And this minimum must occur at section X and is equal to the energy of flow at the critical depth.

3. The total energy line above the weir may be determined accurately and without difficulty. The critical depth may be calculated and hence the energy of flow. This may be set off above the downstream edge of the crest and will give another point on the total energy line.

4. The total energy line over the glacis has now to be determined. The author has examined the five series of experiments upon which the paper is based and extracted the "loss of energy" over the glacis in terms of "total energy available" above the weir.

The Author holds that this is a more accurate and more generally applicable ratio than the one customary in this Department (*viz.*, in terms of H the head acting).

The results are given in Table 12 and average 15 per cent. for the particular combination and dimensions used in the investigation.

5. At this stage the author was faced by an unexpected phenomenon. The loss (presumably due to shock) in passing from the toe of the glacis to the flat bed of the channel downstream, was worked out in a similar manner and was found to average 20 per cent.

6. Beyond the toe of the glacis the loss of total energy is the loss due to friction and determines the slope of the total energy line.

The formula used for calculation of the friction is the Chezy formula as being quite the simplest.

s=slope of total energy line

m=mean hydraulic depth

$$=\frac{\text{cross sectional area}}{\text{wetted perimeter}}$$

V=Mean velocity

C=a constant

then $s=\frac{C V^2}{m}$.

Table 13 sets out the calculations for C for the five series of experiments.

average value=143.

7. A "formal" or theoretical total energy line can now be drawn. (see fig. 23).

Select points A B C D for treatment described in the foregoing paragraphs 3 to 6.

(i) On the vertical A set off the total energy of flow of the stream above the weir, a height A a.

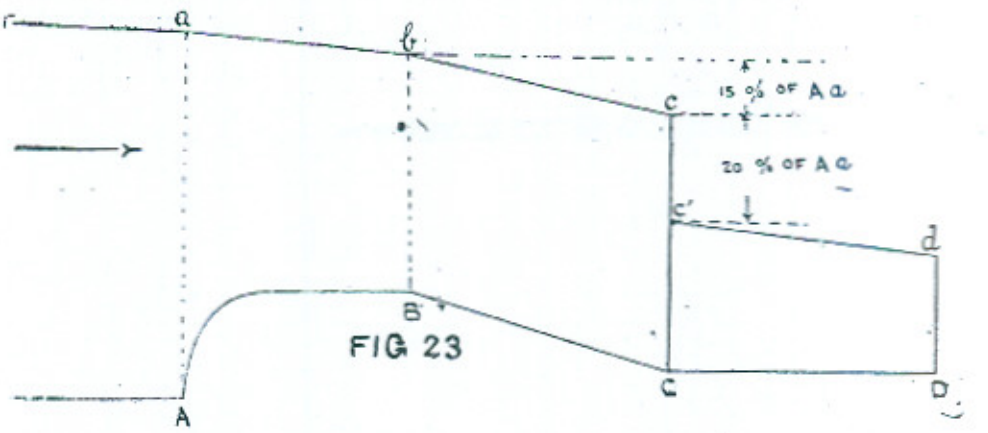


FIG 23

(ii) On the veritical B set off the "minimum energy of flow," a height B b.

(iii) On the vertical C set downwards 15 per cent. of Aa below the level of b to the point c.

(iv) From C set off cc' downwards 20 per cent. of A a.

(v) Calculate the loss in friction on the distance C D from the chezy formula as per para. 6.

The slope of the total energy line c' d is such as to give this loss'

The "ideal" total energy line is now complete.

8. From this total energy line, an ideal water profile may now be drawn in the manner described in para. 8 of Part II.

Since a water profile cannot have abrupt changes, nor under these circumstances, can it be truly vertical, an approximate "natural" profile may be sketched in.

Reference should now be made to Diagram XII. This diagram illustrates the above operations carried out for a discharge of 1·48 cusecs over the experimental weir.

(i) The ideal total energy line is shewn in a dotted blue line.

(ii) The ideal water profile is shewn in a dotted black.

(iii) The "natural" profile is shewn in full green line.

(iv) The actual profile as obtained in series II of the experiments is shown in a full black line.

Nos. **(iii)** and **(iv)** may be compared. The Author submits that the coincidence is sufficiently close for all practical purposes.

9. Before condemning the above process, the alternatives should be considered.

Many authorities were consulted. All agreed that the profile must be calculated from the " back water function." The clearest and fullest exposition of this function is to be found in Vol. III of the Miami Conservancy Report, to which reference may be made.

The result of the general integral is :—

$$L = \frac{D_n}{g} z - D_n \left(\frac{1}{s} - \frac{C^2}{g} \right) \left[\frac{1}{6} - \log_e \frac{z^2 + z + 1}{(z-1)^2} - \frac{1}{\sqrt{3}} \text{cotan}^{-1} \frac{2z+1}{\sqrt{3}} \right]$$

Wher L=Length of channel.

D_n =Depth of normal flow.

z $=D/D_n$.

s =Slope

C =co-efficient of friction in Chezy formula.

This expression is not quite so bad as it looks and can be worked out with fair freedom by the assistance of Bresse's tables for the expression within the square brackets.

10. Nevertheless it is not one which the average engineer would care to handle, if a simpler process can be found. Moreover it requires one known point on which to peg the profile, and it is the assumptions involved in this that renders the profile so obtained, extremely doubtful.

11. Mr. Burkitt in his 1919 Congress paper suggested the use of Manning's formula—an expression similar in form to the Chezy. But the method adopted again involves the knowledge of one point on the profile. Mr. Burkitt's assumption that the critical depth exists on the downstream edge of the crest is incorrect and consequently the profile so obtained is useless for our purpose.

12. It seems appropriate to point out here the serious nature of the error involved in this assumption.

If the critical depth be assumed to exist on the downstream edge of the crest, the velocity so calculated is always below, and may be much very below, the true velocity.

This error persists all the way down the glacis.

Now the depth on the glacis is *less* than the critical depth. Therefore the left hand leg of the "energy of flow" curve applies. If the depth assumed or calculated is *greater than* the actual, the standing wave derived therefrom occurs earlier (*i.e.*, higher up the glacis) and with a smaller destruction of energy than will be the case in actual practice. Reliance on design so derived is therefore dangerous.

13. If it is desired to take great precaution against under estimation of the position of the standing wave, the percentage losses, described in paras. 4 and 5 may be reduced to any extent desired. By so doing the formal profile is lowered, *i.e.*, the depth is decreased and the velocity apparently increased.

By so doing, the subsequent calculations for the standing wave result in its position being determined further downstream. The precautions taken to deal with it in this extreme position result in any desired margin of safety.

The Author therefore submits the conclusions that the method here in advocated for tracing the high velocity profile, combines reasonable accuracy with safety, with the added advantage of extreme simplicity.

PART IV.

Critical examination of the actual results obtained from the Experiments.

1. The assembly of the theorems into a logical whole in Part II and the development of the method of constructing the high velocity profile in Part III was not the straight forward process now recorded. In the same way, the examination detailed in the following paragraphs is really the result of slow and often useless collection of data from the diagrams of actual profiles.

2. The importance of the "total energy" line has been stressed continuously. This is shewn on Diagram II to VI as a full blue line.

It is drawn first for the high velocity stream assuming a remote standing wave. It can also be drawn for each of the conditions downstream; but has been omitted for the sake of clarity. Moreover it is not essential to our enquiry.

The depth being given by the profile, the velocity is easily calculated and also the velocity energy which may be set up above the water surface. (In actual fact, the "total energy" diagram was used as it happened to have been drawn first. See diagrams VII to XI.).

3. The critical depth for each discharge (the cross sections being always the same) was calculated from the formula :—

$$\frac{Q^2}{g} = B^2 D_c^3$$

This was marked off, above the Weir, for each series of experiments *i.e.*, on each diagram, as a matter of interest. It should be noted that the Critical depth is a useful concept when the stream lines have no appreciable curvature, but where there is curvature or no parallelism it has no physical significance.

4. The next step (in point of fact, already done) was to construct, for each discharge, (*i.e.*, each series *i.e.*, each diagram of profiles) a diagram as shewn in fig. 13 Part II.

Various depths were assumed and the velocity energy calculated.

The "Pressure-plus momentum" was also calculated. These calculations are given in full as Tables 7 to 11 and the diagrams are shewn as diagrams VII to XI.

5. From these diagrams, the "ideal" profiles of the low velocity water may be carried back on to the glacis for each experiment of each series. But for the reason given in para. 2 this has not been done.

6. It would be perfectly simple to *test* each standing wave by means of the diagram drawn as described in para 4. But instead of so doing the value of the "pressure-plus momentum" has been plotted on the Diagrams II to VI as a full red line. Where the "pressure-plus

momentum " line, appropriate to the downstream condition, cuts that of the upstream condition should be the position of the standing wave. The results for each of the 35 experiments are also shewn on Diagrams II to VI and are tabulated in table 14.

7. The coincidence, particularly on the sloping glacis, is so close as to convince the Author of the accuracy of the process. The variation observable on the trough floor is due to the extreme sensitiveness of the standing wave when off the glacis——a state of affairs which should never be permitted in practice.

8. As shewn in para. 10 Part III, Mr. Burkitt's method of drawing the high velocity profile involves a serious error. This should not effect the accuracy of his well known formula for the standing wave on a glacis, (derived in the same paper), since the *actual* depth, either upstream or downstream, can be measured off the observed profile

$$i = D_2 - \sqrt{D_1^2 + \frac{2q^2}{g}\left(\frac{D_2 - D_1}{D_2 D_1}\right)}$$

This was calculated (*vide* Tables 15 to 19) and plotted on each experimental profile.

It will be noticed that the points so calculated are incorrect and that the departure is not even consistent save in the fact that the formula always places the standing wave too high up the glacis, and that the error becomes greater as the standing wave is the nearer to the toe of the glacis. The serious effect of this error is that the height of the resulting standing wave is underestimated, and in consequence, the amount of energy destroyed in the standing wave.

The results are tabulated in Col. 5 of Table No. 14.

9. The Author offers the following possible explanations for this result.

(a) The implied assumption that friction may be neglected between the two plane sections is incorrect. Without friction no standing wave could form.

(b) No allowance for the upward slope of water surface due to the continued downward slope of glacis is made (*vide* para. 30 Part II). The resulting " height " of standing wave is, therefore, incorrect.

(c) The assumption that the reaction of the glacis below the standing wave (resolved horizontally) is balanced by the pressure on an equal portion of the lower plane section, is not justified. It is also unnecessary.

10. Both Mr. Robertson and the Author felt that if the results were permitted to stop at this point, nothing would have been done beyond confirming existing theory and, in particular, Mr. Hind's application thereof.

Many relations were tested without result, until a promising line presented itself with the plotting of the downstream depth against the actual position of the standing wave horizontally.

The results obtainable from the thirty-five experiments in five series, were insufficient for the purpose. A further thirteen experiments (corresponding each to a "series") were made to determine the relation between the depth downstream on the floor of the trough, and the actual position of the standing wave.

