PAPER No. 140.

EXPONENTIAL LAW OF SUBSOIL FLOW.

By

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1. Experiments on the Shahpur Minor, Gujranwala and Ferozepore.
2. Deductions from the results—Exponential Law and Extension of this law into a General Law.
3. Application of this law to deduce a generalised form of Darcey's Law—which holds only on the surface—as such the new law is more general.
4. Application of this law to find out the transmission constant of soils.

Early in 1928 a series of experiments with tube wells were started on the Shahpur Minor at Gujranwala and at Ferozepore. The object of the experiments on the Shahpur Minor was primarily to study the behavior of the subsoil water-table during and after pumping from the tube wells and incidentally to find out its radius of influence. Ferozepore experimental pumping was taken in hand to observe the effect of such pumping on the subsoil water-table and to calculate the direction and transmission constant of flow.

Gujranwala Tube Wells.—These tube wells were worked by air-lift. In the three diagrams plates Nos. 1, 2 and 3 of the water-table about Gujranwala, it will be seen that the contours are for three separate periods, before the pumping, during pumping and after pumping. In all, the general flow is towards the chappar; in the pre-pumping period, plate No. 1, this has not been disturbed, in the pumping period, plate No. 2 there is a distinct trough formed, the contour lines have been turned back near the tube-wells by the effect of the pumping and in the after-pumping period plate No. 3 this depression still persists though pumping had stopped for about six months. The effect of pumping on the subsoil water-table is shown very markedly by these contours and the radius of influence of these pumps extend considerably beyond 1,000′. The observation pipes for these tube wells extend to a distance of 1,000′ on either side and the drop in the water-table at this distance is between three to four feet, when pumps are working. These tube wells are too far away from Gujranwala city to have any appreciable effect on the water-table there; but if we calculate the amount of sub-soil flow that is available in a stretch of 3,000′ through the line of the tube-wells, it comes to about alf the amount of the total discharge of the pumps; the other half must e been drawn from the subsoil water-table itself.

Ferozepore Tube wells.—Plate 4 shows an arrangement of the Ferozepore tube wells. The data relate to the tube wells A, B. . . . The wells marked F. F′, E E′, H H′, I I′, J J′, K K′, are for fresh experiments. These tube wells were worked by centrifugal pumps; and it has been found that the direction of subsoil flow turns through 30° from its normal direction of steady flow, as soon as pumping is started.

The experiments consist mainly in taking the undisturbed subsoil water-level before any pumping was undertaken. This is denoted by H (*see* Appendix I).

Then as pumping was started, the heights of the water-table at different distances from the pumps were noted till a steady cone of depression was formed. Pumping was continued in this state for about a week, noting all the time, discharges from the pumps and the water-level in different observation pipes. Then pumping was stopped and the heights of the water-level in the different pipes at definite intervals were noted till the water-table obtained the steady level.

From these experiments it has been possible to deduce an Exponential Law of subsoil Flow which it has been found holds more generally than Darcey's Law, *viz*:

$$v = K\frac{h}{l}$$

where v is the velocity of flow, 'h' is the loss in the hydraulic-head between two points and 'l' is the horizontal distance between these points and K a constant for the soil known as the Transmission constant.

It is well known that Darcey's Law holds only on the surface of the subsoil flow—a very serious limitation to its applicability to general liquid flow in the soil. It was to remove this limitation and to find out a law which would be more generally applicable that the tube well data were examined.

The water-levels for each pipe were tabulated as shown in appendix and the quantity $\log \frac{H-Z_o}{H-Z}$ was calculated assuming for Z_o the water-level at time t_o and Z at t. These were plotted against $t-t_o$ and it was found that they fall for the pipes at Ferozepore along a series of parallel straight lines giving a relation such as

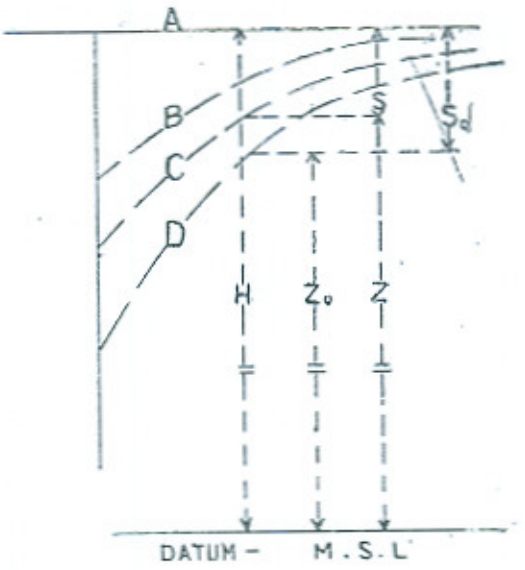
$$\log_e \frac{H-Z}{H-Z_o} = -\lambda (t-t_o) \qquad \text{(i)}$$

Now if $H-Z_o = s_o$ and

$H-Z = s$

we get $s = s_o\, e^{-\lambda (t-t_o)}$ (ii)

where s and s_o are the depths of the water profile, (or as usually understood the draw down curve) from the equilibrium level of the subsoil water at times t and t_o.



The curves A, B, C, D . . . give the profiles of the subsoil water-table at different periods ; A is the steady natural water surface ; D at time t_o. C at t and so on.

$$AD = H\text{-}Zo = s_o$$

$$AC = H\text{-}Z = s$$

The above relation (i) or (ii), it will be seen, holds generally for all the tube wells at the Shahpur Minor as well as at Ferozepore, and for all observation pipes irrespective of their distance from the pumps. Of course the factor λ is different for the experiments at the Shahpur Minor and at Ferozepore. This is what we should expect. This factor λ will be shown later to be a function of K the transmission constant of the soil (*see* equation ix).

It will first of all be proved that starting from this Exponential Law of subsoil flow all the laws of subsoil hydraulics can be deduced ; and that incidentally Darcey's Law will come out as a particular form of the general relation.

Before any further use of the above exponenual relation is made, a substitution will be necessary. This consists in replacing hydrodynamically the flow of water in a porous medium, such as a soil, by the flow of a very viscuous fluid. This substitution will be later on justified by the correctness of the deduction drawn from it.

