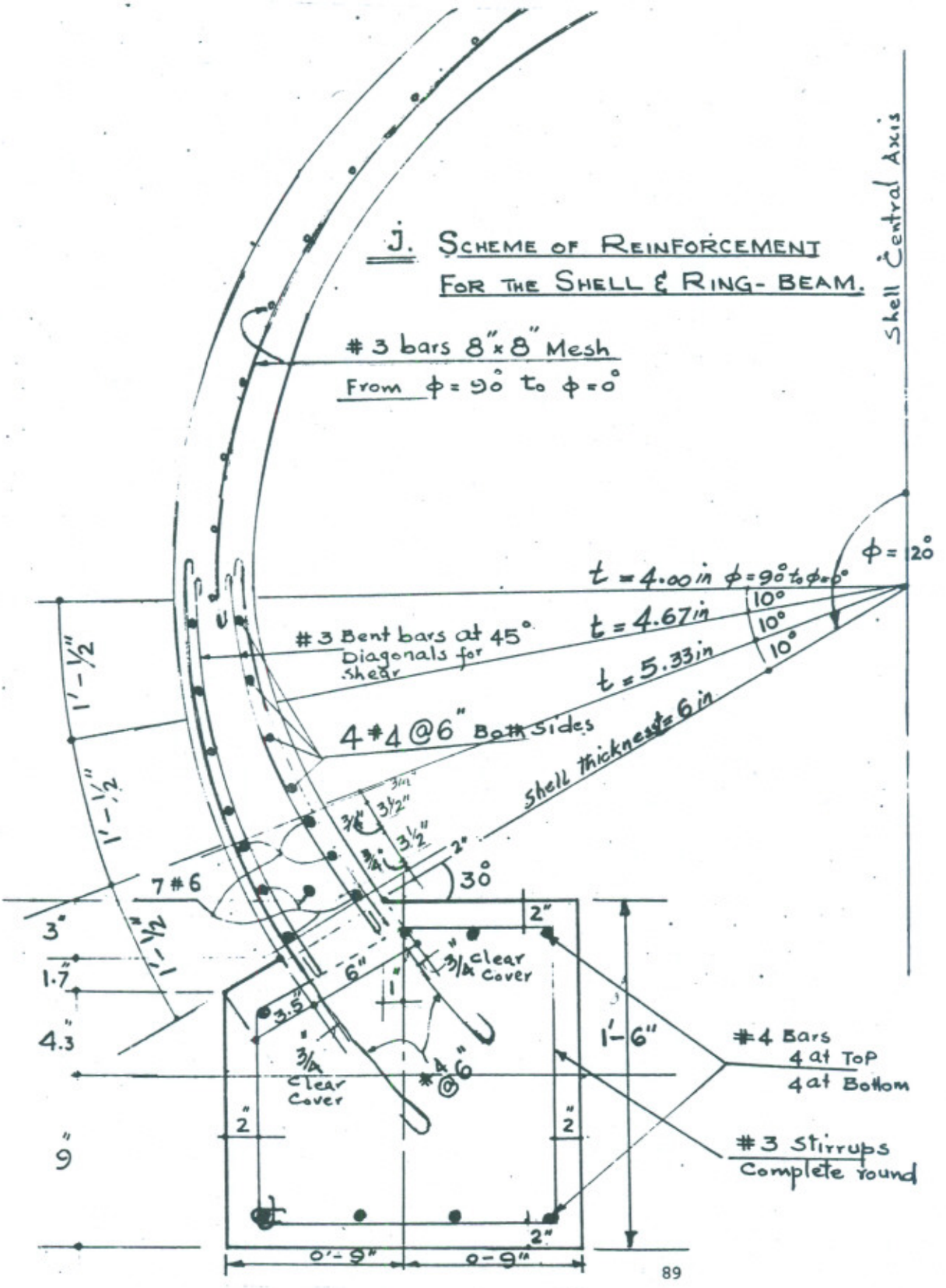
REINFORCEMENT FOR THE SHELL

Segment	Ψ^o	Meridional Force Reinforcement N_ϕ $K/_{FT}$	Meridional Bending Moment Reinforcement M_ϕ $\frac{FT-K}{FT}$	Hoop-Force Reinforcement N_θ $K/_{FT}$
1'-½"	0^o-10^o	$\frac{-1.992 \times .63}{2}$ = -.63K Steel Bars not needed Concrete is sufficient	$(M_\phi)_{max} = +1.699 \frac{FT-K}{FT}$ d = 4.5 in. $A_s = \frac{1.699 \times 12}{20 \times 7/8 \times 4.5}$ $= .254 \frac{in^2}{FT}$ Use 2 #4, A_s=.40 in^2 @ 6" c.c.	Total Force in Seg Seg. 1 $\frac{+46.46+7.45}{2}(1.04)$ = +56.06K $\therefore A_s = \frac{56.06}{20} = 2.803\ in^2$ Use 7#6, A_s = 3.09 in^2
1'-½"	10^o-20^o	- do -	Use 2#4 @ 6"c.c.	Minimum Steel = .0035bd =.0035x12x5.25 =.22 in^2 Use 2#4, A_s=.40 in^2
1'-½"	20^o-30^o	- do -	Use 2#4 @ 6" c.c.	Use 2 # 4

In the upper portion ϕ 0^o - 90^o A mesh of # 3 Bars 8"x8" will be used: $A_s/_{FT}$ = 12/8 (.11) = .165 in^2;

$(A_s)_{MIN}$ = .0035 bd = .0035x12x3.25 = .136 in^2 < .165 in^2

In the Edge - Zone $\Psi = 0^o - 30^o$, Diagonal Reinforcement for Shear will also be used


J. SCHEME OF REINFORCEMENT FOR THE SHELL & RING-BEAM.
#3 bars 8"x8" Mesh
From φ = 90° to φ = 0°
Shell Central Axis
φ = 20°
t = 4.00 in φ = 90° to φ = 0°
t = 4.67 in
t = 5.33 in
10°
10°
10°
#3 Bent bars at 45° Diagonals for shear
4 #4 @6" Both sides
Shell thickness = 6 in
