PAPER No. 141.

ENERGY LOSSES OVER LONG CRESTED WEIRS.

By Messrs. J. D. H. Bedford & A. M. R. Montagu.

CONTENTS.

	Para.
Notation	
Discharge over long crested weir	1
Nature of the constant K	2
True head H	3
Gauge reading differs from H	5
Attempts to reconcile discharge based on G	6
Causes of difference between G and H	10
Measurement of Q	11
Calculation of H	12
Importance of hydraulic mean depth, M	13
Approximate calculation of M	14
Curve connecting M and % loss of H	15
Deduction therefrom	ibid
Description of site of experiment	16 et seq
6 Tables of of observations and calculations	
1 diagram of experimental Weir	
1 graph connecting M and H_L %	

NOTATION.

A	..	Cross sectional area of a stream.
B	..	Widths of crest.
C	..	Empiric constants.
G	..	Gauge reading (above crest level).
g	..	Gravity constant.
H	..	True head acting (*i.e.*, total energy expressed in feet head of water) at downstream edge of Weir crest.
H_L	..	A loss of head between gauge site and downstream edge of crest.
K	..	A constant $\sqrt{g} \times (\frac{2}{3})^{\frac{3}{2}}$ $=3{\cdot}08880....$
M	..	Hydraulic mean depth
Q	..	Total discharge in cusecs
V	..	Velocity : V/a velocity of approach.
W. P.	..	Wetted perimeter.

ENERGY LOSSES OVER LONG CRESTED WEIRS

By Messrs. J. D. H. Bedford & A. M. R. Montagu.

The theoretical discharge over a long crested semi-modular Weir is given by the formula

$$Q = K \times B \times H^{\frac{3}{2}}$$

where Q is the discharge

K is a constant.

B is a width of the weir.

H is a head acting.

2. Some of these terms will repay a brief examination.

The theoretical value of the constant K is $\sqrt{g} \times (\frac{2}{3})^{3/2}$ and is of course a true constant in the same sense that "g" or "π" are true constants. Its appriximate value is 3.08880....and in this paper K invariably represents this value.

3. The "head acting" H is the total energy expressed in "feet head of water" and is the summation of the depth, velocity, pressure energies. This "head" should be measured at the down stream edge of the weir crest where alone the minimum energy can exist.

4. Owing to the presence of curvature in the stream lines, the depth, pressure and velocity energies are all redistributed and it is not possible, at present, to derive this quantity H from any direct measurement at the down stream crest.

In this paper the real or "true" head acting at the down stream crest is designated by H.

5. In practice it is necessary to read a gauge sufficiently far up stream to be clear of the preliminary "drawdown" or curvature of the water surface. In this paper, this gauge reading is designated G and it is the purpose of this paper to discuss the differences invariably found to exist between G and H.

6. The fact that this error does occur, is generally recognized and attempts have been made to correct the calculated discharge by applying an arbitrary co-efficient.

The formula then becomes

$$Q = C \times K \times B \times G^{3/2}$$

This co-efficient C may vary from 0·6 to 0·9999 depending on the circumstances of each individual case.

7. The best work on the subject which is readily available to officers of the Punjab Irrigation is that of Mr. Fane in his paper No. 110 read before this Congress in 1927 and that of Mr. Inglis of the Bombay Irrigation Research Circle. In both cases, a series of curves have been drawn giving the value of $(C \times K)$ plotted for various "drowning ratios" and widths of weir.

8. The result of this work has been to prove conclusively that a separate co-efficient is required for each combination of G and B as well as for different frictional surfaces and curvatures of the weir.

In short, no matter how carefully the work has been done within a given range of discharge, it is not applicable outside that range, and only approximately applicable within that range to weirs built of different materials and of different curvatures.

9. The present writers hold the view that a more promising line of attack lies in measuring the loss of head between the gauge site and the down stream edge of the weir crest.

As indicated in para. 3, the *true* head acting is that which causes the discharge to pass *at* the downstream edge of the crest, and can be calculated from experimental observed discharges.

The true head so found is always less than the gauge reading and the difference is the loss of head (energy) between the gauge site and the downstream crest.

10. The various components of this loss of energy are :—

(*a*) Friction.