The heights of water found in the observation pipes record the hydraulic pressures at the points in the soil at which the strainer points end. Hence though they might not record the actual watertable this being the case at Gujranwala where the watertable is in a clay layer—they record Hydraulic-Pressure-Head at the points which has been shown in a memoir of the Irrigation Research Laboratory equivalent to ϕ. As the observation pipes tap different points in the soil, this experimental relation holds for all points in the fluid. So that we get universally for the fluid medium.

$$\phi = \phi_o e^{-\lambda (t - t_o)} \quad .. \quad (3)$$

The stream function ψ which is a conjugate function to ϕ will therefore satisfy

$$\psi = \psi_o e^{-\lambda (t - t_o)} \quad .. \quad (4)$$

Before we proceed any further it will be well if the two quantities and ψ were defined more fully. Hydrodynamically we have the following relations between the velocities u, v, w and ϕ

$$u = -\frac{\delta \phi}{\delta x} = -\frac{\delta \phi_o}{\delta x} e^{-\lambda (t-t_o)} = -u_o e^{-\lambda (t-t_o)} \quad (5)$$

$$v = -\frac{\delta \phi}{\delta y} = -v_o e^{-\lambda (t-t_o)} \quad .. \quad (6)$$

$$w = -\frac{\delta \phi}{\delta z} = -w_o e^{-\lambda (t-t_o)} \quad .. \quad (7)$$

Accordingly u, v, w the velocity components along the three axes will also vary as $e^{-\lambda (t-t_o)}$.

The direction of flow at any point on the surface ϕ = constant will always tend to be perpendicular to the surface. These lines of flow—orthogonal to the ϕ lines—are known as ψ lines.

Thus we have the two curves ϕ and ψ cutting each other at right angles.

In the case of subsoil flow it will be seen that ϕ curves are identical with the curves of equal hydraulic-head.

Stokes equations of Motion of viscous fluid are the following :—

$$\frac{\delta u}{\delta t} + u \frac{\delta u}{\delta x} + v \frac{\delta u}{\delta y} + w \frac{\delta u}{\delta z} = X - \frac{1}{\rho} \frac{\delta p}{\delta x} + \nu \nabla^2 u \quad ..(8)$$

$$\frac{\delta v}{\delta t} + u \frac{\delta v}{\delta x} + v \frac{\delta v}{\delta y} + w \frac{\delta v}{\delta z} = Y - \frac{1}{\rho} \frac{\delta p}{\delta y} + \nu \nabla^2 v \quad ..(9)$$

$$\frac{\delta w}{\delta t} + u \frac{\delta w}{\delta x} + v \frac{\delta w}{\delta y} + w \frac{\delta w}{\delta z} = Z - \frac{1}{\rho} \frac{\delta p}{\delta z} + \nu \nabla^2 w \quad ..(10)$$

and $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0.$ ..(11)

is the equation of continuity, where $\nu = \frac{\mu}{\rho}$ is known as kinematic viscosity,

When the motion is very small, so that the inertia terms $u\frac{\delta u}{\delta x}$ *etc.*, can be neglected and the external force is only gravity we have,

$$X = Y = O, \text{ and } Z = g$$

and the above equations (8) (9) and (10) reduce to

$$\frac{1}{\nu}\frac{\delta u}{\delta t} = -\frac{1}{\mu}\frac{\delta p}{\delta x} + \nabla^2 u \qquad ..(12)$$

$$\frac{1}{\nu}\frac{\delta v}{\delta t} = -\frac{1}{\mu}\frac{\delta p}{\delta y} + \nabla^2 v \qquad ..(13)$$

$$\frac{1}{\nu}\frac{\delta w}{\delta t} = \frac{g}{\nu} - \frac{1}{\mu}\frac{\delta p}{\delta z} + \nabla^2 w \qquad ..(14)$$

The above equations can also be written in the following form :—

$$\frac{1}{\nu}\frac{\delta u}{\delta t} = -\frac{1}{\mu}\frac{\delta}{\delta x}(p - g\rho z) + \nabla^2 u \qquad ..(15)$$

$$\frac{1}{\nu}\frac{\delta u}{\delta t} = -\frac{1}{\mu}\frac{\delta}{\delta y}(p - g\rho z) + \nabla^2 v \qquad ..(16)$$

$$\frac{1}{\nu}\frac{\delta w}{\delta t} = -\frac{1}{\mu}\frac{\delta}{\delta z}(p - g\rho z) + \nabla^2 w \qquad ..(17)$$

because $(g\rho z)$ being a constant quantity with respect to x and y, $\frac{\delta}{\delta x}(g\rho z)$ and $\frac{\delta}{\delta y}(g\rho z)$ vanish and $\frac{\delta}{\delta z}(g\rho z)$ reduces to $g\rho$.

Now from equation (5)

$$u = u_o e^{-\lambda(t - t_o)}$$

$$\frac{\delta u}{\delta t} = -\lambda u_o e^{-\lambda(t - t_o)} = -\lambda u \qquad ..(18)$$

Substituting this in (15) we get.

$$-\frac{\lambda}{\nu}u = -\frac{1}{\mu}\frac{\delta}{\delta x}(p - g\rho z) + \nabla^2 u$$

$$(\nabla^2 + a^2)u = \frac{1}{\mu}\frac{\delta}{\delta x}(p - g\rho z) \qquad (19)$$

where $a^2 = \frac{\lambda}{\nu}$.