1'-1½"
1'-1½"
1'-1½"
7 #6
3½"
3½"
3½"
2"
30°
3"
1.7"
4.3"
9"
6"
1"
3.5"
¾" clear Cover
¾" clear Cover
#4 @6"
2"
2"
2"
2"
1'-6"
#4 Bars
4 at Top
4 at Bottom
#3 Stirrups
Complete round
0'-9"
0-9"

K. *Scheme of dome support*

The scheme of support for the dome along with the ring-beam is shown below. A grid system of RCC beams has been provided. Near the ends the grid beams are concealed in the roof slab so that apparently a disconnected and unsupported grid system of beams is visible to an on looker. (Fig.4).

7.0 RCC SHELL DOMES A BETTER SUBSTITUTE FOR MASONRY DOMES

RCC shell design for masjid domes is much more economical as compared with masonry domes because;

* *the quantity of the material for the same volume is much less.*
* *the use of a temporary earth mound as a supporting frame instead of wooden forms, reduces the cost appreciably.*
* *comparatively much smaller sections of ring-beam are found adquate.*
* *the structural system supporting the ring-beam and dome shell is, likewise, much smaller in size and hence economical.*
* *from the aesthetic considrations also RCC shells lend themselves more to decoration and desing.*
* *being lighter in weight even large domes will cause comparatively much less rotation and deflection in the supporting structure, and hence assure more safety and stability.*