(*b*) Shock at entry to the weir.

(*c*) Improper curvatures of the bed and sides.

It is not the purpose of this paper to differentiate between the effects of the different sources of loss, but to indicate that the total effect is better represented by correcting the value of the reading of G than by a purely arbitrary variation of the " Constant."

11. To this end, discharges varying from 1·5 cusecs to 45 cusecs, have been passed over weirs varying from 0·5 feet wide to 6·25 feet wide. In describing the form of the experiments, (*vide* paras. 16 *et seq*) it has been indicated that the volume of the discharge has been measured fairly accurately, probably to within one per cent.

12. As soon as the rate of discharge was ascertained, the true head acting (H) was calculated from the formula

$$Q = K \times B H^{3/2}$$

this head H acts only at the downstream edge of the weir.

The gauge reading G at some point above the weir (beyond the effect of the draw-down) had been recorded. G has been corrected for velocity of approach V_a, of course, whereas H automatically includes this correction, representing as it does, the total available energy of the water at the downstream crest of the weir. The difference between G (including correction for V_a) and H represents the total loss of energy between the two points.

13. Most of the hydraulic flow formulæ are related to the hydraulic mean depth M, and the enquiries of the Authors lead to a belief that the phenomena under investigation were similarly related.

M, hydraulic mean depth

$$= \frac{\text{A (Cross sectional area)}}{\text{W. P. (wetted perimeter)}}$$

But how to obtain the average of these quantities between the gauge site and the downstream crest of the weir?

14. In what follows, the Authors freely admit the consequences of approximation, based on the theoretical assumption, the results of which now to be discussed, are themselves the justification of the method adopted.

It is known that, theoretically, the depth over the crest should be $\frac{2}{3}$ of the true head. Mr. Montagu's profiles (Congress paper No. 126 of 1929) clearly indicate that this is not so, and why? But it is nearly accurate to assume that the average depth over the weir crest is $\frac{2}{3}$ the true head, and to continue the calculations on this basis.

15. From the true head H, derived as per para. 12 the depth, area, wetted perimeter and M were all calculated simply enough on this assumption.

Various quantities were plotted one against another, some of which revealed experimental errors which had to be corrected subsequently.

At last the following curve was traced :—

Vertically the hydraulic mean depth.

Horizontally the percentage loss of head, *viz* :—

$$\frac{G-H}{H} \times 100$$

This curve is now presented.

It will not be denied that the points plotted hereon indicate some relation————a relation which is the more remarkable in view of the admitted approximation regarding calculation of M (**vide** para. 14).

From this curve the authors submit the following deductions :—

(*i*) All the observations lie very approximately on one curve which is independent of discharge, or of the ratio of H (or G) to B of the weir.

In this respect, alone, the curve indicates a great advance over all other experiments of this nature.

(*ii*) The discharge, through the hydraulic mean depth, selects the region of the curve and automatically takes care of the ratio of H (or G) to B of the weir. This is also an advance.

(*iii*) It cannot be said to what extent the material of the weir will affect the results. This is matter for further experiment.

(*iv*) As M falls, the % loss increases: when M is small, the % loss increases rapidly.

This indicates that for small discharges smoothness of weir and material is of first importance and that a portion of the losses is of the nature of a constant, possibly dependent on the distance of the gauge site above the weir.

(*v*) As M increases the % loss falls. When M is big, the % loss decreases very slowly indeed.

This indicates that for large discharges friction is of less importance than curvature. At the worst, the "Curvature" can only be the worst possible, and the loss in that case a maximum. For large discharges this would appear to be in the neighbourhood of three per cent.

Friction over this type of weir can add but little to the loss.

(*vi*) Since curvature at large discharges is of importance, experiments on a large scale are clearly desirable.

16. The experiments were carried out at Malikpur at mile 6 of the Upper Bari Doab Main Canal. The accompanying sketch shows the constructional details of the flume. The maximum discharge of the supply channel is about 50 cusecs. The approach channel on to the flume is 100 feet long, and at the upstream there is an escape weir that was utilized to eliminate any small variation of supply entering the experimental channel at the head. The maximum depth of water on crest was taken to be 4·75 feet and the length of crest built. $4{\cdot}75 \times 3$ feet = 14·25 feet. Actually the greatest depth obtained on the crest did not exceed 4·0 approximately.