Similarly we get

$$(\nabla^2 + a^2)\, v = \frac{1}{\mu} \frac{\delta}{\delta y}(p - g\rho z) \quad .. \quad (20)$$

$$(\nabla^2 + a^2)\, w = \frac{1}{\mu} \frac{\delta}{\delta z}(p - g\rho z) \quad .. \quad (21)$$

Now differentiating (19) (20) and (21) with respect to x, y and z respectively we get

$$(\nabla^2 + a^2) \frac{\delta u}{\delta x} = \frac{1}{\mu} \frac{\delta^2}{\delta x^2}(p - g\rho z)$$

$$(\nabla^2 + a^2) \frac{\delta v}{\delta y} = \frac{1}{\mu} \frac{\delta^2}{\delta y^2}(p - g\rho z)$$

$$(\nabla^2 + a^2) \frac{\delta w}{\delta z} = \frac{1}{\mu} \frac{\delta^2}{\delta z^2}(p - g\rho z)$$

Adding these we get

$$(\nabla^2 + a^2)\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}\right) = \frac{1}{\mu} \nabla^2 (p - g\rho z)$$

Now from equation (11) we have, for continuity,

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$$

so that $$\nabla^2 (p - g\rho z) = 0 \quad .. \quad (22)$$

Now $$u = \frac{1}{a^2 \mu} \frac{\delta}{\delta x} (p - g\rho z)$$

is a particular solution of (19) for, by substituting this in the latter we have

$$(\nabla^2 + a^2) \frac{1}{a^2 \mu} \frac{\delta}{\delta x} (p - g\rho z) = \frac{1}{\mu} (p - g\rho z)$$

or $$\frac{1}{a^2 \mu} \frac{\delta}{\delta x} \nabla^2 (p - g\rho z) + \frac{1}{\mu} \frac{\delta}{\delta x} (p - g\rho z) = \frac{1}{\mu} \frac{\delta}{\delta x} (p - g\rho z)$$

which is an identity by (22). Thus we get as a particular solutions of the differential Equations (19) (20) and (21).

$$u = \frac{1}{a^2 \mu} \frac{\delta}{\delta x} (p - g\rho z) \quad .. \quad (23)$$

$$v = \frac{1}{a^2 \mu} \frac{\delta}{\delta y} (p - g\rho z) \quad .. \quad (24)$$

$$w = \frac{1}{a^2 \mu} \frac{\delta}{\delta z} (p - g\rho z) \quad .. \quad (25)$$

These expressions give the most general formulae for subsoil velocities.

The differential form of Darcy's law

$$u = K \frac{\delta h}{\delta x}$$

can easily be deduced from these. On the free surface we have p = cons. so that from (23) we get

$$u = \frac{1}{a^2 \mu} \frac{\delta}{\delta z} (- g \rho z)$$

$$= - \frac{g\rho}{a^2 \mu} \frac{\delta z}{x}$$

$$= - k \frac{\delta z}{\delta x}$$

Thus Darcy's Law holds only on the free surface and confines itself only to the horizontal components of the subsoil velocity.

In conclusion the paper may be summed up with the following remarks. It is the **Exponential law**

$$s = s_o e^{-\lambda (t - t_o)}.$$

which is really the **fundamental equation** in subsoil hydraulics ; all other relations such as the above equations (23) (24) and (25) or the Darcy's relation can be deduced from it mathematically.

APPENDIX.

These data are collected from measurement of water-levels after pumping had been stopped. Those water-levels (z) have been generally considered that were observed after sometime had elapsed since the pumps were stopped, as it was found that the water-level records were rather very irregular just after the cessation of pumping. The height of the natural water-surface, H, was taken that steady value of the water-surface which was observed just before pumping started or about a fortnight after it stopped, if they agreed among themselves, but if they varied very much the latter data only was considered. It was found that they differed when there was a general upward tendency of the water-level before the pumping started.

Well levels on the Shahpur Minor Gujranwala Tube Well A. 2-29.

$H = 734{\cdot}95$ feet.

Date.	Interval since pumping ceased.	Time.	Z	H—Z	$\log_{10}(H-Z)$	$\log \frac{H-Z}{H-Z_o}$
2-2-29 ..	3 hours.	8 p.m.	734·0	0·95	—·022	
3-2-29 ..	15 ,,	8 a.m.	734·25	0·70	—·155	—·133
3-2-29 ..	27 ,,	8 p.m.	734·45	0·50	—·301	—·279
4-2-29 ..	39 ,,	8 a.m.	734·57	0·38	—·420	—·398
4-2-29 ..	51 ,,	8 p.m.	734·65	0·30	—·523	—·501
5-2-29 ..	63 ,,	8 a.m.	734·73	0·22	—·658	—·636
5-2-29 ..	75 ,,	8 p.m.	734·78	0·17	—·770	—·748
6-2-29 ..	87 ,,	8 a.m.	734·83	0·12	—·921	—·899
6-2-29 ..	99 ,,	8 p.m.	734·87	0·08	—1·097	—1·075

$$\frac{H-Z}{H-Z_o} = e^{-\lambda (t-t_o)}$$

$$\text{Log}_e \frac{H-Z}{H-Z_o} = -\lambda (t-t_o)$$

$$\lambda = \frac{\log_e \frac{H-Z}{H-Z_o}}{t-t_o} = \frac{\log_{10} \frac{H-Z}{H-Z_o}}{(t-t_o) \times \cdot 43}$$

from plate No. 5. $= \frac{\cdot 8^*}{74 \times \cdot 43} = \cdot 025$ where unit of time is hour.

$= \cdot 7 \times 10^{-5}$ in F. G. S. units.

**Note.*—0·8 is taken off the curve A 2·29 Gujranwala, $t-t_o = 74$, λ is obviously the slope of this curve.

Well levels near Ferozepore A. 1·29.

$H = 635{\cdot}99$

Time interval = 24 hours.

Z	H—Z	Log (H—Z)	log $\frac{H-Z}{H-Z_o}$	
635·37	·62	$\bar{1}{\cdot}792$		
			—·120	log H—Zo = $\bar{1}{\cdot}792$
·52	·47	$\bar{1}{\cdot}672$		
			—·287	
·67	32	$\bar{1}{\cdot}505$		
			—·361	
·72	·27	$\bar{1}{\cdot}431$		
			—·412	
·75	·24	$\bar{1}{\cdot}380$		
			—·450	$\bar{1}{\cdot}792$
·77	·22	$\bar{1}{\cdot}342$		
			—·491	$\bar{1}{\cdot}230$
·79	·20	$\bar{1}{\cdot}301$		
			—·562	—·562
·82	·17	$\bar{1}{\cdot}230$		
			·713	
·87	·12	$\bar{1}{\cdot}079$		

$\log \frac{H-Z}{H-Z_o} = -{\cdot}425$ Significant

Standard deviat $\sigma = {\cdot}166$ where the interval is 24 hours.

$$\lambda = \frac{{\cdot}7}{7{\cdot}5 \times {\cdot}43 \times 60 \times 60 \times 24} \text{ in F. G. S.}$$

$$= 0{\cdot}25 \times 10^{-5} \text{ in F. G. S.}$$

$$t = t_o = \frac{1}{\lambda'} \times \log \frac{H - Z_o}{H - Z}$$

$$\lambda' = \frac{1}{t - t_o} \times \log \frac{H - Z_o}{H - Z}$$

$$\lambda' = \lambda \times \quad ,$$

Ferozepore Aa$_1$ 1·29.