8.0 REFERENCES:

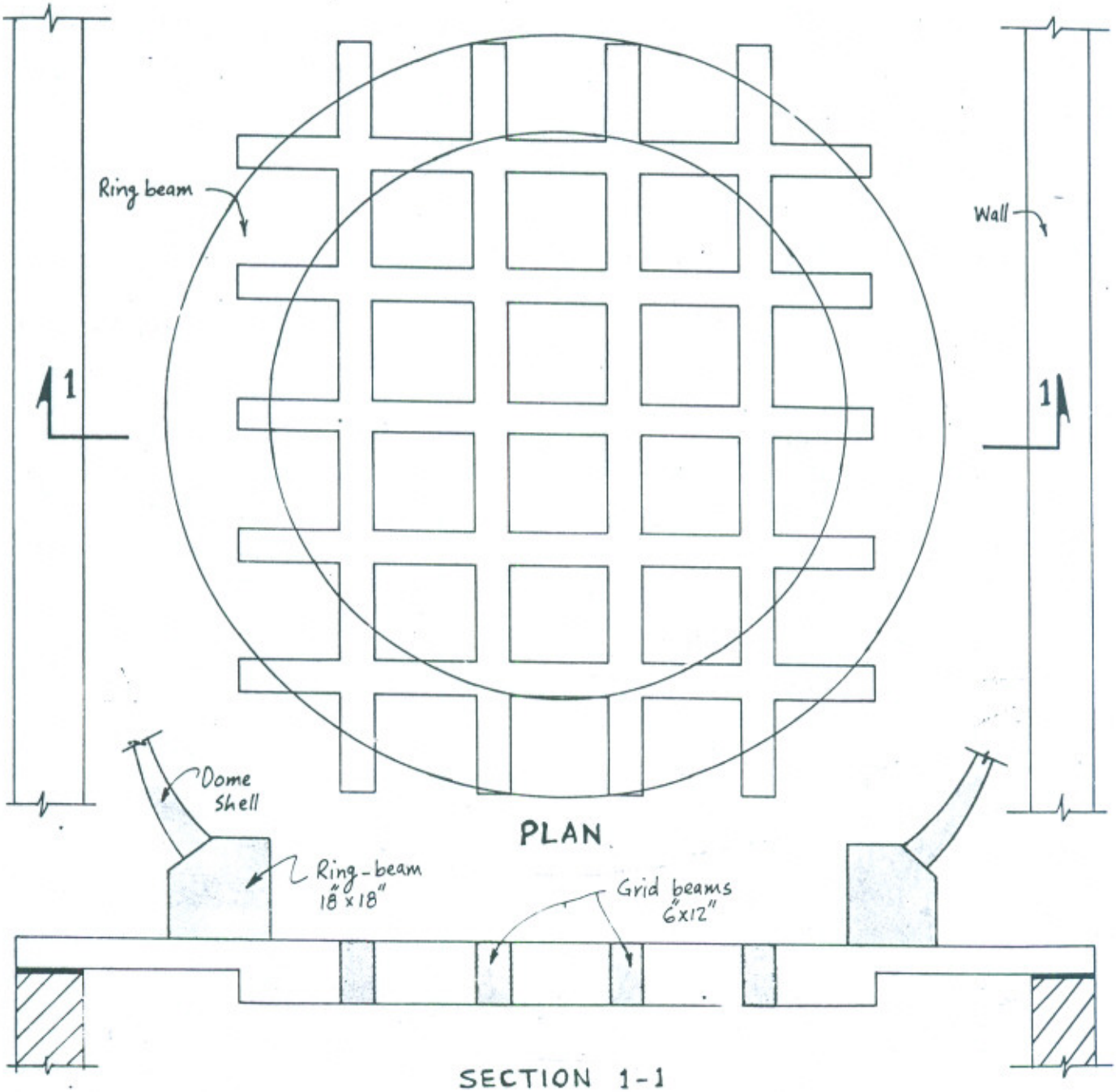
1. Timoshenko, S.P, and Woinowsky- Kricger: " Theory of Plates and Shells ".

   2nd ed., McGraw-Mill Book Company,New York,1959.

2. Billington, D.P., "Thin Shell Concrete Structures" McGraw-Hill Book Company, New York, 1965.

3. Harry Kraus, "Thin Elastic Shells " John Wiley & Ions Inc. New York, 1967.

(Fig.4)


Ring beam
Wall
1
1
Dome Shell
PLAN
Ring-beam 18"x18"
Grid beams 6"x12"
SECTION 1-1

ECONOMICAL DESIGN OF MASJID DOMES

APPENDIX

(DESIGN COMPUTATIONS)

SUPER-SCRIPTS: A "LETTER-SUPERSCRIPT" INDICATES • NATURE OF CAUSE OR • A REFERENCE TO
A "MARK-SUPERSCRIPT" INDICATES • A DISTINCTION

SUBSCRIPTS: A "LETTER-SUBSCRIPT" INDICATES • THE LOCATION OF EFFECT OR • A DISTINCTION
A "NUMERAL-SUBCRIPT" INDICATES • A DISTINCTION

SYMBOLS:

A_R	CROSS-SECTIONAL AREA OF RING-BEAM
A_S	CROSS-SECTIONAL AREA OF STEEL
DL	DEAD LOAD
E	MODULUS OF ELASTICITY
$\overset{\circ}{F}$	
F	FLEXIBILITY MATRIX
F_Y	YIELD STRESS OF STEEL
H_R	HORIZONTAL FORCE NORMAL TO RING-BEAM CROSS-SECTION
M_R	BENDING MOMENT IN THE RING-BEAM
M_ϕ	MERIDONAL BENDING MOMENT IN THE SHELL
M_θ	LATITUDINAL BENDING MOMENT IN THE SHELL
N_ϕ	MERIDONAL FORCE IN THE SHELL
N_θ	HOOP FORCE IN THE SHELL
$N_{\phi\theta} = N_{\theta\phi}$	LATITUDINAL SHEAR FORCE IN THE SHELL
X_1	HORIZONTAL REDUNDANT FORCE AT THE SHELL-EDGE
X_2	MOMENT AT THE SHELL-EDGE