These results are tabulated as Table 20 Experiments Nos. 36 to 48.

11. From these results Diagram XIII was constructed.

Depth downstream was plotted vertically. Position is plotted horizontally, the zero point being the toe of the glacis.

The result is that *for any given discharge* the position of the standing wave for any depth downstream can be read off directly.

This, of itself, is of some value. From the information available in the Department it becomes possible to construct diagrams, shewing in terms of discharge per foot width of cill, the position of the standing wave for all depths downstream. Further reference to this matter is made in para. 10 V. of Part V.

12. **Definition.**—For purposes of what follows the neutral depth is defined as that depth downstream which will bring the standing wave over the toe of the glacis, for any given discharge.

13. On the same Diagram XIII is also plotted the neutral depth against discharge. The resulting curve was suggestive and was then plotted to double logarithmic scale to get the law. This curve is shewn as Diagram XIV constructed from Table No. 21.

The law so determined is

$$D_n = 0{\cdot}408\ Q^{\cdot 55}$$

14. The author does not suggest that the above expression holds for all weirs, but does submit that though the constants may alter, the expression will remain similar in form and of the same dimensions.

The precise significance is very puzzling. The expression is not rational, and the only deduction which it appears safe to make from it is as follows.

15. In any normal channel of depth, D, the discharge Q varies roughly as $D^{5/3}$

Substitute $C\,D^{5/3}$ for Q in the above expression, then

$$D_n = C'\,D^{0{\cdot}92}$$

From which it appears that D the channel depth rises more quickly than Dn, the minimum depth required to retain the standing wave on the glacis.

Or put another way, once the standing wave is on the glacis any further increase of discharge results in the wave advancing up the glacis. This, of course, is born out by observation.

16. One more result of the observations must be discussed briefly: the relation of the height of the standing wave to its length.

The theoretical position of the standing wave has been found and marked as described in para. 6. The "height" of the standing wave has been taken as the intercept between the observed water surface at that point and the glacis.

The length of the standing wave has been taken as the observed distance between the trough preceding the wave and the observed end of disturbed water as recorded in Tables Nos. 2 to No. 6.

17. The results are tabulated in Table No. 22 and are interesting. It would appear that the "physical" method of observing the end of the standing wave has broken down, somewhat, as the results differ widely.

It may be said with safety that the ratio of length of standing wave to height varies between three and five with a tendancy to the higher figure.

More observations are required on this head, although the above are very much in accordance with the observations of the Miami Project Engineers.

PART V.

Conclusions and Suggestions.

1. In the preceding parts, it has been shewn that it is possible to draw the profile over a Weir and deduce the position, the height and energy consumption of the standing wave below it, for any given discharge.

The problem usually before the irrigation engineer is slightly different. The discharge varies with the head acting and the depth downstream varies with the discharge. It is required to find the position of the worst standing wave and to design the work accordingly.

2. The discharge Q varies as $H^{3/2}$ over the weir and roughly as $D^{5/3}$ in the channel below it.

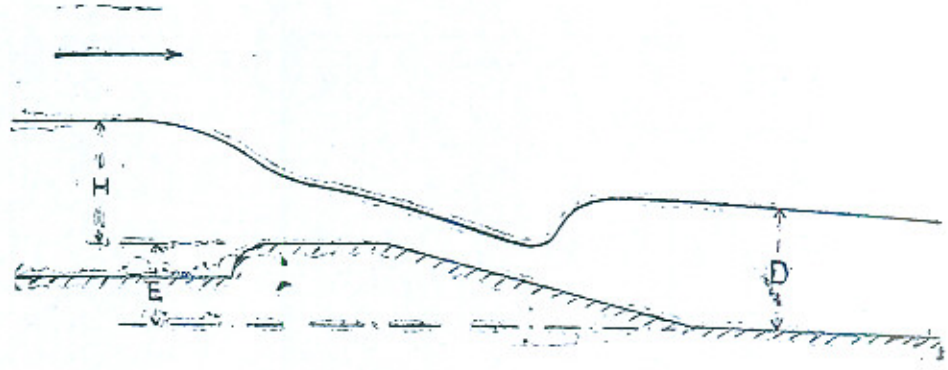


Fig 24

Let H be the total "still pond" head acting over the weir of width B.

D be the "still pond" depth of channel downstream of the weir

E the elevation of the weir crest above the bed downstream.

Q the discharge in cubic feet per sec.

W the weight of 1 c. ft. of water.

The loss of energy per second involved in passing from the high level channel, over the weir into the low level channel

$$= W\,Q\,(H + E - D).$$

But $Q = C_1 H^{3/2}$ $\quad \therefore H = \left\{\frac{Q}{C_1}\right\}^{2/3}$

$= C_2 D^{5/3}$ $\quad \therefore D = \left\{\frac{Q}{C_2}\right\}^{3/5}$

$\therefore$ Loss of energy

$$= W\,Q\,(E + H - D)$$

$$= W\,E\,Q + \frac{W}{C_1^{2/3}} \cdot Q^{5/3} - \frac{W}{C_2^{3/5}} \cdot Q^{8/5}$$

When Q=O, H=O and loss of energy is zero.

When H+E=D again the loss of energy is zero. Between these two limits there must be a maximum. The simplest way obviously is to take a few values of Q and plot them against the resultant loss of head.

3. From this, the following have been obtained.

(*a*) the discharge which gives the worst standing wave.

(*b*) the depth in the channel downstream.

(*c*) the loss of energy involved.

From the above information the profile of the high velocity Stream may be determined and the height and position of the standing wave.

4. But it is more than likely that the position of the standing wave will be unsuitable, *i.e.*, too far down the glacis or even on the bed of the channel.

Clearly the correct remedy is to increase the depth downstream by interposing a cistern.

Consider first the *exit* from the cistern. This exit should be smooth to avoid further turbulence on entering the normal channel below it.

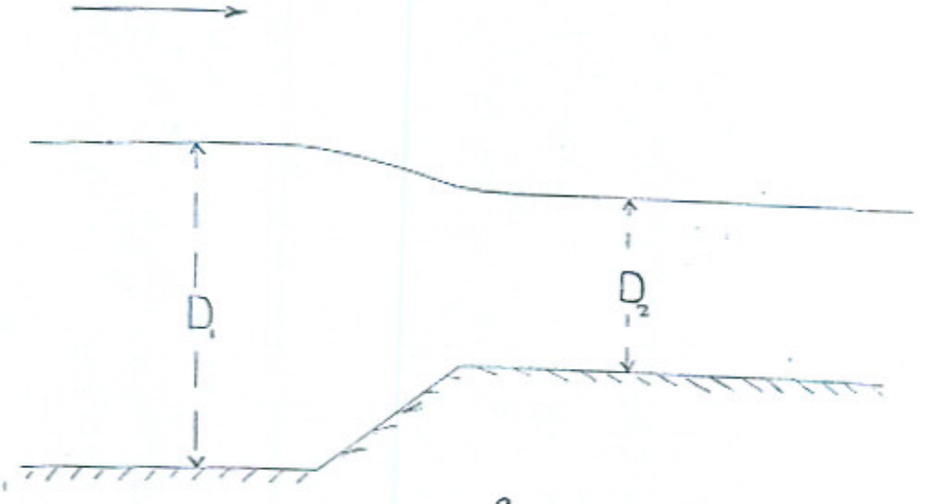


fig. 25

In fig. 25 the water is shewn passing from cistern with depth D_1 to the normal channel with depth D_2. Since the depth decreases, the velocity must increase and therefore there must be a corresponding fall of surface.

In speeding up the stream lines (*i.e.*, in passing from a larger cross section to a smaller) there is less likelihood of disturbance than in the reverse process, as has been shewn already (para. 22 Part II).

5. Assume a suitable depth of cistern and recalculate the down-stream profile and the standing wave. This is not such a lengthy

process as might appear, once the "energy of flow" and "pressure plus momentum" curves have been drawn.

The length of the cistern is not determinable mathematically, but from para. 17 Part IV, it would appear to be necessary to build it about five times the height of the standing wave.

6. A brief consideration of Mr. Colyer's "Energy Dissipator" or "Biff-Wall" appears indicated at this point.

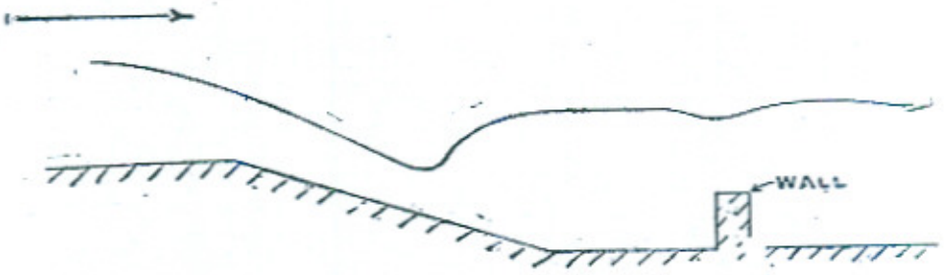


fig. 26

The biff wall is designed to assist in the dissipation of energy of the standing wave.

In fig. 26 the wall, being an abrupt obstruction, is bound to cause eddy and turbulence apart from any other consideration.

There are three definite cases afforded by the relation of the height of this wall to full supply.

7. **Case i.**—Whatever the height of this biff wall, it is bound to cause a depression in the water surface above it.

If the depth of water *above* the biff wall is greater than the critical depth, the depth of water upstream of the wall is the same as that downstream, and the biff wall has no effect on the standing wave itself.

Under such circumstances the effect is confined to turbulence below it.

Case ii.—If the depth of water above the biff wall is designed to be the critical depth the state of affairs is violently unstable. At one moment the biff wall is acting as a secondary weir and at another, is drowned. The turbulence in such a case is bound to be far more pronounced. The checking effect on the standing wave is confined to the moments when the biff wall is acting as a secondary weir.

Case iii.—If the depth of water above the biff wall is designed to be *less* than the critical depth (in relation to the normal channel depth downstream of the biff wall) then the biff wall certainly assists in checking the standing wave by heading up above it (the wall).

The destruction of energy so resulting is reduced at the site of the main wave and is transferred to the site of the secondary wave formed by the biff wall.

And this is undesirable.

8. From the above considerations it appears certain that a properly designed cistern is the proper remedy.

The author designed two, tentatively, one for about 700 cusecs and one for about 30 cusecs on Nokhar Branch in 1925-26. Both are reported as working excellently, particularly the one for 700 cusecs which was actually built with slight modifications by his successor.

9. This paper has grown to such dimensions that the Author has postponed for future consideration some practical examples which have been kindly provided by officers of the Department.

With these examples it may be possible at some future date, to present the practical results of some works designed on the lines indicated by the paper.

10. The Author submits the following suggestions as to the course of future enquiry:

i. Loss by friction in the different parts of flumes and Weirs, due to enhanced velocities.

ii. Departure of the actual profiles from "ideal" profiles.