H = 635·90

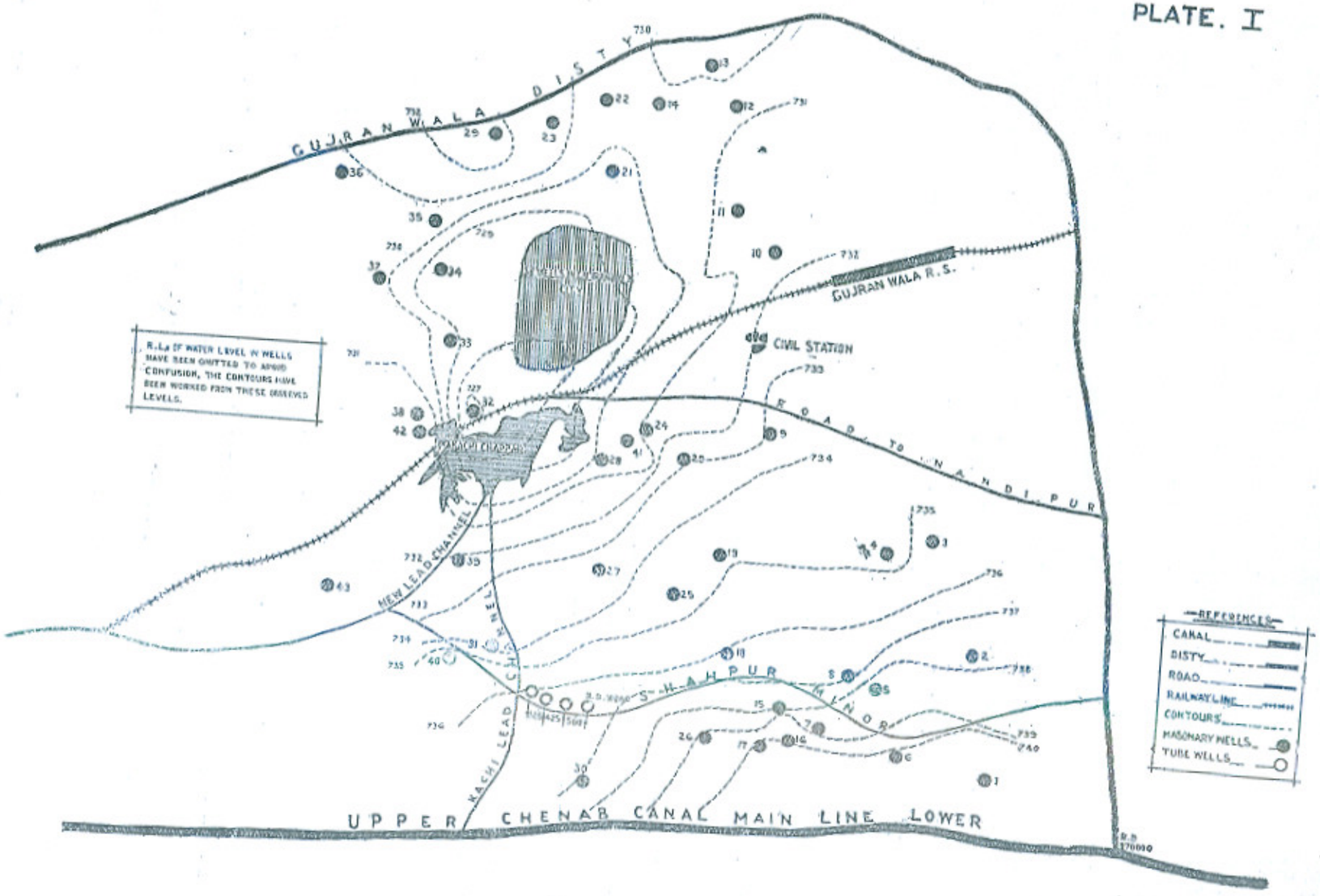
Time interval =24 hours.

Z	H—Z	log (H—Z)	log $\frac{H-Z}{H-Z_o}$
635·30	·60	$\bar{1}$·778	
·40	·50	·699	—·079
·55	·45	·653	—·125
·60	·30	·477	—·301
·65	·25	·398	—·380
·67	·23	·362	—·416
·70	·20	·301	—·477
·75	·15	·176	—·602
·77	·13	·114	—·664

$$\lambda = \frac{\cdot 5}{5\cdot 6 \times 24 \times 60 \times 60 \times \cdot 43} \text{ in seconds.}$$