I_R	MOMENT OF INERTIA OF THE RING-BEAM
$I_{xx} = I_R$	
f_s	ALLOWABLE STRESS OF STEEL
f_c'	YIELD STRESS OF CONCRETE
f_c	ALLOWABLE STRESS OF CONCRETE IN BENDING.

b	bending
e	eccentricity
f_{11}	Linear flexibility due to Unit force
f_{12}	Linear flexibility due to Unit Moment
f_{21}	Angular flexibility due to Unit force
f_{22}	Angular flexibility due to Unit Moment
h	height of the ring-beam
h'	height of a spherical Segment
m	membrane
h_i'	inside height of a spherical Segment

h_o'	outside height of a spherical segment
p_H	horizontal wind pressure
q_{cr}	Critical buckling Load for a ring
r	radius of the Shell to the middle Surface
r_o	radius at the Shell-edge to the middle Surface
r'	radius of a spherical segment
r_i'	inside radius
r_o'	outside radius
s	Shell
t 97	Thickness of the shell

T temperature
u_j absolute movement of point j
v_j meridonal movement of point j
w_j radial movement of point j
jd lever arm for internal moment in a concrete beam

R ring beam
w wind
means the word "number"
wt. weight
vol. volume
w_θ weight per foot on slab along the ring-beam circumference.

α_t COEFFICIENT OF THERMAL EXPANSION $= 6 \times 10^{-6}$

β ANGLE BELOW THE HEMISPHERICAL PORTION IN THE DOME-SHELL

λ A CONSTANT $= \sqrt[4]{3(1-\nu^2)(r/t)^2}$

ψ $= \beta$ MEASURED IN REVERSE DIRECTION

ϕ_c SEMICENTERAL DOME-ANGLE $\doteq 120^\circ$

ϕ MERIDONAL ANGLE

θ LATITUDINAL ANGLE FOR PARALLE HORIZONTAL PLANES AT $\phi_{1\ldots n}$

ν POISSON'S RATIO

Δ_1 LINEAR DEFORMATION IN A HORIZONTAL PLANE

Δ_2 ANGULAR DEFORMATION

ΔT TEMPERATURE DIFFERENTIAL

Computation of Displacements

Shell assumed severed from the Ring-Beam

Membrane displacements of the Shell

Dead Load

1. $\Delta_1^s = \frac{wr^2}{Et}\left(\frac{1+\nu}{1-\sin\beta} + \sin\beta\right)\cos\beta$

$= \frac{.100\times 6^2}{E\times \frac{1}{2}}\left(\frac{1+.167}{1-.500}+.500\right)(.866)$

$= \frac{17.63}{E}$

2. $\Delta_2^s = \frac{wr}{Et}(2+\nu)\cos\beta = \frac{.100\times 6}{E\times\frac{1}{2}}(2+.167)(.866) = \frac{2.26}{E}$

Temperature $\Delta T = +40^\circ F$ $\alpha_t = 6.0\times 10^{-6}\frac{F_T}{F_T - F^\circ}$

3. $\Delta_1^s = 6\times 10^{-6}\times 40\times 5.2 = 1248\times 10^{-6}$

4. $\Delta_2^s = 0.00$

Membrane displacements of the RING-BEAM for Dead Load

5. $\Delta_1^R = \frac{wr\sin\beta(r_0)^2}{(1-\sin\beta)A^R E} = \frac{.100\times 6\times .500\times 27.04}{(1-.500)\times 2.10E} = \frac{7.73}{E}$

6. $\Delta_2^R = 0.00$

Horizontal Gap between Shell-Edge & Ring-Beam due to

7. Dead Load: $\Delta_1 = \Delta_1^s + \Delta_1^R = -\frac{17.63}{E} - \frac{7.73}{E} = -\frac{25.36}{E}$

And the Angular-Gap due to Dead Load:

8. $\Delta_2 = \Delta_2^s + 0.0 = \Delta_2^s = +\frac{2.26}{E}$

9. INFLUENCE COEFFICIENTS $\quad \begin{matrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{matrix}$

$$f_{11} = f_{11}^{S} + f_{11}^{R} \;,\quad f_{21} = f_{21}^{S} + f_{21}^{R} \;,$$

$$f_{12} = f_{12}^{S} + f_{12}^{R} \;,\quad f_{22} = f_{22}^{S} + f_{22}^{R}$$

10. Numerical Computation of Flexibilities

$$f_{11} = \frac{2r\lambda\cos^2\beta}{Et} + \frac{4r_0^2}{A_R E} = \frac{100.8}{E} + \frac{4(27.04)}{2.10E} = \frac{100.8}{E} + \frac{51.5}{E} = +\frac{152.3}{E}$$

$$f_{21} = -\frac{2\lambda^2\cos\beta}{Et} + \frac{3r_0^2}{h/2\,A_R E} = -\frac{108}{E} + \frac{51.5}{E} = -\frac{56.5}{E}$$

$$f_{12} = f_{21} = -\frac{56.5}{E} \;;\; f_{22} = \frac{4\lambda^3}{Et^3} + \frac{3r_0^2}{(h/2)^2 A_R E} = +\frac{234.4}{E} + \frac{68.67}{E} = +\frac{303.1}{E}$$