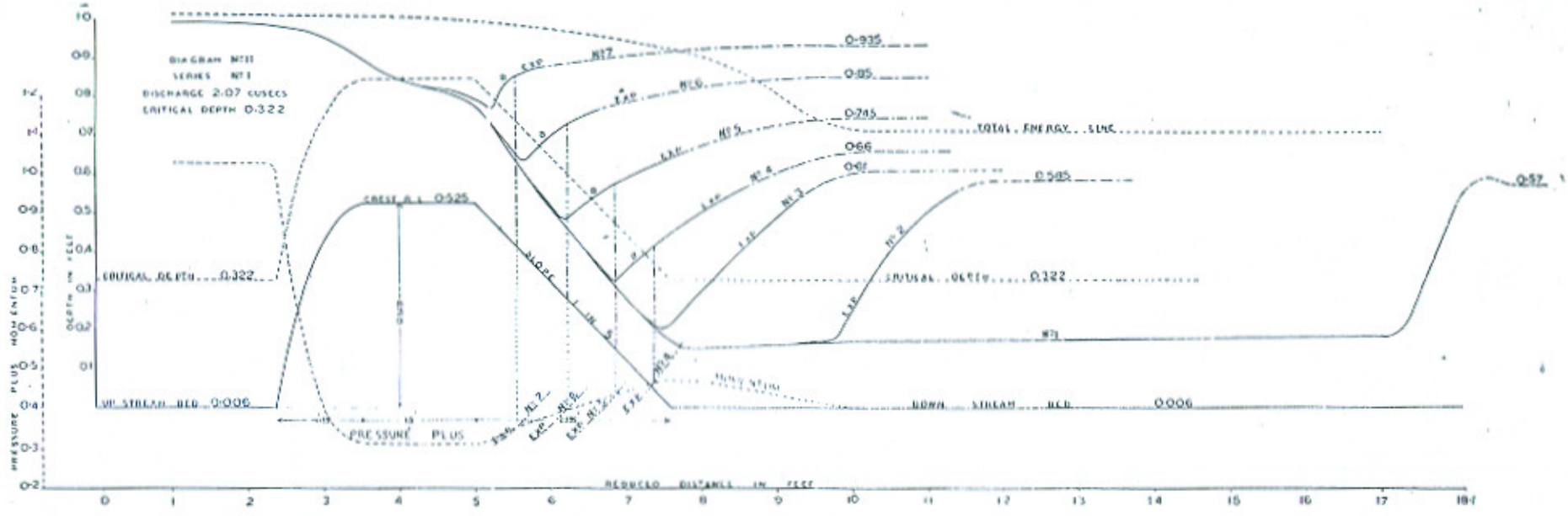
iii. More accurate determination of the *length* of the standing wave and its relation to the slope of the glacis.

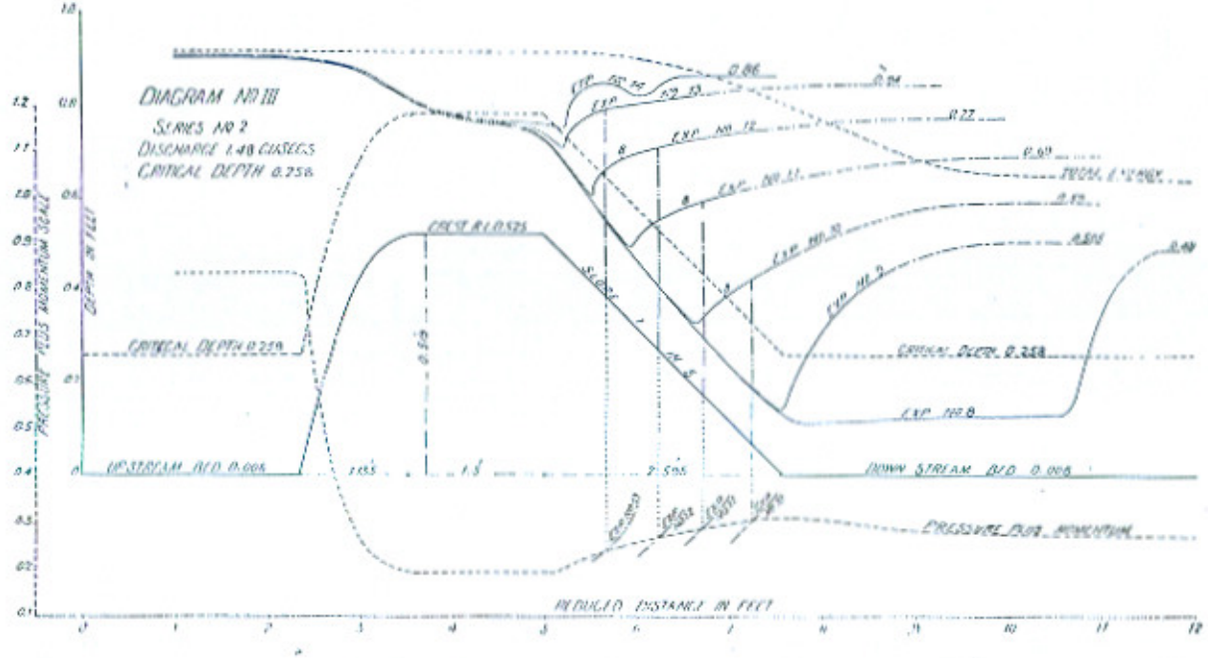
iv. Preparation of Diagrams giving "Energy-of-flow" and "Pressure-plus-momentum" curves for rectangular sections on a basis of discharge per foot run of crest.

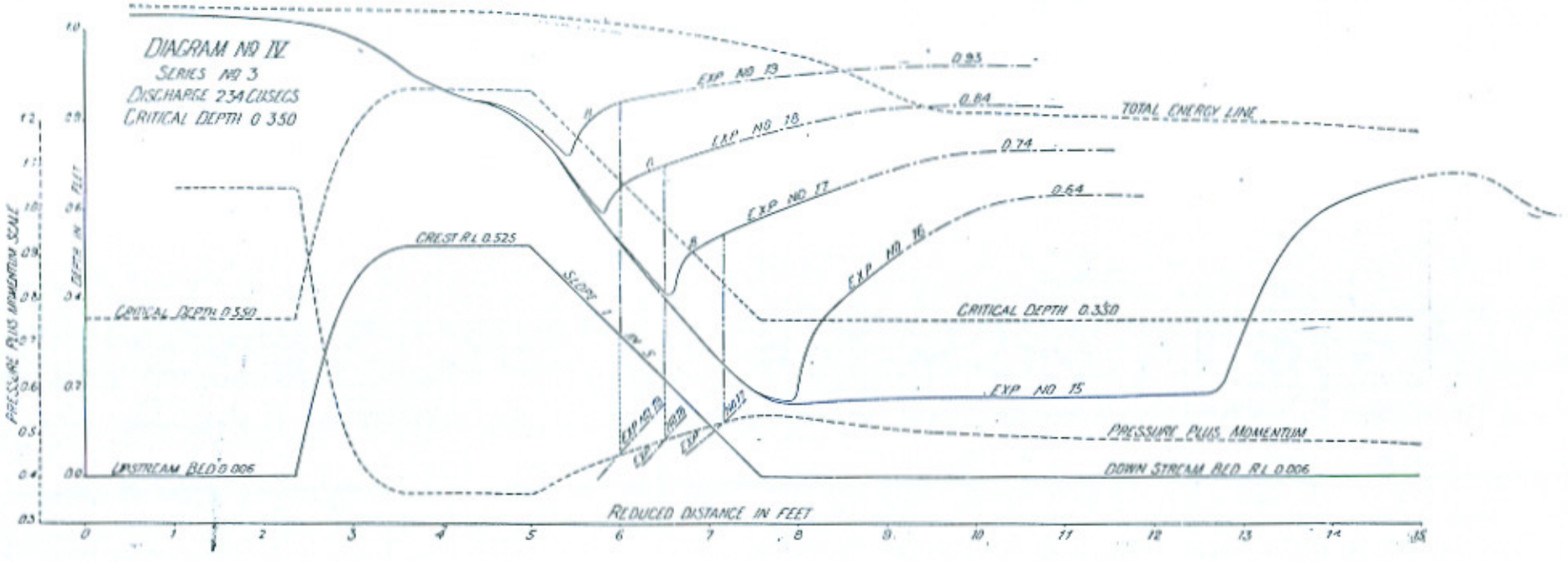
v. Preparation of Diagrams giving the position of the standing wave against depth downstream, on a basis of discharge per foot run for weirs of standard glacis slope and standard divergence.

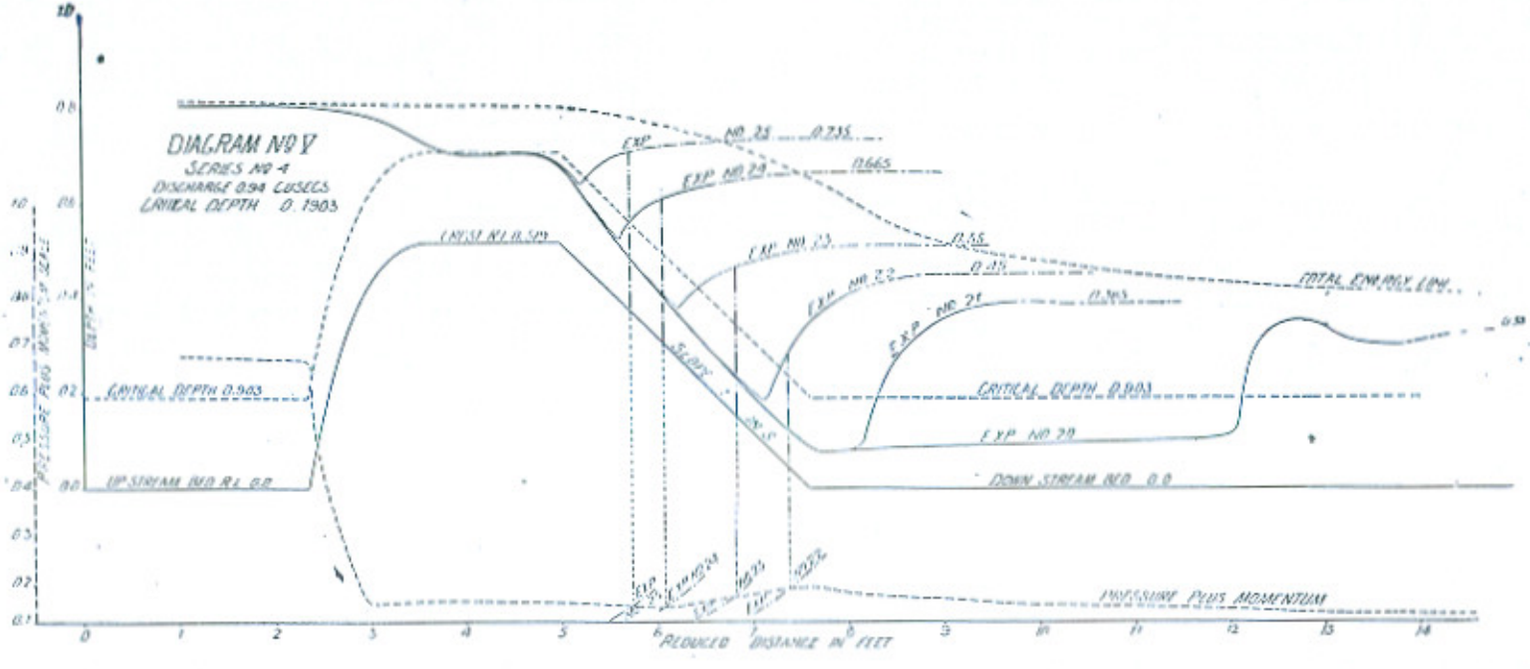
These last would be invaluable in the rapid determination of the depth of cistern required below weirs.