17. The maximum width of crest on which experiments were made was 6·25. After each set of experiments the side walls were dismantled and rebuilt to the width required, the curve of side walls being the same for all experiments. The floor was made truly horizontal with cement plaster and the width between side walls were made accurate in all cases with cement plaster; the largest variation in width on the whole length of side walls did not exceed ·008 feet.

18. All gauge readings were done by means of telescopes. The water passing over the flume was accurately measured in a pacca tank that had previously been calibrated. A two way water switch existed downstream of the weir so that water could be switched into or out of the masonry tank. Six sets of experiments were carried out for width of flume as follows:—

6·25′, 3·5′, 2·2′, 1·5′, 1·1′, ·5′.

19. The results of the experiments are given in tabular form in the statements attached.

Signed for Messrs. J. D. H. Bedford and A. M. R. Montagu.

A. M. R. MONTAGU,

29th November 1929.

B=6·25′

$$H^{3}/_{2}=\frac{Q}{6\cdot25\times3\cdot089}=\frac{Q}{19\cdot3}$$

G	Q	$H^{3}/_{2}$	H	H_L =G—H	$\frac{H}{H}\times100$	$\frac{2}{3}H$	WP= B+$^{4}/_{3}$ H	A= B×$\frac{2}{3}$H	M= $\frac{A}{Wr}$
1·813	44·912	2·327	1·756	0·057	3·245	1·17	8·59	7·312	0·85
1·536	34·964	1·812	1·485	0·051	3·434	0·990	8·23	6·187	0·752
1·332	28·169	1·46	1·286	0·046	3·58	0·857	7·964	5·356	0·672
1·0584	19·922	1·032	1·021	0·0374	3·664	0·681	7·612	4·256	0·559
0·778	12·414	0·643	0·746	0·032	4·285	0·497	7·244	3·106	0·429
0·527	6·775	0·351	0·4975	0·0295	5·93	0·332	6·914	2·075	0·301
0·299	2·784	0·1443	0·275	0·024	8·73	0·183	6·616	1·144	0·173

B=3·5′

$$H^{3}/_{2}=\frac{Q}{3\cdot5\times3\cdot089}=\frac{Q}{10\cdot811}$$

G	Q	$H^{3}/_{2}$	H	H_L =G—H	$\frac{H_L}{H}\times100$	$\frac{2}{3}H$	WP= B+$^{4}/_{3}$H	A= B×$\frac{2}{3}$H	M= $\frac{A}{WP}$
2·407	38·184	3·53	2·32	0·087	3·75	1·547	6·594	4·425	0·671
2·133	31·745	2·936	2·05	0·083	4·05	1·367	6·234	3·905	0·625
1·868	25·898	2·396	1·79	0·078	4·36	1·193	5·886	3·409	0·593
1·6435	21·359	1·976	1·574	0·069	4·38	1·049	5·598	2·999	0·536
1·342	15·763	1·458	1·285	0·057	4·43	0·857	5·214	2·449	0·471
1·187	13·107	1·213	1·137	0·050	4·48	0·758	5·016	2·166	0·432
0·909	8·686	0·804	0·865	0·044	5·08	0·577	4·654	1·649	0·354
0·621	4·825	0·447	0·584	0·037	6·33	0·389	4·278	1·111	0·260
0·348	1·913	0·177	0·315	0·033	10·48	0·210	3·920	0·60	0·153