Solution for Redundants X_1 and X_2

The above computed influence-Coefficients may be represented by a matrix $[F]$ thus: $[F] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$ ← Flexibility Matrix

Now if Δ_1 and Δ_2 are gaps in the directions of X_1 and X_2 due to any Condition of Loading

Then $\quad f_{11}X_1 + f_{12}X_2 + \Delta_1 = 0$

and $\quad f_{21}X_1 + f_{22}X_2 + \Delta_2 = 0$

or $\quad \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -\Delta_1 \\ -\Delta_2 \end{bmatrix}$

And the redundants $\{X\}$ may be found as: $\{X\} = [F]^{-1}\{-\Delta\}$

11. Flexibility Matrix

$$[F] = \frac{1}{E}\begin{bmatrix} +152.3 & -56.5 \\ -56.5 & +303.1 \end{bmatrix}$$

The inverse Matrix

$$[F]^{-1} = \begin{bmatrix} +.00705 & +.00131 \\ +.00131 & +.00354 \end{bmatrix}$$

"E" cancels out in DL

Calculation of Redundants

12. Due to Dead Load

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} +.0071 & +.0013 \\ +.0013 & +.0035 \end{bmatrix}\begin{bmatrix} +25.36 \\ -2.26 \end{bmatrix} = \begin{bmatrix} +.177 \\ +.025 \end{bmatrix}$$

13. Due to Temperature

"E" Considered

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} +.0071 & +.0013 \\ +.0013 & +.0035 \end{bmatrix}\begin{bmatrix} +539.1 \\ 0.00 \end{bmatrix} = \begin{bmatrix} +3.83 \\ +.70 \end{bmatrix}$$

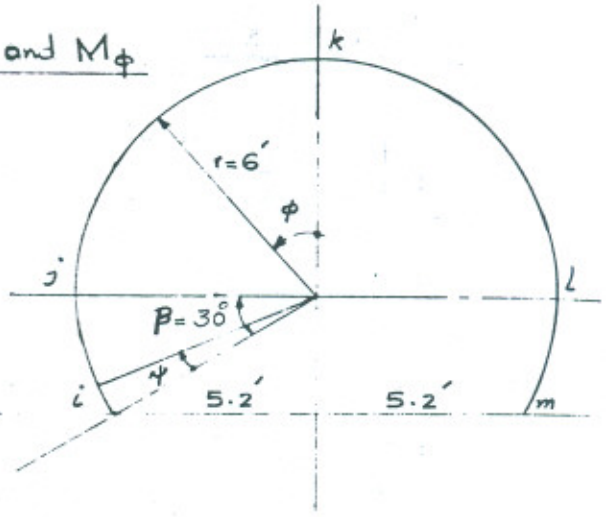
M Computation of N_ϕ, N_θ and M_ϕ

Due to Dead Load:

Zone k-j $\phi = 0^\circ \rightarrow \pi/2$

$\omega_{DL} = .065\ ^{k}/_{FT^2}$, $r = 6.0'$

Temperature effect in the zone k-j and k-l is found to be "ZERO" due to the fast damping effect.



ϕ°	$N^m_\phi = -\frac{wr}{1+\cos\phi}$	$N^m_\theta = wr\left[\frac{1}{1+\cos\phi} - \cos\phi\right]$	$M^m_\phi = 0$
10°	-.196	-.187	0.000
20°	-.201	-.166	
30°	-.209	-.129	
40°	-.221	-.078	
50°	-.237	-.013	
60°	-.260	+.065	
70°	-.291	+.157	
80°	-.332	+.265	
90°	-.390	+.390	0.000