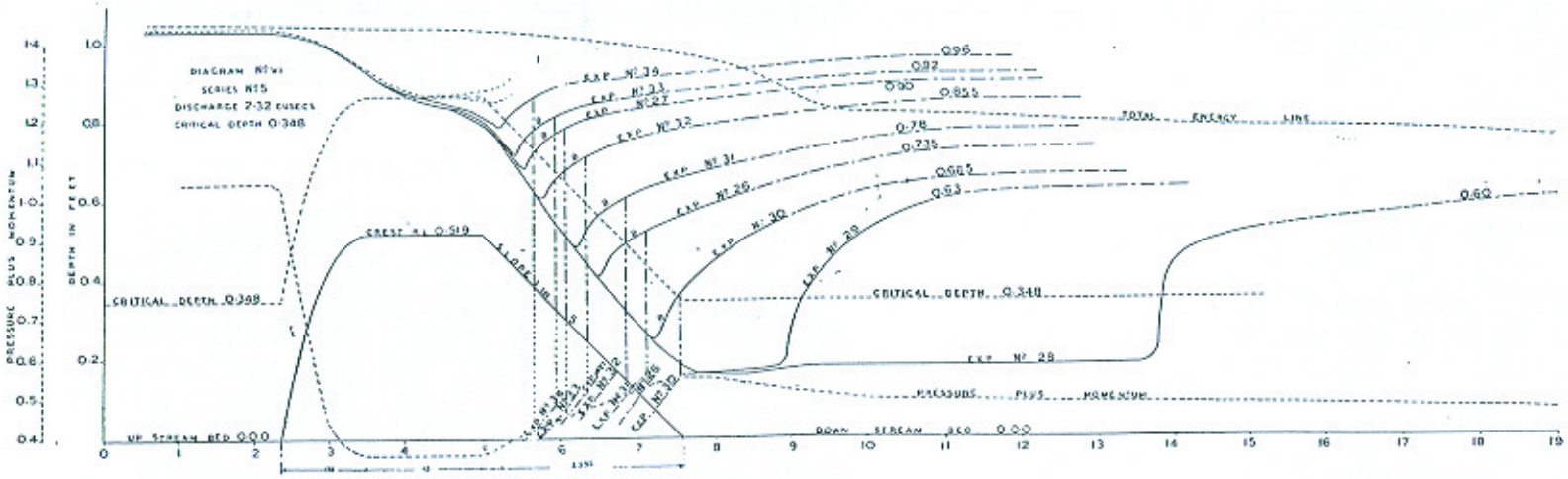
11. In conclusion the Author wishes to record his sincere thanks to Mr. A. N. M. Robertson for his invaluable assistance, amounting to guidance, in the laboratory and to express his regret that circumstances have deprived the Author of Mr. Robertson's co-operation in the preparation of this paper.

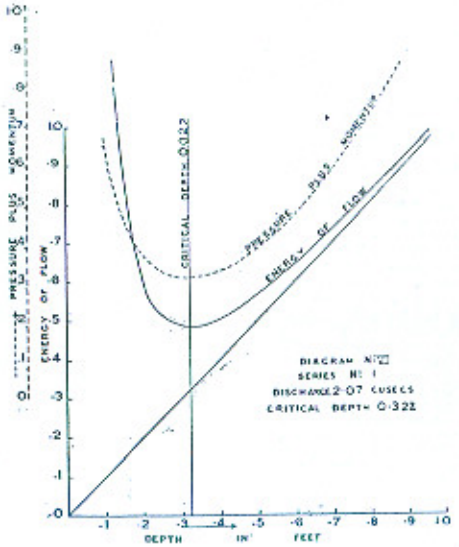

DIAGRAM NO II
SERIES NO 1
DISCHARGE 2.07 CUSECS
CRITICAL DEPTH 0.322
CREST R.L 0.525
SLOPE 1 IN 5
CRITICAL DEPTH 0.322
UP STREAM BED 0.006
DOWN STREAM BED 0.006
PRESSURE PLUS
TOTAL ENERGY LINE
0.935
0.85
0.785
0.66
0.61
0.585
0.57
EXP NO 7
EXP NO 6
EXP NO 5
EXP NO 4
EXP NO 3
EXP NO 2
EXP NO 1
PRESSURE PLUS MOMENTUM SCALE
DEPTH IN FEET
REDUCED DISTANCE IN FEET

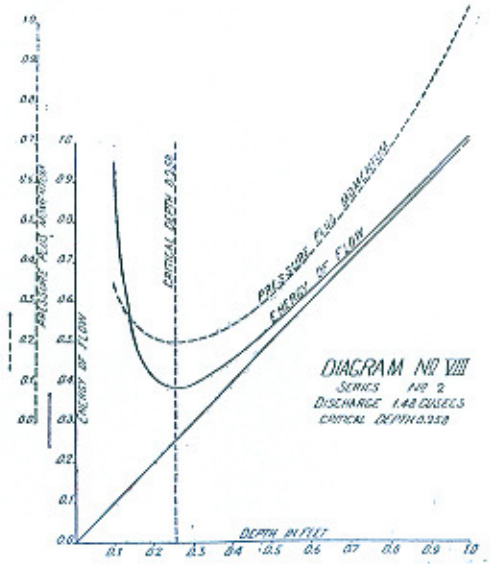

DIAGRAM NO III
SERIES NO 2
DISCHARGE 1.48 CUSECS
CRITICAL DEPTH 0.258
CREST R.L 0.525
SLOPE 1 IN 5
CRITICAL DEPTH 0.258
UP STREAM BED 0.006
DOWN STREAM BED 0.006
PRESSURE PLUS MOMENTUM
0.86
0.84
0.77
0.69
0.62
0.516
0.49
EXP NO 13
EXP NO 12
EXP NO 11
EXP NO 10
EXP NO 9
EXP NO 8
PRESSURE PLUS MOMENTUM SCALE
DEPTH IN FEET
REDUCED DISTANCE IN FEET

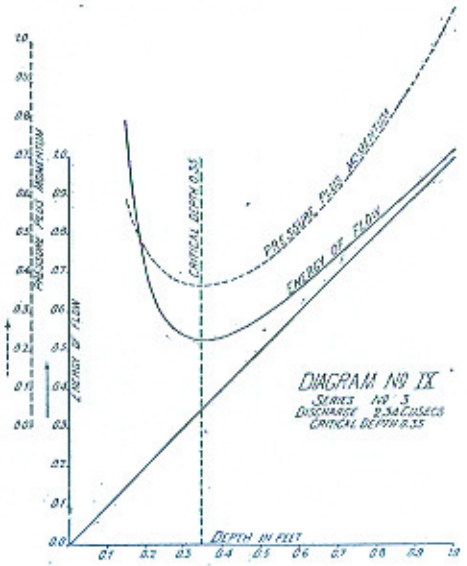

DIAGRAM NO IV.
SERIES NO 3
DISCHARGE 234 CUSECS
CRITICAL DEPTH 0 350
PRESSURE PLUS MOMENTUM SCALE
DEPTH IN FEET
CREST R L 0.525
CRITICAL DEPTH 0.350
UPSTREAM BED 0.006
SLOPE 1 IN 5
EXP NO 19
EXP NO 18
EXP NO 17
EXP NO 16
EXP NO 15
0.95
0.84
0.74
0.64
TOTAL ENERGY LINE
CRITICAL DEPTH 0.350
PRESSURE PLUS MOMENTUM
DOWN STREAM BED R L 0.006
REDUCED DISTANCE IN FEET

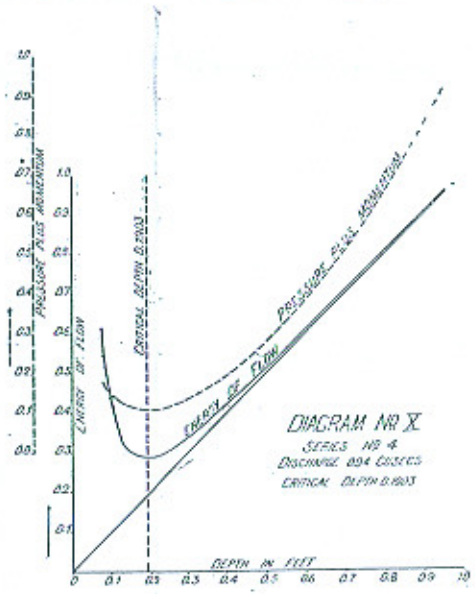

DIAGRAM NO V
SERIES NO 4
DISCHARGE 0.94 CUSECS
CRITICAL DEPTH 0.1903
CREST R L 0.525
CRITICAL DEPTH 0.963
UP STREAM BED R L 0.0
SLOPE 1 IN 5
EXP NO 25
EXP NO 24
EXP NO 23
EXP NO 22
EXP NO 21
EXP NO 20
0.735
0.665
0.55
0.45
0.363
TOTAL ENERGY LINE
CRITICAL DEPTH 0.903
DOWN STREAM BED 0.0
PRESSURE PLUS MOMENTUM
REDUCED DISTANCE IN FEET

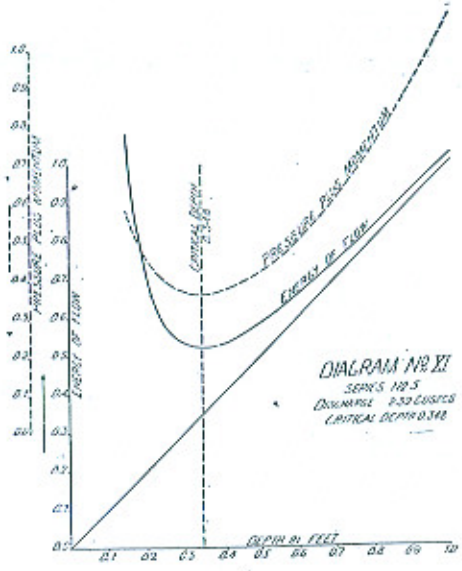

DIAGRAM N°VI
SERIES N15
DISCHARGE 2·32 CUSECS
CRITICAL DEPTH 0·348
CREST at 0·519
CRITICAL DEPTH 0·348
CRITICAL DEPTH 0·348
TOTAL ENERGY LINE
PRESSURE PLUS MOMENTUM
UP STREAM BED 0·00
DOWN STREAM BED 0·00
DEPTH IN FEET
PRESSURE PLUS MOMENTUM
EXP N° 39
EXP N° 33
EXP N° 27
EXP N° 32
EXP N° 31
EXP N° 30
EXP N° 29
EXP N° 28
0·96
0·92
0·90
0·85½
0·79
0·73½
0·68½
0·63
0·60