B=2·2′

G	Q	$H^{3}/_{2}$	H	$H_L = G-H$	$\frac{H_L}{H} \times 100$	$D = \frac{2}{3}H$	$WP = B + \frac{4}{3}H$	$A = B \times \frac{2}{3}H$	$M = \frac{A}{WP}$
2·980	32·59	4·790	2·840	0·140	4·93	1·894	5·99	4·17	0·696
2·572	26·34	3·875	2·470	0·102	4·13	1·646	5·49	3·62	0·659
2·345	22·92	3·370	2·240	0·105	4·69	1·490	5·18	3·28	0·633
2·133	19·75	2·905	2·040	0·093	4·56	1·360	4·92	2·99	0·609
1·850	15·99	2·350	1·765	0·085	4·82	1·176	4·55	2·59	0·569
1·533	12·05	1·722	1·465	0·068	4·64	0·976	4·15	2·15	0·518
1·166	7·96	1·170	1·110	0·056	5·04	0·739	3·68	1·625	0·441
0·518	2·25	0·331	0·479	0·039	8·14	0·319	2·84	0·703	0·248
3·478	41·07	6·140	3·350	0·128	3·82	2·230	6·66	4·91	0·740
2·986	32·78	4·820	2·850	0·136	4·78	1·900	6·00	4·18	0·697
3·214	36·46	5·360	3·060	0·154	5·03	2·040	6·28	4·49	0·715
2·560	26·06	3·830	2·450	0·110	4·49	1·634	5·47	3·60	0·658
0·757	4·07	0·599	0·711	0·046	6·46	0·474	3·15	1·043	0·331

B=1·5′

G	Q	$H^{3}/_{2}$	H	$H_L = G-H$	$\frac{H_L}{H} \times 100$	$D = \frac{2}{3}H$	$WP = B + \frac{4}{3}H$	$A = B \times \frac{2}{3}H$	$M = \frac{A}{WP}$
3·607	29·690	6·400	3·445	0·162	4·71	2·296	6·09	3·440	0·564
3·106	23·720	5·110	2·965	0·141	4·76	1·976	5·45	2·960	0·543
2·548	17·670	3·810	2·440	0·108	4·43	1·626	4·75	2·440	0·514
2·304	15·060	3·245	2·190	0·114	5·120	1·460	4·42	2·190	0·496
1·941	11·700	2·520	1·845	0·096	5·20	1·230	3·96	1·845	0·466
1·573	8·490	1·839	1·498	0·075	5·01	0·999	3·50	1·500	0·428
1·150	5·330	1·149	1·095	0·055	5·02	0·730	2·96	1·095	0·370
0·773	2·900	0·625	0·730	0·043	5·89	0·487	2·47	0·730	0·295