ψ DEG.	ψ RAD.	$\lambda\psi$ DEG.	$\lambda\psi$ RAD.	$e^{-\lambda\psi}$	$\sin(\lambda\psi)$	$\sin(\lambda\psi-45^\circ)$	$\sin(\lambda\psi+45^\circ)$	$\sin(\lambda\psi-90^\circ)$	$\tan(\beta-\psi)$	$(\beta-\psi)$
0	0.00	0.00	0.00	1.000	0.000	−.707	+.707	−1.000	.577	30°
1	.0175	5.60	.097	.900	.0987	−.640	+.773	−.995	.554	29°
2	.0349	11.20	.195	.823	.1942	−.556	+.831	−.981	.532	28°
3	.0524	16.80	.293	.744	.2890	−.472	+.880	−.957	.509	27°
5	.0873	28.00	.490	.613	.4694	−.292	+.956	−.883	.466	25°
7	.1220	39.20	.685	.503	.6320	−.101	+.995	−.775	.424	23°
10	.1750	56.00	.977	.375	.8290	+.191	+.982	−.559	.364	20°
15	.2620	84.00	1.465	.232	.9945	+.629	+.777	−.104	.268	15°
20	.3500	112.00	1.955	.141	.9272	+.920	+.391	+.375	.176	10°
25	.4360	140.00	2.442	.087	.6428	+.996	−.087	+.766	.087	5°
30	.5250	168.00	2.930	.053	.2079	+.838	−.545	+.978	0.000	0.0°

TABLE OF THE FUNCTIONS USED

$\lambda = 5.60$

$\beta = 30^\circ$

4. Computation of N_ϕ, N_θ and M_ϕ due to Dead Load and Temperature.

Zone i-j

$$N_\phi^{*b} = \frac{2\lambda}{\gamma}\tan(\beta-\psi)e^{-\lambda\psi}\sin(\lambda\psi)X_2 + \sqrt{2}\tan(\beta-\psi)e^{-\lambda\psi}\cos\beta\sin(\lambda\psi-45)X_1$$

$$N_\theta^{*b} = -\frac{2\sqrt{2}\lambda^2}{\gamma}e^{-\lambda\psi}\sin(\lambda\psi-45)X_2 - 2\lambda\cos\beta e^{-\lambda\psi}\sin(\lambda\psi-90)X_1$$

$$M_\phi^{*b} = \sqrt{2}e^{-\lambda\psi}\sin(\lambda\psi+45)X_2 + \frac{\gamma}{\lambda}e^{-\lambda\psi}\cos\beta\sin(\lambda\psi)X_1$$

Knowing that $X_1^T = \pm 3.85 \frac{K}{F_T}$, $X_2^T = \pm .700 \frac{F_T-K}{F_T}$; $X_1^\omega = .177 \frac{K}{F_T}$, $X_2^\omega = .025 \frac{F_T-K}{F_T}$

ψ°	N_ϕ^b K/FT		N_θ^b K/FT		M_ϕ^b $\frac{F_T-K}{F_T}$	
Deg.	DL $\omega = .100 \frac{K}{F_T}$	Temp $\Delta T = \pm 40^\circ F$	DL $\omega = .100 \frac{K}{F_T}$	Temp. $\Delta T = \pm 40^\circ F$	DL. $\omega = .100 \frac{K}{F_T}$	Temp. $\Delta T = \pm 40^\circ F$
0°	-.092	$\mp$ 1.90	+ 1.98	$\pm$ 44.47	+ .025	+ .700
1°	-.068	$\mp$ 1.43	+ 1.75	$\pm$ 39.23	+ .039	+ 1.011
2°	-.049	$\mp$ 1.03	+ 1.55	$\pm$ 35.33	+ .050	+ 1.251
3°	-.034	$\mp$ .69	+ 1.35	$\pm$ 30.08	+ .058	+ 1.412
5°	-.012	$\mp$ .21	+ 1.00	$\pm$ 21.96	+ .067	+ 1.610
7°	+.002	$\pm$ .08	+ .69	$\pm$ 15.01	+ .069	+ 1.630
10°	+.011	$\pm$ .27	+ .33	$\pm$ 7.12	+ .063	+ 1.472
20°	+.006	$\pm$ .14	- .14	$\mp$ 3.31	+ .023	+ .519
30°	0.000	0.00	- .10	$\mp$ 2.39	+ .001	+ .010

In making the above calculations, the expressions for N_ϕ, N_θ and M_ϕ were first simplified by substituting for $\lambda, \gamma, \cos\beta, X_1, X_2$ as under.