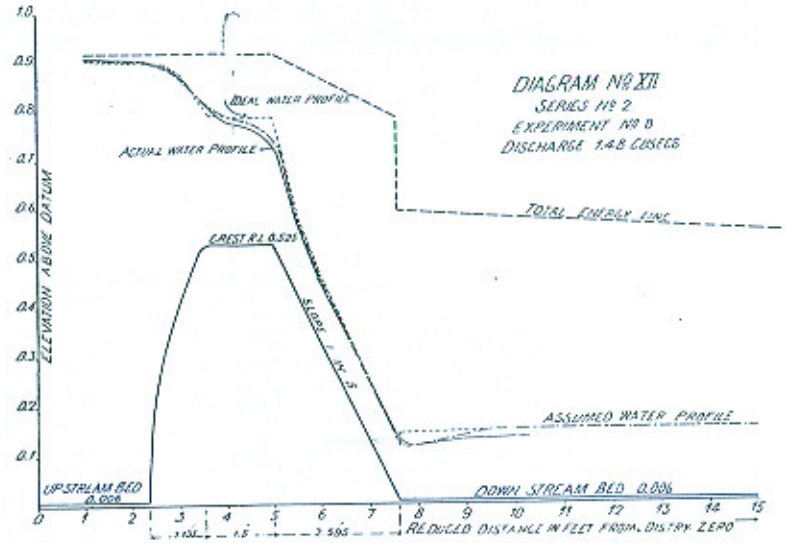

DIAGRAM N°VII
SERIES N° 1
DISCHARGE 2·07 CUSECS
CRITICAL DEPTH 0·322
CRITICAL DEPTH 0·322
PRESSURE PLUS MOMENTUM
ENERGY OF FLOW
DEPTH IN FEET

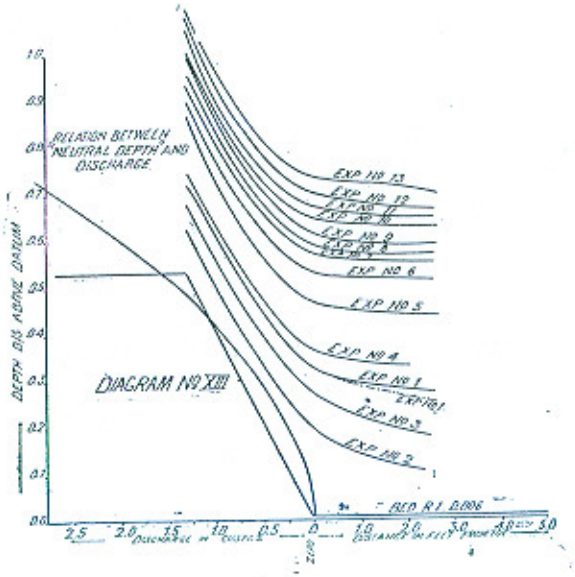

DIAGRAM N° VIII
SERIES N° 2
DISCHARGE 1·48 CUSECS
CRITICAL DEPTH 0·258
CRITICAL DEPTH 0·258
PRESSURE PLUS MOMENTUM
ENERGY OF FLOW
DEPTH IN FEET

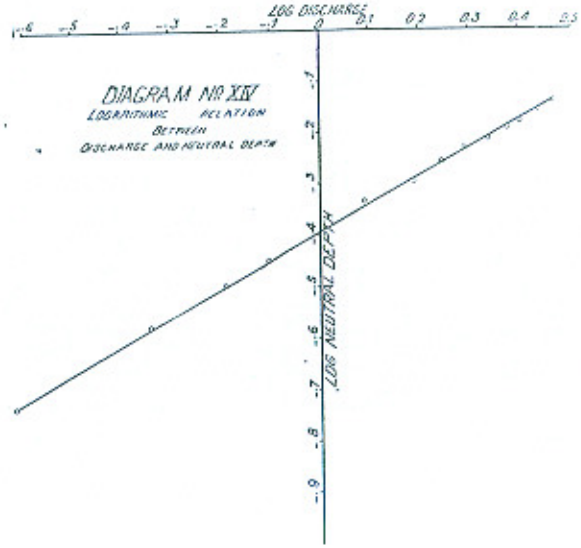

DIAGRAM N° IX
SERIES N° 3
DISCHARGE 2·34 CUSECS
CRITICAL DEPTH 0·35
CRITICAL DEPTH 0·35
PRESSURE PLUS MOMENTUM
ENERGY OF FLOW
DEPTH IN FEET


DIAGRAM Nº X
SERIES Nº 4
DISCHARGE 0.94 CUSECS
CRITICAL DEPTH 0.1903
PRESSURE PLUS MOMENTUM
ENERGY OF FLOW
CRITICAL DEPTH 0.1903
DEPTH IN FEET


DIAGRAM Nº XI
SERIES Nº 5
DISCHARGE 0.53 CUSECS
CRITICAL DEPTH 0.148
PRESSURE PLUS MOMENTUM
ENERGY OF FLOW
CRITICAL DEPTH 0.148
DEPTH IN FEET


DIAGRAM Nº XII
SERIES Nº 2
EXPERIMENT Nº 9
DISCHARGE 1.48 CUSECS
IDEAL WATER PROFILE
ACTUAL WATER PROFILE
TOTAL ENERGY LINE
CREST R.L 0.521
SLOPE 1 IN 5
ASSUMED WATER PROFILE
UP STREAM BED 0.006
DOWN STREAM BED 0.006
ELEVATION ABOVE DATUM
REDUCED DISTANCE IN FEET FROM DISTRY ZERO


DIAGRAM Nº XIII
RELATION BETWEEN NEUTRAL DEPTH AND DISCHARGE
EXP Nº 13
EXP Nº 12
EXP Nº 11
EXP Nº 10
EXP Nº 9
EXP Nº 8
EXP Nº 6
EXP Nº 5
EXP Nº 4
EXP Nº 1
EXP Nº 3
EXP Nº 2
DEPTH DIS ABOVE DATUM
DISCHARGE IN CUSECS
DISTANCE IN FEET FROM ZERO