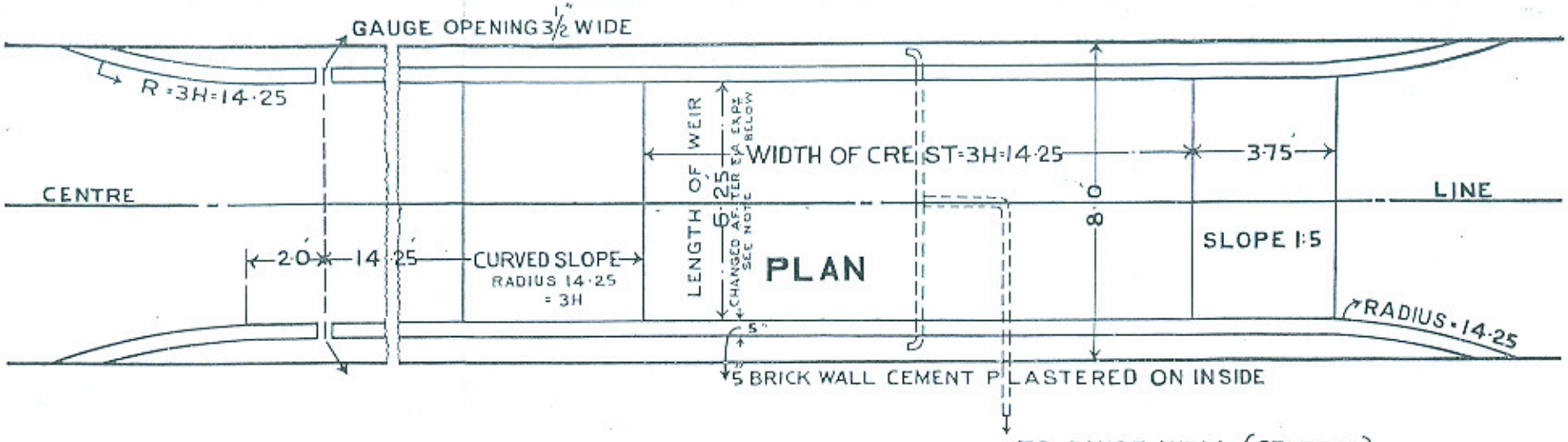
B=1·1′

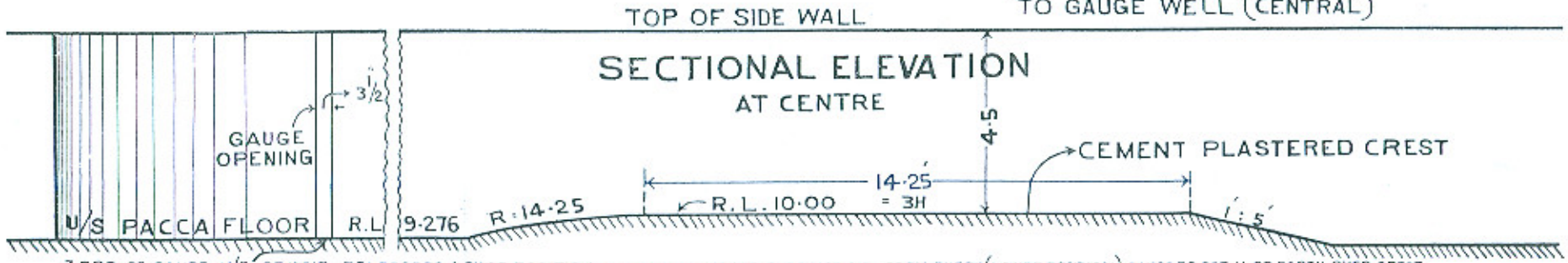
G	Q	$H^{3/2}$	H	$H_L = G - H$	$\frac{H_L}{H} \times 100$	$D = \frac{2}{3}H$	$WP = B + \frac{4}{3}H$	$A = B \times \frac{2}{3}H$	$M = \frac{A}{WP}$
3·733	22·70	6·680	3·550	0·183	5·15	2·36	5·82	2·60	4·47
3·450	20·14	5·930	3·280	0·170	5·19	2·19	5·48	2·41	4·40
3·146	17·48	5·150	2·980	0·166	5·57	1·99	5·08	2·19	4·32
2·726	14·03	4·130	2·570	0·156	6·08	1·71	4·52	1·88	0·416
2·748	14·20	4·180	2·590	0·158	6·11	1·73	4·56	1·90	0·416
2·289	10·82	3·185	2·160	0·129	5·97	1·44	3·98	1·58	0·397
1·924	8·32	2·445	1·820	0·104	5·71	1·21	3·52	1·33	0·378
1·463	5·48	1·610	1·370	0·093	6·79	0·91	2·92	1·00	0·343
1·016	3·15	0·927	0·952	0·064	6·73	0·63	2·36	0·69	0·292
0·675	1·69	0·497	0·627	0·048	7·65	0·42	1·94	0·46	0·237

B=0·5′

G	Q	$H^{3/2}$	H	$H_L = G - H$	$\frac{H_L}{H} \times 100$	$D = \frac{2}{3}H$	$WP = B + \frac{4}{3}H$	$A = B \times \frac{2}{3}H$	$M = \frac{A}{WP}$
3·576	9·046	5·480	3·250	0·326	10·0	2·166	4·83	1·08	0·224
3·214	7·700	4·970	2·920	0·294	10·0	1·946	4·39	0·97	0·221
2·773	6·152	3·970	2·500	0·273	10·9	1·666	3·83	0·83	0·217
2·232	4·431	2·860	2·020	0·212	10·5	1·346	3·19	0·67	0·210
1·817	3·268	2·108	1·640	0·177	10·8	1·093	2·69	0·55	0·205
1·079	1·473	0·952	0·966	0·113	11·7	0·644	1·79	0·32	0·179
1·530	2·513	1·620	1·380	0·150	10·9	0·920	2·34	0·46	0·196
3·723	9·621	6 210	3·380	0·343	10·1	2·254	5·01	1·13	0·22

PROPOSED H=4·75





ZERO OF GAUGE U/S OF WEIR, TELESCOPE LOWER POSITION = 9·260 R.L. HENCE SUBTRACT 0·14 FROM EVERY (LOWER POSITION) GAUGE TO GET H OR DEPTH OVER CREST

" " " UPPER POSITION = 12·046 R.L. HENCE ADD 2·786' TO EVERY UPPER READING IN ORDER TO CONVERT TO LOWER

R.L OF U/S FLOOR = 9·276 R.L HENCE SUBTRACT 0·016 FROM EVERY LOWER READING TO GET DEPTH OF WATER U/S OF WEIR

NOTE: THE LENGTHS OF WEIR EXPERIMENTED UPON WERE 6·25', 3·5', 1·5', 0·5, 2·2' AND 1·1' THE 1ST TWO SETS WERE REPEATED

THE RADII OF THE END APPROACHES WERE KEPT CONSTANT E.G: 14·25 IN ALL CASES.