Dead Load:

$$N_\phi = e^{-\lambda\psi}\tan(\beta-\psi)\left[.047\sin(\lambda\psi) + .22\sin(\lambda\psi-45)\right]$$

$$N_\theta = e^{-\lambda\psi}\left[-.37\sin(\lambda\psi-45) - 1.72\sin(\lambda\psi-90)\right]$$

$$M_\phi = e^{-\lambda\psi}\left[.035\sin(\lambda\psi+45) + .164\sin(\lambda\psi)\right]$$

Temperature

$$N_\phi = e^{-\lambda\psi}\tan(\beta-\psi)\left[1.31\sin\lambda\psi + 4.67\sin(\lambda\psi-45)\right]$$

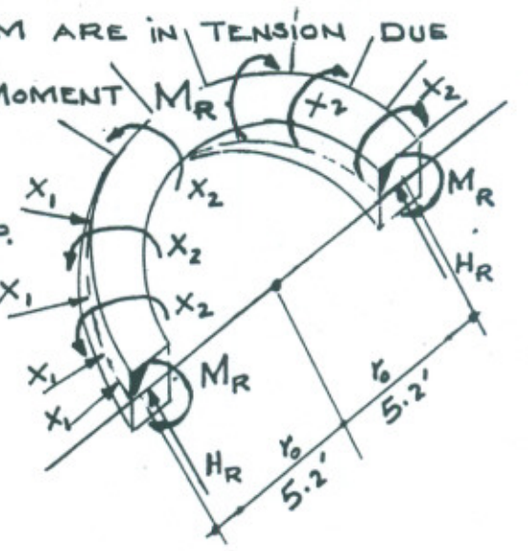
$$N_\theta = e^{-\lambda\psi}\left[-10.35\sin(\lambda\psi-45) - 37.15\sin(\lambda\psi-90)\right]$$

$$M_\phi = e^{-\lambda\psi}\left[1.00\sin(\lambda\psi+45) + 3.55\sin(\lambda\psi)\right]$$

O. Ring-Beam Reinforcement

The Ring-Beam is in Compression due to the direct thrust force H_R acting normally at its cross-section

The top fibers of the Ring-Beam are in Tension due the cross-sectional Bending Moment M_R



Now $H_R = (X_1^T + X_1^{DL})\, r_0$

$= 4.01 \times 5.2 = 20.85^{K}$ Comp.

$M_R = (X_2^T + X_2^{DL})\, r_0$

$= .725 \times 5.2 = 3.77$ Ft-K

Check for Ring-Buckling

$$q_{cr} = \frac{3EI_R}{r_R^3} = {}^{K}/_{FT}$$

$$= \frac{3 \times 4.32 \times 10^5 \times .37}{(4.9)^3} = 4075\ {}^{K}/_{FT} \text{ vs } 4.01\ {}^{K}/_{FT}\text{; Quite Safe}$$

Compressive Strength of the Ring-Beam $= A_R f_c$

$= 2.10 \times .750 \times 144 = 226.8$ K vs 20.85 K; Quite Safe

Bending: $A_s = \frac{M_R}{f_s j d} = \frac{3.77 \times 12}{20 \times 7/8 \times 15} = .172\ in^2.$

$(A_s)_{min} = .0025bd = .0025 \times 18 \times 15 = .670\ in^2 > .172\ in^2$

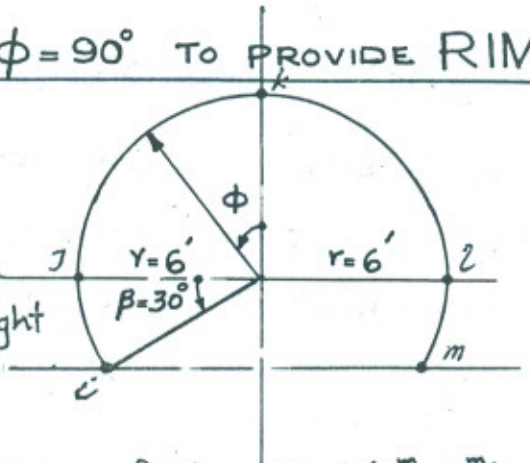
Use $(A_s)_{min} = .670\ in^2$. Put 4 #4 at top and 4 #4 at Bottom $\Big]\ A_s = .88\ in^2$

Also Use #3 Stirrups Complete Round To Account for Torsion also.