$$= 0\cdot 25 \times 1^{-5} \text{ in F. G. S.}$$

PLATE. I

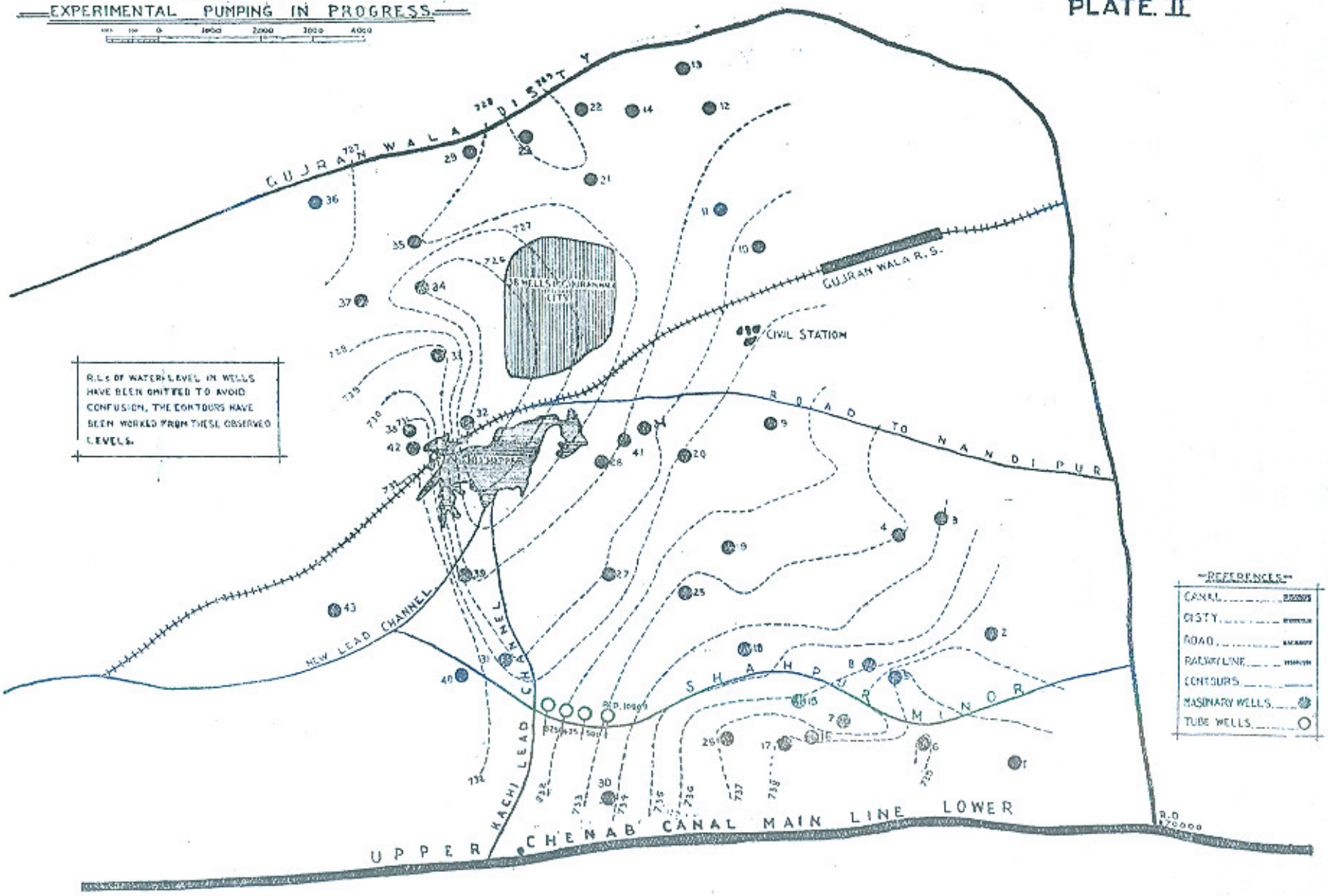

GUJRANWALA TUBE WELLS
SEPT 1927
BEFORE EXPERIMENTAL PUMPING COMMENCED
GUJRANWALA DISTY
GUJRAN WALA R.S.
CIVIL STATION
ROAD TO NANDIPUR
NEW LEAD CHANNEL
KACHI LEAD CHANNEL
SHAHPUR MINOR
UPPER CHENAB CANAL MAIN LINE LOWER
R.L.s OF WATER LEVEL IN WELLS HAVE BEEN OMITTED TO AVOID CONFUSION, THE CONTOURS HAVE BEEN WORKED FROM THESE OBSERVED LEVELS.
REFERENCES
CANAL
DISTY
ROAD
RAILWAY LINE
CONTOURS
MASONARY WELLS
TUBE WELLS