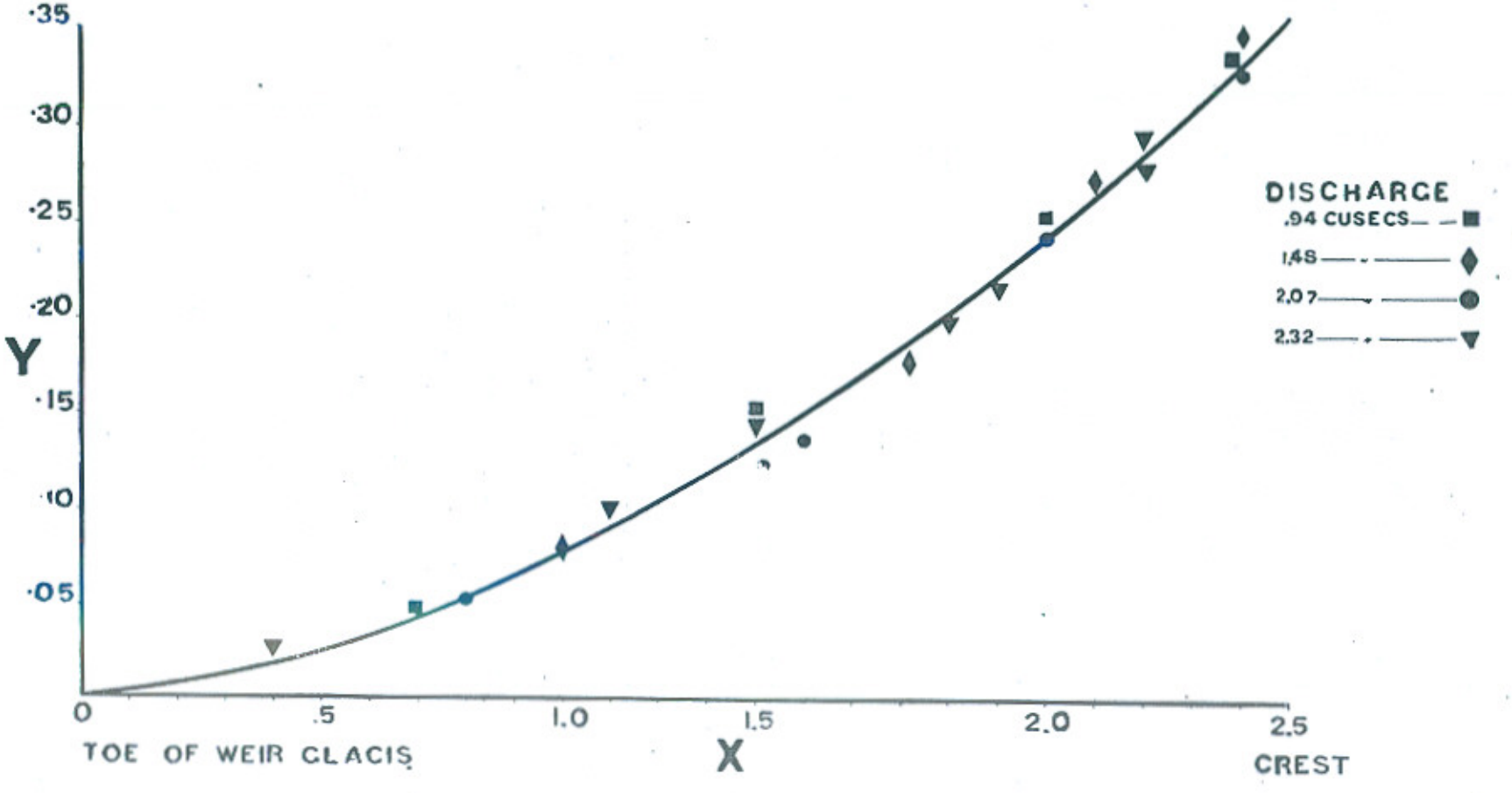

DIAGRAM Nº XIV
LOGARITHMIC RELATION BETWEEN DISCHARGE AND NEUTRAL DEPTH
LOG DISCHARGE
LOG NEUTRAL DEPTH

CURVE $Y = .0767\ X^{\frac{5}{3}}$

SHOWING RELATION BETWEEN INCREASE OF DOWNSTREAM DEPTHS AND POSITION OF STANDING WAVE ON GLACIS

VERTICAL SCALE SHOWS INCREASE OF DOWNSTREAM DEPTHS OVER ZERO DEPTHS

ZERO DEPTH BEING THE DOWNSTREAM DEPTH AT WHICH STANDING WAVE REACHES TOE OF GLACIS

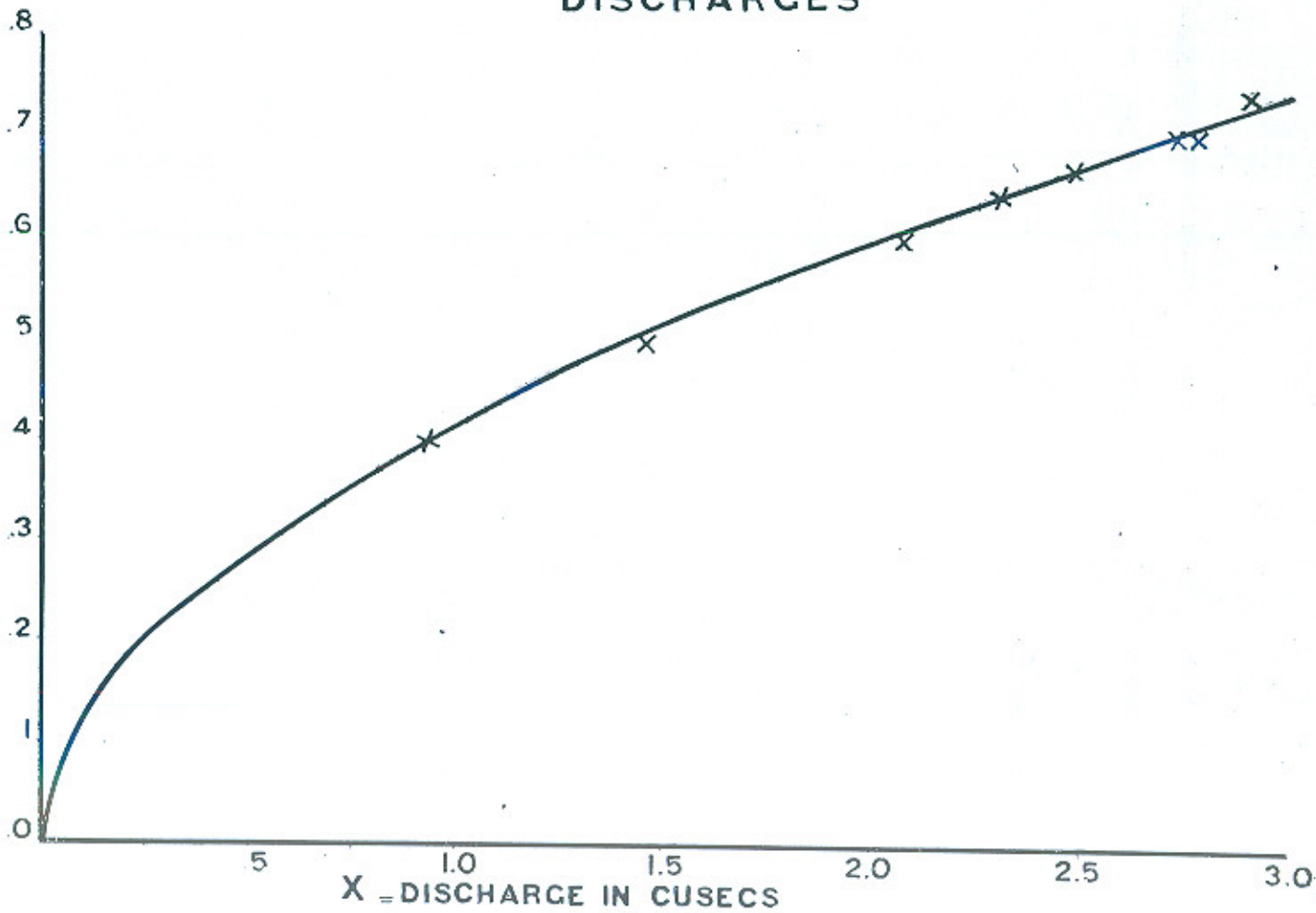


CURVE $Y = .42 X^{\frac{1}{2}}$

SHOWING RELATION BETWEEN ZERO DEPTHS

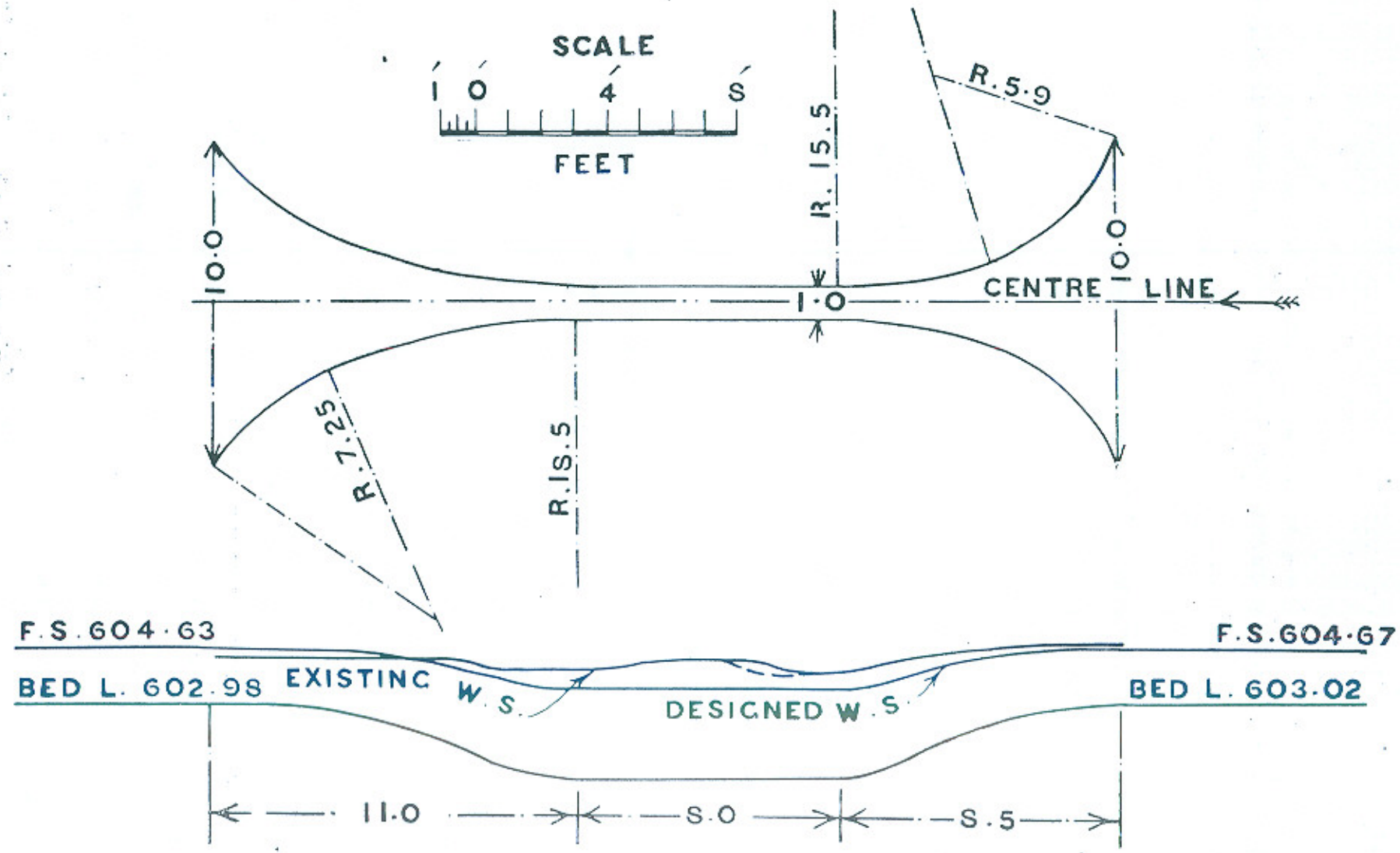
&

DISCHARGES



HYDRAULIC SURVEY AT THE SITE OF A FLUMED BRIDGE R.D. 1140 RENALA I. R. B. DISTRY.

See Mr. Blench's remarks.



DISCUSSION.

MR. ROBERTSON, said that having been associated with Mr. Montagu in the experiments carried out in London in the summer of 1927, there are some points of interest which he would like to bring before the Congress.

After making the experiments, they had been associated for several weeks with Mr. Dixon, Assistant Professor of Mathematics in an endeavour to obtain a method of determining the position of the standing wave on a weir glacis.

Mr. Montagu has given the results of the work on page 54 of his paper and in Diagram 13.

The speaker showed a diagram obtained for the earlier experiments by plotting the position of the standing wave against the increase of downstream depth above what Mr. Montagu calls the 'neutral' depth *i. e.*, that at which the wave reaches the toe of the glacis. This has been referred to as the 'zero' depth in the diagram. For a number of experiments ranging between ·94 to 2·32 cusecs, it will be seen that for the same increase of depth above the 'zero' depth, the wave moves an equal distance along the glacis, independently of the discharge, and the resulting relation is given by the formula as shown on the Diagram.

Further experiments with smaller discharges, where friction had a greater effect, did not give such a regular curve, the relation is an interesting one and it is worth while investigating if such a relation holds when applied to experiments on a large scale.

The speaker also showed a diagram showing the relation between zers depth's and discharges, which approximates to the results from Mr. Montague's Diagram No. 13.

DISCUSSION.

MR. BLENCH said that he had had the luck to build the first new type flume, based on Mr. Montagu's theory, in the Head Quarter's Compound of Renala Sub-Division, therefore he was in the fortunate position of being able to turn what appeared to be a complete failure into a first class vindication of the theory. The work was a very bold first attempt, fluming a 25 cusecs channel of 10 ft. bed width and 1·6 ft. depth to a breadth of one foot in a convergence of some 10 ft. long, running the water out by a curved divergence some 12 ft. long. The bed being dropped 2½ ft. below channel bed.

It was assumed that there would be one hissa loss of head due to friction, and the water surface would follow in a smooth curve without any standing wave.

Actually, two standing waves formed, the first approximately in front of the junction of the slope of the bed with the horizontal portion of the flume bottom; the other in front of the next rapid change of curvature. The loss of head through the flume was about 4 hissas; and a strong jet formed at exit. Unlike the subsequent works, which worked perfectly well, it was ocularly a failure. The designer primarily blamed a miscalculation of friction, causing his design to give a velocity higher than the critical, with consequent jump, and loss of enregy. He, the speaker, plotted the actual water profile carefully on the design; and from it obtained the Total Energy Line by marking up velocity energy from the water surface.

The result was a Total Energy Line showing an increase of energy over the standing waves instead of the reverse, and the discovery that the so-called standing wave occurred at a depth above the critical so that there could be no jump or drop of energy.