CHECK FOR DEFLECTION AT $\phi = 90°$ TO PROVIDE RIM

Considering the point "j"

If v_j and w_j represent its meridonal and radial movements, respectively, under the self weight of the Shell Then:



$$v_j = \sin\phi \left(\int \frac{f(\phi)}{\sin\phi} d\phi + C \right), \text{ where } f(\phi) = \frac{r(1+\nu)(N_\phi^m - N_\theta^m)}{Et}$$

$$w_j = v_j \cot\phi - \frac{r}{Et}(N_\theta^m - \nu N_\phi^m)$$

Now after Integration and Evaluation of the Constant C"

$$v_j = \frac{r^2\omega(1+\nu)}{Et}\left[\frac{1}{1-\sin\beta} - \frac{1}{1+\cos\phi} + \ln\frac{1+\cos\phi}{1-\sin\beta} \right]\sin\phi$$

$$= \frac{6^2 \times .065 \times 1.167}{E(1/3)}\left[\frac{1}{1-.5} - \frac{1}{1+0} + \ln\frac{1+0}{1-.5} \right](1) = \frac{13.87}{E}$$

$$w_j = \frac{13.87}{E}(0) - \frac{r^2\omega}{Et}\left(\frac{1+\nu}{1+\cos\phi} - \cos\phi \right)$$

$$= 0 - \frac{6^2 \times .065}{E(1/3)}\left(\frac{1.167}{1+0} - 0 \right) = -\frac{8.19}{E}$$

Thus the absolute outward movement of "j" is:

$$u_j = \sqrt{v_j^2 + w_j^2} = \frac{1}{E}\sqrt{(13.87)^2 + (-8.19)^2} = \frac{16.11}{E}$$

$$= \frac{16.11 \times 12}{4.32 \times 10^5} = .0004475 \text{ inch.}$$

No Need For RIM

Q. Self Weight of the Shell

Volume of a Sphere $= \frac{4}{3}\pi r^3$

Volume of a Sphere-Segment $= \frac{1}{3}\pi h'^2(3r-h')$, h' = Height of the sphere-segment

The thickness of the Shell for the most part is $= 4'' = \frac{1}{3}'$. The radius of the middle surface of the shell is $= 6$ Ft. If the subscript "0" designates the out-side dimension and "i" the inside clear dimension Then:

$r_o' = 6.167$ Ft., $r_i' = 5.833$ Ft., $h_o' = 3.167$ Ft. and $h_i' = 2.833$ Ft

$$\therefore (\text{Vol})_{net} = \left[\left\{\frac{4}{3}\pi r_o'^3 - \frac{1}{3}\pi h_o'^2(3r_o' - h_o')\right\} - \left\{\frac{4\pi}{3}r_i'^3 - \frac{1}{3}\pi h_i'^2(3r_i' - h_i')\right\}\right]$$

$$= \left[\left\{\frac{4}{3}\pi 6.167^3 - \frac{1}{3}\pi 3.167^2(3\times 6.167 - 3.167)\right\} - \left\{\frac{4}{3}\pi 5.833^3 - \frac{1}{3}\pi 2.833^2(3\times 5.833 - 2.833)\right\}\right]$$

$$= \frac{\pi}{3}\left[\left\{4\times 234 - 10(18.5 - 3.167)\right\} - \left\{4\times 198.5 - 8.03(17.5 - 2.833)\right\}\right]$$

$$= 35.48\pi \text{ Ft.}^3$$

And Wt. $= 35.48\pi \times .150 = \underline{5.322\pi}$ K $= 17.35$ K

Area of the Shell-Surface $= 4\pi r^2 - 2\pi r h'$

Where r is the radius to the middle-Surface $= 6.0$ Ft

And h' is the height of the Sphere-Segment from the middle-Surface $= 3.0$ Ft.

Now $A = 4\pi\times 6^2 - 2\pi\times 6\times 3 = \underline{108\pi}$ Ft.^{2} $= 339.42$ Ft.^{2}

Check at wt/Ft.^{2}: $w = \frac{5.322\pi}{108\pi} = .0492$ K/Ft.^{2} vs $.050$ K/Ft.^{2}

As assumed in the begining for 4" Shell-thickness.

R. Self Weight of the Ring-Beam

Total Weight of the Ring-Beam $= A_R(\pi d_R)(.150)$

$= 2.10 \times \pi \times 9.8 \times .150 = 3.087\pi$ k

$= 9.70$ k

Wt./$_{FT}$ on the Roof-Slab along the Ring-Beam Circumference using mean radius i.e. $(5.2+4.9)/2 = 5.05$ FT. at $\phi = 120^\circ$

Thus $\omega_\theta = 2.10 \times 1.00 \times .150 + \frac{5.322\pi}{\pi \times 10.10} = .315 + .526 = .841$ K/$_{FT}$

Since the shell-thickness varies in the Edge-Zone from 6" at $\phi = 120^\circ$ to 4" at $\phi = 90^\circ$ and thence on being constant equal to 4" to $\phi = 0^\circ$, it is recommended that ω_θ be taken $= 1.000$ K/$_{FT}$ along the Ring-Beam Circumference