GUJRAN WALA TUBE WELLS

MARCH 1928

EXPERIMENTAL PUMPING IN PROGRESS

PLATE. II

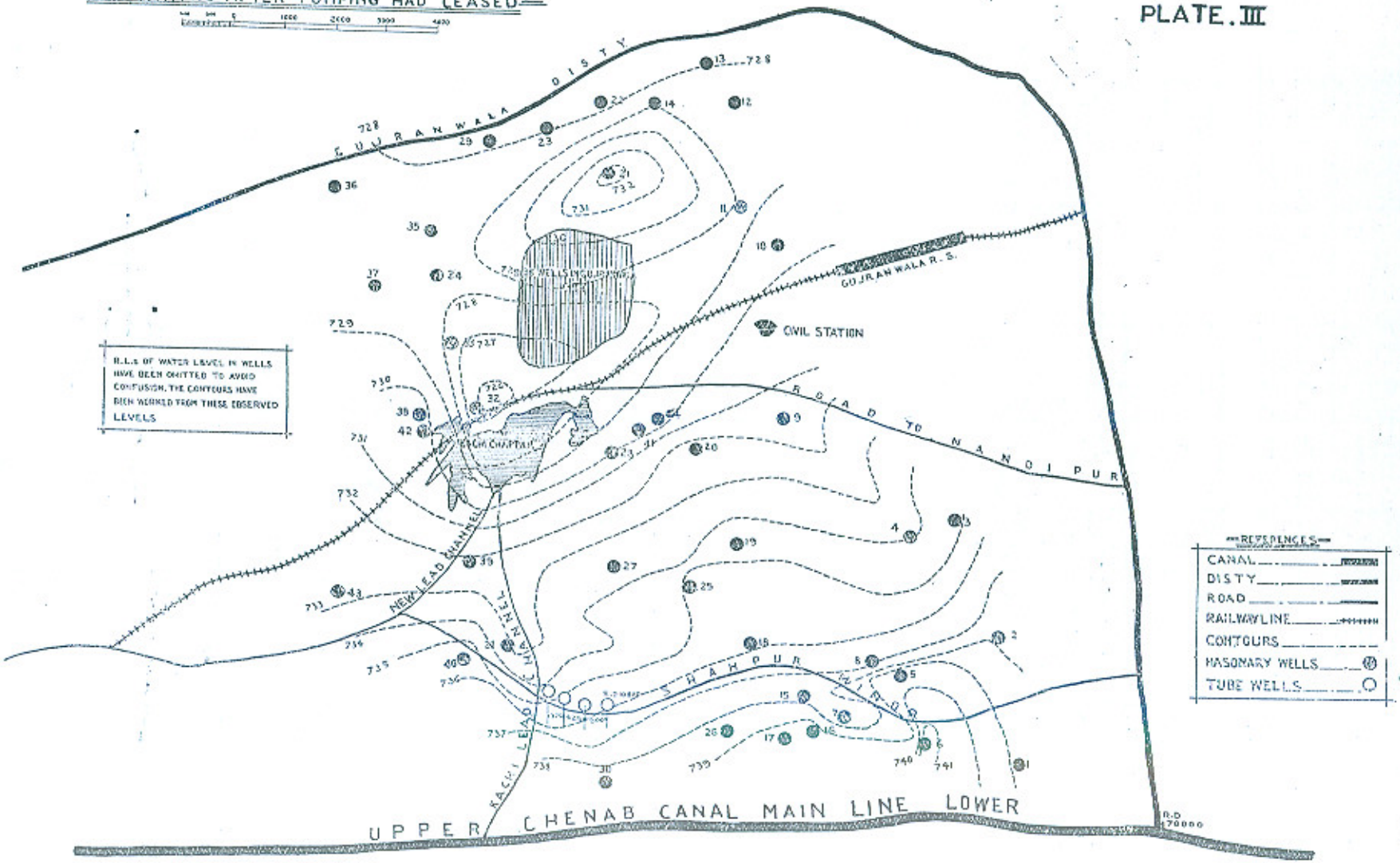


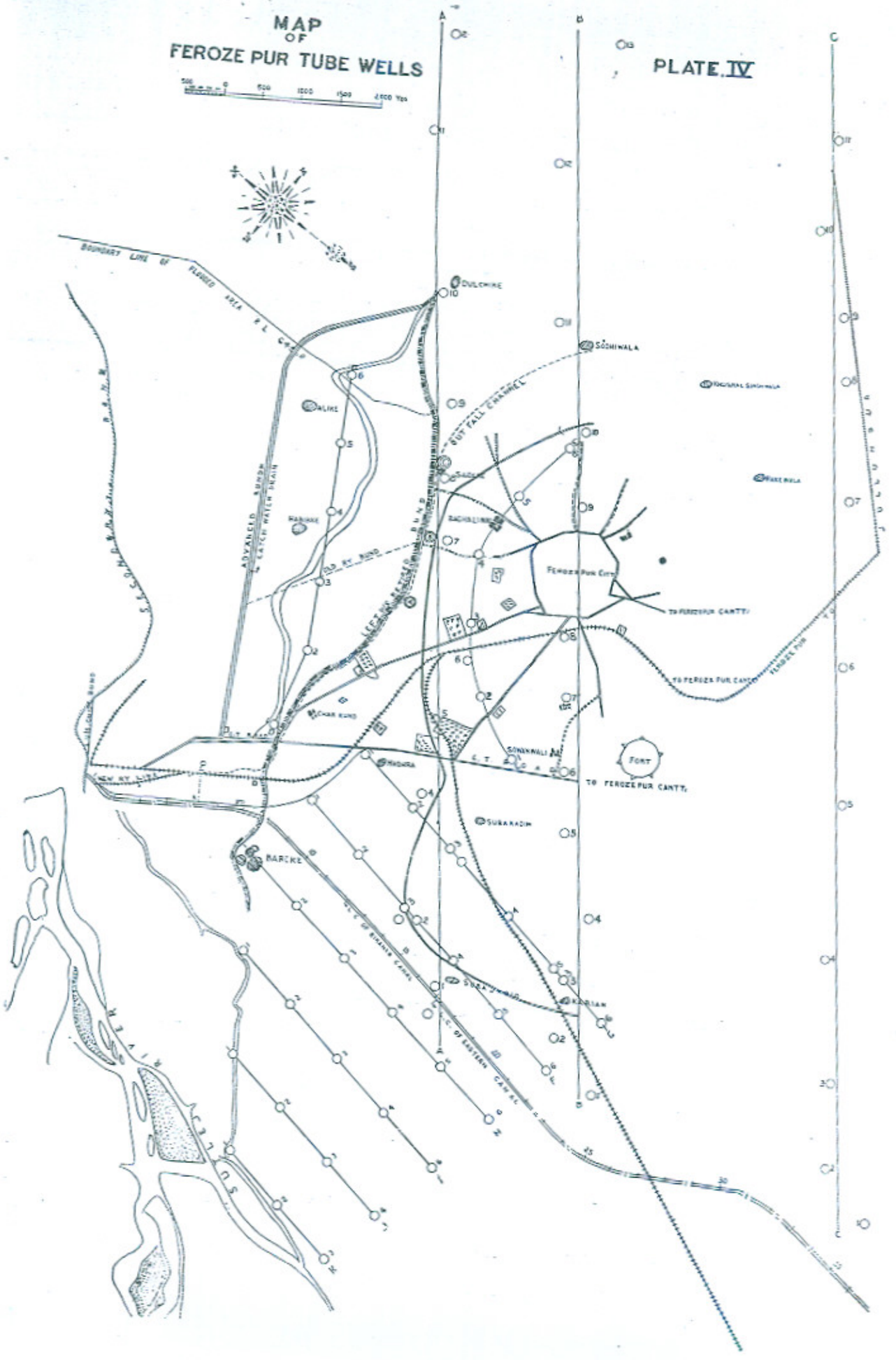
GUJRAN WALA TUBE WELLS

OCTOBER 1929.