In view of the abrupt curvature this result is not surprising. The effect of curvature in modifying pressure energy must be considerable. If the water surface is convex upwards there is a negative pressure gradient due to it from the surface to bed, therefore the pressure energy is less than we take it, therefore total energy is less than it was calculated. Therefore if the Total Energy Line was plotted from the water surface, a Total Energy Line was obtained which was too high. The order of the error may be estimated by decreasing pressure energy ordinarily calculated in the ratio of $\frac{g-a}{g}$ where a is the vertically downward acceleration of the water. A rough calculation showed that the order of the error is of the order of the correction. It can therefore be stated, with a quantitative reason, as well on a theoretical one, that the elevation of Total Energy Line was due to neglect of curvature effect in estimating pressure energy: and it may be allowed for in design. The standing wave is found to be, apparently, a false one. It arises because the depth increases at entry to the flume; and any sharp change in the bad entry must produce a sharp change in the water surface, if the bed rather rapidly becomes level, the

water surface must rise above its former less abrupt slope, and may actually ascend.

Dealing now with another curious feature of the water surface; a couple of feet before the standing wave the water surface is convex upwards in a plane, perpendicular to the flow. Just before the wave it is concave upwards. A difference of level between centre and sides of about $\frac{1}{8}$" is involved. Upward convexity can only be caused by a curvature of the stream lines in a horizontal plane corresponding to centres of curvature outside the work, *i.e.*, the water has left the sides of the converging flume entry and formed a "vena contracta." The discharge per foot breadth in such an area will be greater than that calculated in the design; and as the critical depth is greater as the discharge per foot increases; the water velocity may actually pass through the critical.

A reference to Mr. Montagu's curves shows that this probably occurs, but the actual passage beyond the critical point is not great. A small jump will occur and a small loss of energy. There are two of these little waves in the flume; and if a loss of one hissa energy in each be assumed the loss of 4 hissas over the flume becomes 2 hissas due to friction and 2 due to jump. Here again abrupt convergence is to blame for the standing wave. The speaker took it that the pseudo-standing wave masks the real one, which is small.

The excessive loss of head is now reconciled with theory by the fact that the so-called standing wave referred to above hides a small but a true one and we see that abrupt curvature is to blame; and the secondary calculations required by it were neglected in design. The flume is now vindicated up to the jet.

The question of jet leads one to look into the forces at play. The water cannot be expected to follow the lines required by the energy theory, if only on the grounds that energy is not derivable from a potential in these problems. The equations involving momentum (for which Mr. Montagu has a system of curves, similar to his energy ones) must be considered. Take a section before the jet and one after and assume the motion without jet formation. The forward forces over the volume enclosed by the boundaries and the sections are those from the water behind, and from the side walls. The backward, those from the water in front; from friction (of which I should ask Mr. Montagu what use he has made, for I know he has made some) and from the floor. Now the floor pressure; to the first order, is proportional to D, whilst sides pressure are proportional to $\frac{D}{2}$ herefore floor pressure is twice as important per unit of area as the side pressure. Because there is deceleration of the water, and as there is receding curvature and upwards slope the pressure over the bed is given by something less than the depth; and the preponderance of backward pressure from bed, over forward pressure from sides is reduced. It may happen therefore, that on making a secondary correction to boundary pressure we find the forward pressure exceed the backward, which is inconsistent with deceleration. The

motion we have assumed is impossible. A possible motion must be of the jet form, because this implies less deceleration and the forward pressure from one wall is balanced by the backward pressure from the still water outside the jet; and the floor boundary pressures increase due to less dece'eration.

It appears, therefore that bed curvature is more important than side; and the flume could be put right, perhaps, by merely altering the bed.

So the flume action has now been explained in terms of Mr. Montagu's theory, making an estimate of the secondary effects originally neglected, pointing to how future designs may be rendered more accurate; and the secondary effects reduced. The point regarding bed curvature, I consider important, as the comment on the flume by most observers I have met, is that the horizontal divergence only is too rapid.

MR. KHOSLA said that, unfortunately the advance copy of Mr. Montagu's paper as of other Congress papers came too late to permit of a critical examination in the light of observations carried out on existing large works. There existed in Marala Division and in many another Division, quite a mass of data against which the conclusions of Mr. Montagu's paper can be tested.

The great feature of this paper, to his mind, was the attempt at a correct determination of the water profiles, which will, in turn, enable correct design of works and their protection downstream. The great value of the experimental results—apart from other aspects—lies in the close agreement between the calculated and the observed profiles. It remained now to test these calculations against actual profiles of weirs, flumes, etc., of which quite a large number were now scattered all over the canals. The results obtained from a model have their value in giving the correct lead but in full size works, a large number of other factors come in which may considerably modify the conclusions derived from the models. The real work is to determine these factors and the relative effect of each one of them on what may be called, the ideal profiles of the model.

Up till now there has been a good deal of discussion on the co-efficients of discharge over weirs, flumes and constructions of various hapes, but they had resolved themselves into a discussion of water rofiles. The speaker had a distinct feeling that there was a definite elation between the water profile and the co-efficient of discharge over any structure. The factors that govern one in actual works, also govern the other.

Among the many factors the speaker considered the ratio of width of eir to depth of water on crest of the greatest importance. The latter ctor, to his knowledge, had not been studied so far, although it had been eferred to from time to time.

From a scrutiny of over 70 observations the following conclusions were arrived at for the Bombanwala Regulator :—

i. The velocity of approach head was correctly represented by a well gauge in the central pier which opened in the direction of flow.

ii. The co-efficient of discharge (allowing for velocity of approach was 2·93.

iii. The co-efficient of discharge which allowed for the velocity of approach was constant whereas the one which did not allow for this velocity of approach was variable.

The width of each bay is 8·5' and the depths over crest ranged) from 5' to 8'.

In the case of Nokhar Weir which is 50 ft. wide the co-efficient worked out to 3·04 (allowing for velocity of approach) the depth of water over crest ranging from 2' to 2·5'.

The Bambanwala Regulator was originally designed for a fall of 2·56' but after the lowering of the bed of canal downstream the fall was 6·56' Originally the regulating weir had an approach glacis of 1 : 3, a crest width of 8·5'.

In 1928 to convert it into a meter, the crest was raised by one foot and made 18·8' long with a approach radius of 14·0 ft. The piers were extended and curved to a radius of 30 ft. The conversion of the fall into a meter, increased the intensity of the water jet as it left the downstream edge of crest and the resulting inpact gave rise to pulsations downstream which were likely to damage the downstream protection. Coupled with this, the bed of the canal downstream was lowered. This led to severe action, damaged the downstream protection and worked a scour hole about 15' deep at the end of the loose protection. Under orders of the Superintending Engineer sloping wooden grids were placed below the crest in each bay and the result was that in a short time the scour hole was filled up, the destructive pulsation was stopped and there was no further anxiety.

In this fall it might be noted that the cistern slopes up gradually to meet the downstream floor. This is what Mr. Montagu recommends but for which he has not offered any facts to prove. He personally thought that the cistern would have been more effective if it has vertical wall at the end with a back projection in the crest as adopted in the design for the Nokhar Weir.

The Nokhar Weir for 720 cusecs, referred to by Mr. Montagu, was originally designed by him and modified in the design of the cistern and other details by the Superintending Engineer (Mr. Murphy) and executed by the speaker. The design of the cistern seemed to be a very happy one. Under all supplies, low and high, this weir worked beautifully and a distinct standing wave formed with absolutely no pulsation beyond the cistern wall which is flush with the downstream floor and does not project

up as advocated by Mr. Colyer. Nor yet it is a slope up as now recommended by Mr. Montagu. Apparently the proportions of the cistern and the end wall are the two great features of this design.

The design as it now stood, was a result more of engineering instinct than calculations or even of applications of proportions from other existing works.

There are other weirs in the Nokhar Sub-Division which had a distinct pulsation extending far down below the standing wave. The introduction of a cistern had cured these.

DR. BOSE said the whole problem can be reduced to Bernoulli's equation,

$$p = c - \tfrac{1}{2}\rho\,(u^2+v^2) - g\,\rho^2.$$

If velocity V be assumed uniform over any cross-section and if Q be the discharge per unit width of the canal

$$V = \frac{Q}{Z}$$

$$\text{or } p = C - \tfrac{1}{2}\rho\,\frac{Q^2}{Z^2} - g\,\rho^2.$$

As the velocity increases 'Z' becomes smaller and smaller, so that p disminishes and a point is reached when the fluid pressure is not sufficient to contain the air that is in it or to stand the surface tension, and the liquid will burst out giving froth and foam and relieving a part of its energy; and thereby coming back to a stable state again. If the above explanation of the phenomena of standing wave be correct, then we can expect it to form at the point where the liquid pressure becomes minimum which would be given by

$$\frac{\delta p}{\delta Z} = \rho\,\frac{Q^2}{Z^3} - g\,\rho = 0 \text{ or } Z^3 = \frac{Q_2}{g}$$

or and pressure at this point is given by

$$Z \text{ min} = \frac{1}{\sqrt[3]{g}} + Q^{2/3}$$

$$\rho \text{ min} = C - \tfrac{3}{2}\,g\,\rho Z.$$

This point has been called by Mr. Montague as the "critical point." But according to him the standing wave does not form at this point but may form before it, which he tries to explain by his "Energy of flow" and "Pressure plus momentum" curve, though the exact physical connection of these curves he does not explain. The fact that the standing waves does not form actually at the critical point can be explained as follows.

If the minimum pressure be considerably less than the internal air pressure and the surface tension, a standing wave will form much in advance of the critical stage; and if the minimum pressure be greater than no standing wave will form. So that if for a certain discharge Q the slope of the glacis be greater than that required to produce the critical velocity a standing wave will form.

Mr. MONTAGUE'S two curves can be analytically given by

$$y = \tfrac{1}{2}\rho \cdot \frac{Q^2}{Z^2} + g\rho Z \qquad \text{and}$$

$$y = \tfrac{1}{2} g \rho Z^2 + p \frac{Q^2}{Z} \qquad \text{or}$$

$$g\rho Z^3 - Z^2 y + \tfrac{1}{2}\rho Q^2 \qquad \text{and} \quad \tfrac{1}{2} g \rho Z^3 - y r + \rho Q^2 = o$$

The energy curve has $y = g p Z$ and $z = o$. as the asymptotes and the momentum curve $z = o$ only as the asymptote. It will be seen that the curves are cubic and not parobolas, which by the way cannot have any asymptote.

The above is the simplest theoretical treatment of the subject. For more exact treatment, a solution of the hydrodynamical equations with the oper boundary conditions will be necessary.

MR. BURKITT said that he had been accused by Mr. Montagu of saying that the two-thirds depth occurred at the downstream end of a weir. He certainly thought so in 1919 but changed his opinion in 1920. He wished to point out to members of the Congress that Mr. Montague's experiments only dealt with a weir with a glacis of one in five and with a very broad crest. Where the crest is not broad, a depth greater than the two-thirds depth may occur at the downstream edge. In the design of the Ferozepore Weir, when calculating the height to which the gate must travel, it was assumed that the depth would be three-quarters, instead of two-thirds, at the downstream edge of the weir in high flood, still pond level being nineteen feet above weir crest and the breadth of the weir being four feet.