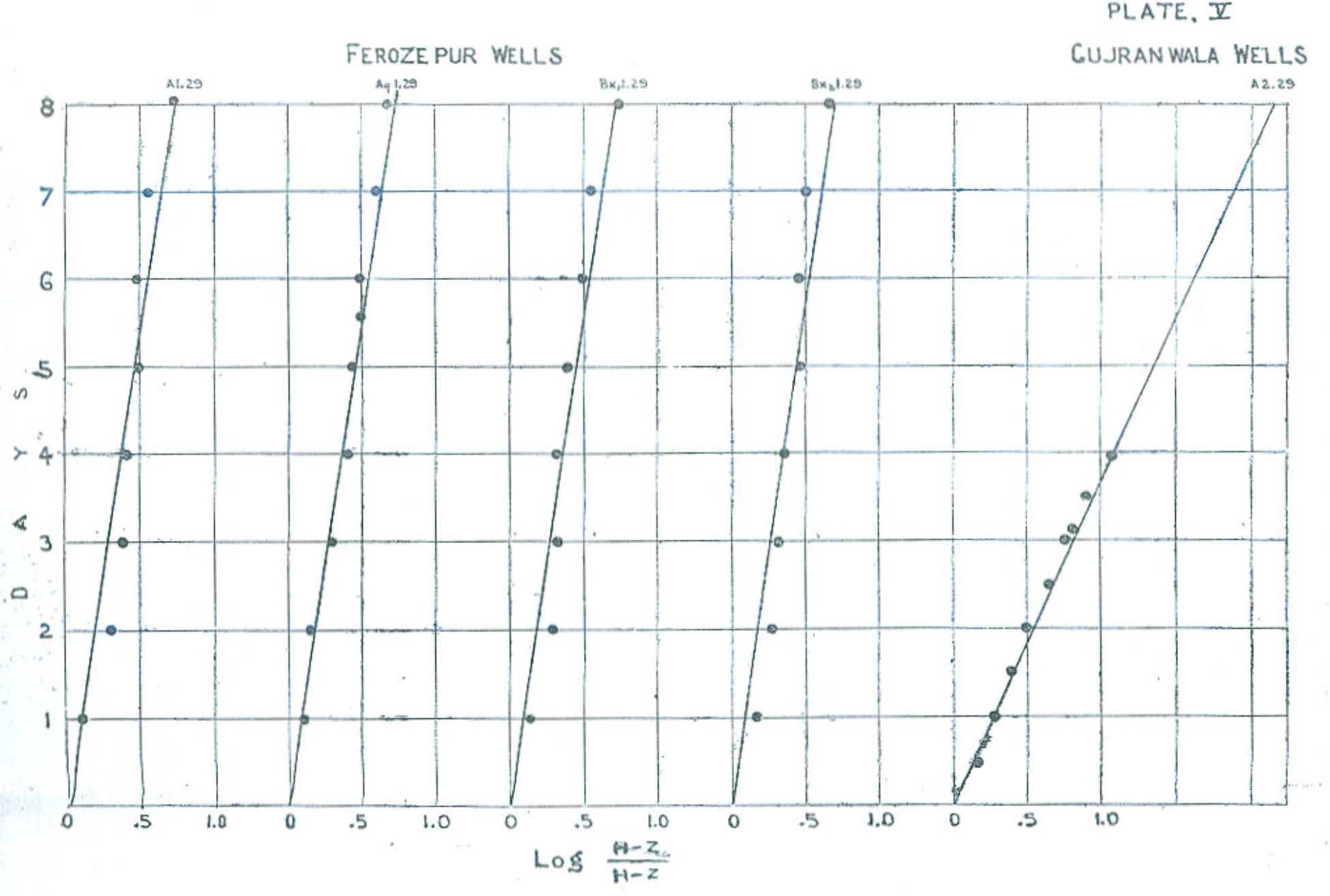
SIX MONTHS AFTER PUMPING HAD CEASED

PLATE. III




MAP
OF
FEROZE PUR TUBE WELLS
PLATE IV
DULCHIKE
SODHIWALA
ALIKE
SUT FALL CHANNEL
RACHBLINKE
FEROZE PUR CITY
TO FEROZE PUR CANTT.
FORT
SUBAKADIM
BARCKE
KARIAN
ADVANCED BUNDH
CATCH WATER DRAIN
OLD RY BUND
NEW RY LINE
C.T. ROAD
C.C. OF BIKANER CANAL
C.C. OF EASTERN CANAL
SUTLEJ RIVER

PLATE, V


FEROZEPUR WELLS
GUJRANWALA WELLS
A1.29
Aq 1.29
Bx1.29
Bx2 1.29
A2.29
8
7
6
5
4
3
2
1
DAYS
0
.5
1.0
Log (H-Zo)/(H-z)

Ferozepore Bx$_1$ 1·29.

H = 636·60.

Time interval = 24 hours.

Z	H—Z	log (H—Z).	t	(t—t_o)	log $\frac{H-Z}{H-Z_o}$
636·18	0·42	$\bar{1}$·623	t_1		
·28	·32	·505	1	t_1 1	—0·118
·38	·22	·342	2	t_1 2	—0·281
·40	·20	·301	3	t_1 3	—0·322
·40	·20	·301	4	t_1 4	—0·322
·43	·17	·230	5	t_1 5	—0·393
·46	·14	·141		6	—0·482
·48	·12	·079		7	—0·544
·52	·08	$\bar{2}$·903		8	—0·720

$\lambda = 0{\cdot}25 \times 10^{-5}$ F. G. S. units.

Ferozepore Bx $_2$ 1·29.

H = 636·20.

Time interval=24 hours.

Z	H—Z	log (H—Z)	log $\frac{H-Z}{H-Z_o}$	t—t$_o$
635·88	·32	$\bar{1}$·505		
635·98	·22	$\bar{1}$·342	—0·163	1
636·03	·17	$\bar{1}$·230	—0·275	2
636·03	·17	$\bar{1}$·230	—0·275	3
636·05	·15	$\bar{1}$·176	—0·329	4
636·08	·12	·079	—0·426	5
636·08	·12	·079	—0·426	6
636·10	·10	$\bar{1}$·000	—0·505	7
636·13	·07	$\bar{2}$·845	—0·660	8

$\lambda = 0{\cdot}25 \times 10^{-5}$ F. G. S.

DISCUSSION.

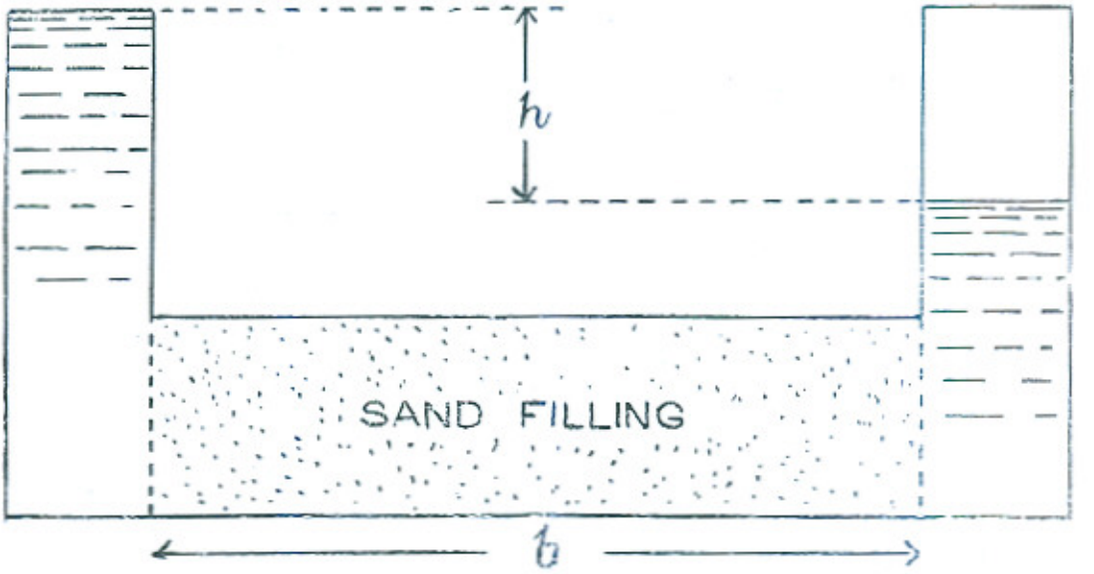
In introducing the paper to the audience Dr. Bose made the following remarks :—

In discussing the problems of subsoil hydraulics frequent reference to Darcy's law have been made. Some have even expressed it as being :—

$$v = K \frac{dh}{dx}$$

For a true appreciation of this paper it was essential briefly to review Darcy's Law, with its scope and limitations.

In 1856 H. Darcy carried out a series of experiment in order to find out some empirical relation for the flow of water in a porous medium like the soil.



The above diagram illustrated the nature of his experiments. There was an arrangement for keeping the water level constant at the upstream and downstream tanks ; there was also a device for measuring the total volume of flow. So that knowing the discharge per second and the sectional area, a value for the average velocity could be calculated. Noting this for different values of head " h " Darcy found that

$$v = \frac{Q}{A} = K \frac{h}{l}$$

where v was the average velocity, Q the discharge A the cross section at area, h the loss in head and l the total length of sand.

The limitation of this Law was apparent. This relationship had been deduced only from horizontal flow ; and as such could only apply

to laminar flow in horizontal directions, or horizontal components of such flow; the other components being incapable of determination by this law. The same relationship was deduced by C. S. Schlichter in 1899 in the Annual Report of the United States Geological Survey by geometrical treatment of the problem of capillary flow in a porous medium. He put it in the form

$$v = 771 \frac{d^2}{K} j \text{ cms per sec.}$$

where d was the average diameter of the soil particles, j the slope $\frac{h}{l}$ and k a constant which depended on the degree of packing.

It was in order to do away with this serious limitation of Darcy's law that the tube well data were collected and analysed. Darcy's law strictly speaking was simply

$$v = K \frac{h}{l},$$

and the idea of putting this in the differential form

$$v = K \frac{dh}{dl}$$

was more a result of intuitional perception than of experimental research or mathematical analysis.

The exponential law of his paper was differential in form and general in application as it had been derived from data which have been obtained by taking points in a liquid medium where the stream lines were neither restricted to the horizontal nor to the free surface. Since any point had been taken in the body of the liquid for deducing this law, its application was general through out the liquid without directional or dimensional limitations.

With these remarks the Author introduced his paper.

Mr. Khosla in opening the discussion expressed the opinion that Dr. Bose's paper was a most important contribution to a knowledge of subsoil hydraulics. The general equation of flow worked out by the author at the end of his paper must be taken as a fundamental equation and not merely as a generalisation of the well known Darcy's law. Dr. Bose in his introductory remarks had already brought out the limitation of Darcy's experiments and his deductions thereform. In brief, Darcy's experiments related only to horizontal flow between parallel planes and therefore, strictly speaking his relation $v = K \frac{h}{l}$ could not be applied beyond the limits of his experiments. Darcy's pressure gradient line had

one definite slope and connoted a constant velocity and therefore a constant discharge under any one set of experiments. Its application to conditions of varying rate of change of pressure or of flow in any but the horizontal direction had no justification either in theory or from experimental data. Dr. Bose's equation gave the necessary justification. It would therefore not be out of place to name this most important and fundamental equation as the "Bose Law." The application of this theory to the various engineering problems dealt with in the speaker's paper and to the fresh problems introduced in the discussions thereon by Messrs. Haigh and Montagu had been fully dealt with by the speaker in his replies to their criticism.

The mathematical treatment was based on actual observations, the only substitution being his replacement, hydrodynamically of the flow of water in a porous medium by the flow of a very viscuous fluid. The deductions justified this substitutions.

Dr. Bose had dealt with the theory of subsoil flow in general and from there he had deduced the particular law of Darcy for the free surface. In this connection it might be mentioned that the filter points of the observation pipes were fixed some distance below the surface. The observations recorded the pressures at those points. During the observations those points remained unchanged so that the rise of water level in the pipes was entirely due to the period of time between the various sets of observations. Thus the investigation was a true representation of the condition of flow at any point in the body mass. As such, the treatment was general and the conditions of the free surface were only a particular case.

The recuperation curves of page 175 which formed the basis of Dr. Bose's exponential law indicated the same phenomenon as described in the speaker's paper on Hydraulic Gradients, namely, that as soon as water level was raised in a drain, the pressure in all the stand-pipes rose correspondingly, the rise being a maximum at the point of application of pressure, and diminishing with the increase of distance from that point.

The Author of the paper had rendered a service of great importance to the engineering profession, and in particular to Irrigation Engineers, by producing a general equation which could be applied equally to problems of water-logging and pumping, and to those governing the stability of structures in saturated soils. Dr. Bose's law was of world-wide importance as it was the first general mathematical deduction of this subject, and was based upon a series of reliable experimental results. While equation (*ii*) was the fundamental equation, called the exponential law of subsoil hydraulics, the equations, 23, 24 and 25 gave the general equations of subsoil velocities which were the main concern of the engineer.