MR. WILSDON said that he would confine his remarks to the experimental part of the paper. The theoretical treatment did not differ from that of Hinds. All Mr. Montague's experiments had been made in a flume where the height of the weir and the slope of the galcis were kept constant. It was, therefore, essential to decide how far observations made under these conditions were applicable generally to full scale works. Mr. Montague himself had admitted in paragraph 13 of his paper that he had no reason to suppose that the expressions developed would hold for all weirs. In such cases Mr. Wilsdon stated that the study of models under dynamically similar conditions was essential. It was necessary to decide what dimensional quantities enter into the phenomena which are the subject of measurements.

The object of the measurements was to determine the point at which a jet of water will break up to form a standing wave. What factors were likely to determine this critical point? It was almost certain that the controlling factor was a pressure. At the free surface this pressure might be identified with the surface tension of the fluid. In the body of the fluid the internal cohesive forces were likely to be greatly modified by the presence of dissolved air. The point at which the wave would break up would then be determined by the cohesive forces becoming smaller than produced by the acceleration of the stream.

On such assumptions it was possible to derive by arguments from dimensions, formula in which all factors entered into a dimensionless function, *e.g.*, if, as was reasonable, we considered the surface tension was concerned as well as gravity velocity or discharge and density, a lineal dimension would enter. No two experiments would then be similar unless it were possible to assign corresponding values to all dimensional quantities. In other words, until we could introduce into our formula all the correct dimensions which control conditions, we could not apply the results of model experiments to full scale works. Mr. Wilsdon had attempted from the curves published by Mr. Montague to test a formula derived in this manner.

The AUTHOR in reply to Mr. Blench's criticism, explained that the flume was designed to produce velocities below the critical everywhere, but to approach the critical in the throat in order to economise waterway and therefore masonry.

When the bridge was first opened the two crisp little waves to which Mr. Blench referred were accompanied by a "heading up" upstream of the flume. As the water surface in the neighbourhood of the wave was actually just below the mark on the flume wall indicating the designed water surface, the Author concluded (*a*) that excessive friction had caused the heading up and that (*b*) in consequence, the velocity had overstepped the critical on entering the throat.

At this time, the Author was at work on the effects of bed curvature on pressure and energy-of-flow and the failure of this flume to function precisely as designed gave him an opportunity of testing the calculatous. The result has been briefly but clearly described by Mr. Blench and needed no endorsement by the Author.

In regard to Mr. Blench's suggestion regarding a reduction of curvature in subsequent designs the Author had secured considerable success by lowering the throat velocity and then reducing the curvatures. This has the effect of lengthening the work somewhat.

Referring to Mr. Khosla's contribution to the discussion, the Author pointed out that along with other critics he had lost sight of the principles of Dimensional and Dynamical Similarity. These applied strictly to the equations for energy of flow and momentum, so that they may be applied also to the method of deducing the position of the standing wave. The experiments were a small scale verification of the theory and presumably would apply to large scale also.

MR. Khosla had suggested a verification of the Author's theoretical methods on large scale works. Naturally this was to be desired, but the application of empirical coefficients as suggested in the paper and to which Mr. Khosla took exception could only be avoided by applying proper corrections for curvature.

MR. Khosla was perfectly justified in drawing attention to the importance of the Bed width to Depth ratio.

In regard to the co-efficients for the Bambanwala Regulator the Author referred Mr. Khosla to his reply to Mr. Burkitt.

MR. Khosla along with other enquirers seemed bent on measuring the length of disturbance following the standing wave. The Author submitted that such a dimension must always remain indeterminate theoretically, as it depended local factors. Also, the limits of disturbance were nebulous, and what might be harmful under one set of circumstances, could safely be neglected elsewhere.

In reply to Dr. Bose, the Author pointed that the advantage of the momentum treatment advocated by him over the pressure theory stated by Dr. Bose is that the former permits of direct measurements (and therefore of engineering design). The latter (so far as the Author can judge at short notice) appears to offer a qualitative explanation without the possibility of determining the position of the standing wave and its height in feet and inches.

In this connection it is interesting to note that the engineers of the Miami Conservancy Board made many experiments to discover the law relating to the height of the standing wave on a basis of air content, but the theoretical results were not supported by experimental evidence, and had to be abandoned in favour of the momentum theory un-adulterated.

In regard to Dr. Bose's final criticism, the Author admits to a looseness of expression. The curves are, of course, cubics and not parabolas.

DR. Bose, as did Mr. Blench, assumes that Bernouilli's law exhausts the hydro-mechanical treatment and turns to the hydrodynamical equations for further progress. The hydrodynamical equations are ordinarily out of the province of engineers and belong more to the province of the pure physicist.

The above remarks also covered Mr. Wilsden's contribution to the discussion, save in the matter of the dimensional homogeneity.

The Author disclaimed any intention of accusing Mr. Burkitt of anything.

MR. Burkitt. had published his 1919 views, regarding the $\frac{2}{3}$ depth on the downstream edge, but so far as the Author was aware, his conversion had not been broadcasted.

CORRESPONDENCE.

Mr. G. Lacey.

MR. Montagu deserves the thanks of the Congress for a lucid exposition of the fundamental principles underlying the problem of the energy of flowing water. He very properly makes no claim to originality for this portion of his paper and expresses his indebtedness to the classic article by Mr. Julian Hinds in the Engineering News Record, dated November 25th, 1920 (page 1034, Vol. 85, No. 22). Mr. Hinds in his paper disavowed all intention of adding to the fundamental theories already advanced and attempted merely to show how the established principles could be applied to the design of canals and canal structures. It is not possible critically to examine Mr. Montagu's paper without a previous study of the work of Julian Hinds.

MR. Montagu had pointed out in a discussion on another paper that the use of Hind's diagrams, or elaborations of them, would save much laborious calculation when headless meters or for that matter any kind of meter was being designed. The writer suggested to Mr. Montagu that a "log" plotting would be useful and discovered that Mr. Montagu had this in mind.

MR. Montagu in the latter part of his paper gives an original method of distributing losses over the glacis, and also at the toe due to shock. These losses are deduced from experiments performed under special conditions which must be reproduced before the calculated water profile can be held to apply to other works designed to a different profile.

APPENDIX.

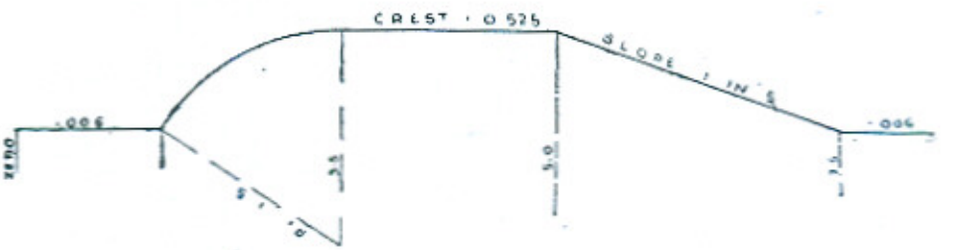
In the following tables, the information upon which the paper is based, is summarised. Those desiring full details of the actual observations from which the profiles are plotted and the subsequent calculations, are invited to apply direct to the Author.

1. *Table I.*

SUMMARY OF EXPERIMENTS.

Series Nos.	Experiments Nos.	Discharge.	Depths d/s.
I ..	1 to 7	2·07 cusecs	0·570 to 0·935
II ..	8 to 14	1·48 ,,	0·505 to 0·860
III ..	15 to 19	2·34 ,,	0·590 to 0·930
IV ..	20 to 27	0·94 ,,	0·330 to 0·735
V ..	28 to 34 & 35	2·32 ,,	0·600 to 0·960 & 1·01

2. Dimensions of test equipment.



SECTION OF WEIR.

The trough : maximum depth .. 1·5′
Width .. 2·0′
Slope .. Nil

Distance of zero from floating plane about 40′
Distance of zero from grid about 45′
Distance of vertical delivery pipe about 50′

3. The "energy of flow" and "pressure plus momentum" Curves, were plotted from data calculated in the form of tables as follows

Series.....Discharge.....Cusecs
Critical Depth.....feet

D	$V=\frac{Q}{2D}$	$\frac{V^2}{2g}$	$P=D^2$	$\frac{Q}{g}\times V$	V Col 4 + col 5.	Col 1 + Col 3.
1	2	3	4	5	6	7

4. A table is published below comparing the observed results (*i. e.*, position of standing wave) with the calculated position from the momentum formula.

Mr. Burkitt's method does not admit of the position of the wave being indicated when off the glacis. The space is left blank.

As the position of the standing wave is extremely sensitive when on the flat channel bed, the results have been shewn as *depths* above and below the wave.

Table of R. D.'s of Standing Wave determined by various methods.

Series.	Experiment No.	Reduced Distance of Standing Wave.		
		Physical observation by finger tips.	By Momentum Formula.	By Burkitt's method.
		Depths		
I ..	1 ..	u/s 0·18 d/s 0·56	u/s 0·18 d/s 0·525	
	2 ..	u/s 0·17 d/s 0·58	u/s 0·17 d/s 0·545	
	3 ..	7·9	7·75	7·35
	4 ..	7·4	7·36	7·09
	5 ..	6·8	6·86	6·54
	6 ..	6·1	6·23	5·89
	7 ..	5·5	5·55	5·39
II ..	8 ..	u/s 0·13 d/s 0·48	u/s 0·13 d/s 0·455	
	9 ..	7·8	8·00	7·38
	10 ..	7·4	7·23	6·96
	11 ..	7·0	6·70	6·22
	12 ..	6·1	6·22	5·86
	13 ..	5·7	5·65	5·40
	14 ..	..	..	..

Series.	Experiment No.	Reduced Distance of Standing Wave		
		Physical observation by finger tips.	By Momentum By Formula.	Burkitt's method.
III ..	15 ..	u/s 0·18 d/s 0·60	u/s 0·18 d/s 0· 59	..
	16 ..	8·1	8·38	..
	17 ..	6·8	7·16	6·86
	18 ..	6·7	6·50	6·32
	19 ..	6·0	6·00	5·63
V ..	20 ..	u/s 0·10 d/s 0·33	u/s 0·10 d/s 0·33	..
	21 ..	u/s 0·08 d/s 0·385	u/s 0.08 d/s 0·38	..
	22 ..	7·2	6·38	7·16
	23 ..	7·0	6·82	6·60
	24 ..	6·1	6·08	5·92
	25 ..	5·5	5·34	5·46
V ..	26 ..	7·3	7·13	6·94
	27 ..	6·2	6·08	5·78
	28 ..	u/s 0·19 d/s 0·60	u/s 0·19 d/s 0·58	..
	29 ..	u/s 0·17 d/s 0·63	u/s 0·17 d/s 0·62	..
	30 ..	7·5	7·55	7·32
	31 ..	7·1	6·86	6·60
	32 ..	6·5	6·35	6·08
	33 ..	5·9	5·95	5·67
	34 ..	5·75	5·66	5·44