(SD:) A.M.R. MONTAGU
EXECUTIVE ENGINEER
OKARA DIVISION L.B.D CANAL

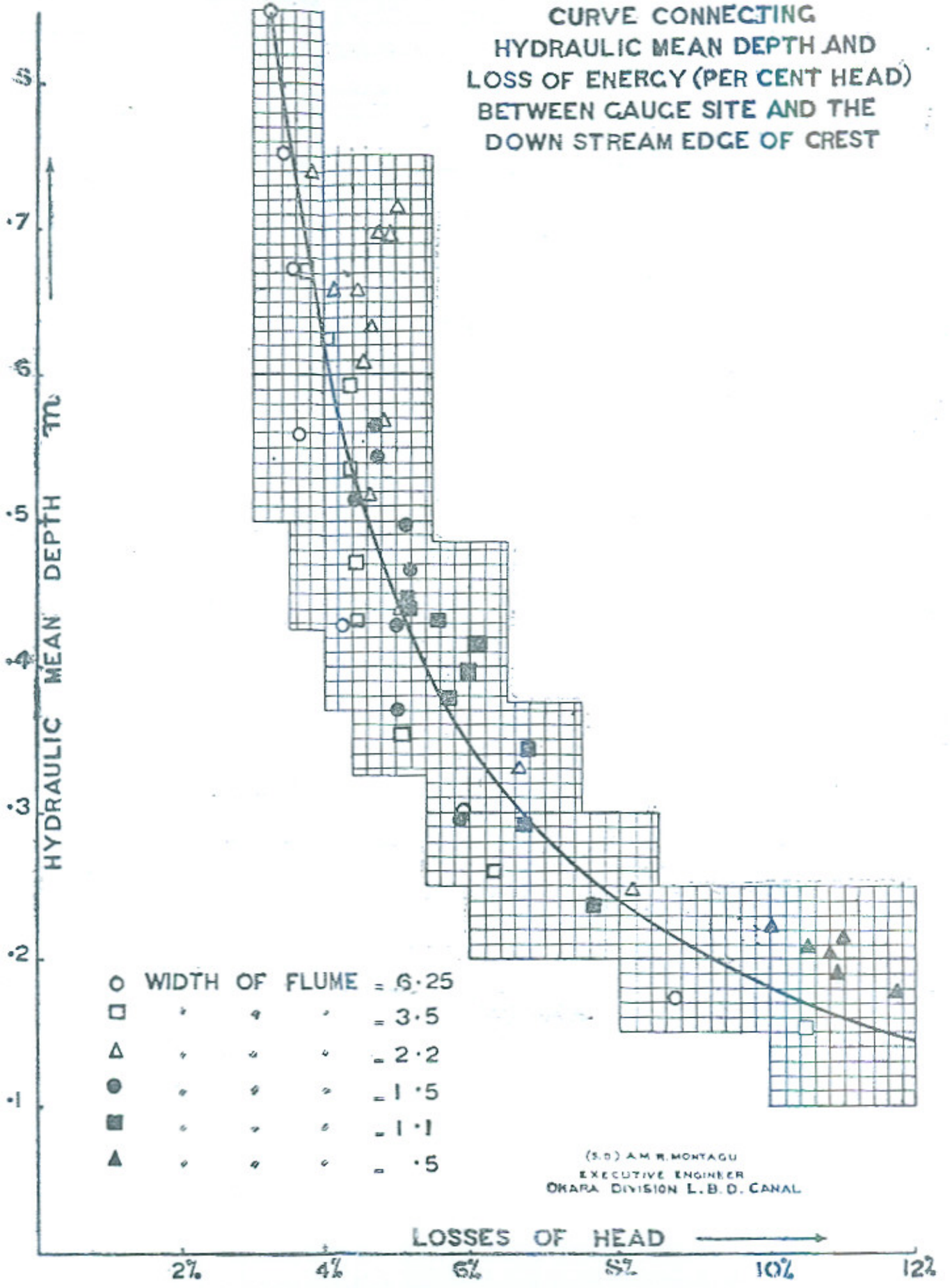
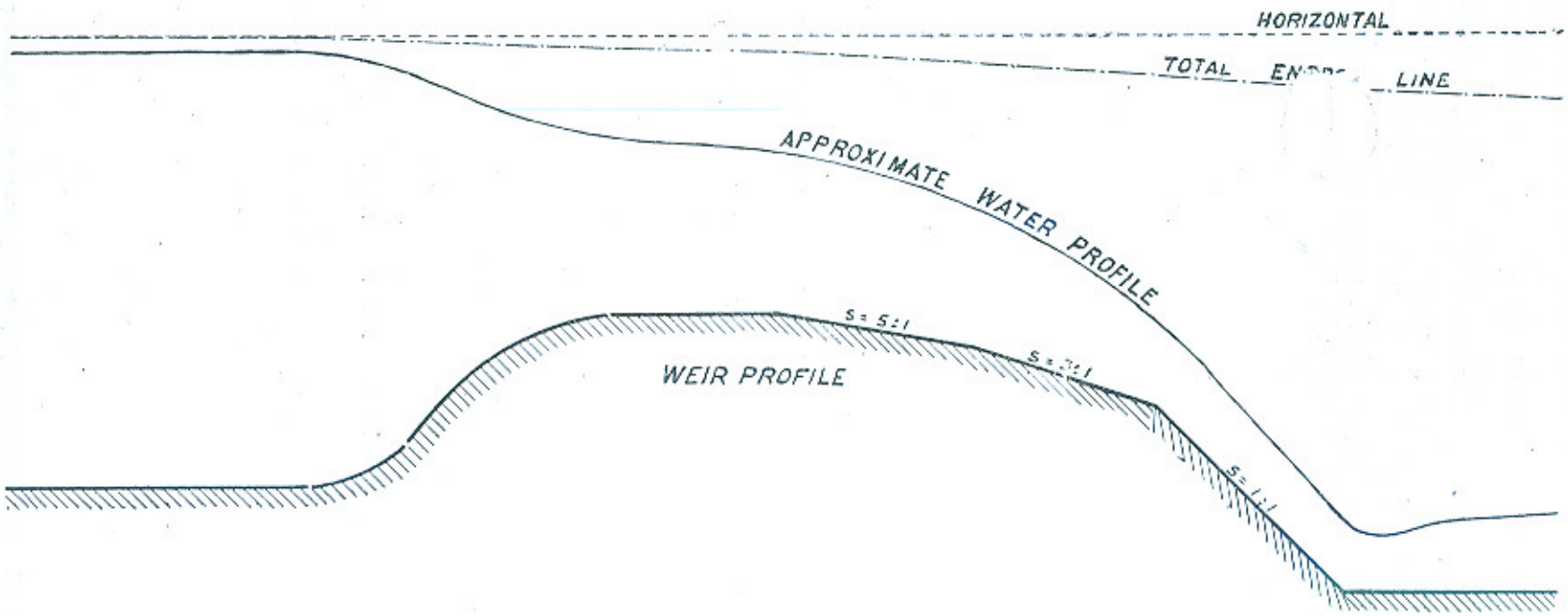

CURVE CONNECTING
HYDRAULIC MEAN DEPTH AND
LOSS OF ENERGY (PER CENT HEAD)
BETWEEN GAUGE SITE AND THE
DOWN STREAM EDGE OF CREST
HYDRAULIC MEAN DEPTH m
·8
·7
·6
·5
·4
·3
·2
·1
WIDTH OF FLUME = 6·25
= 3·5
= 2·2
= 1·5
= 1·1
= ·5
(S.D.) A.M.R. MONTAGU
EXECUTIVE ENGINEER
OKARA DIVISION L.B.D. CANAL
LOSSES OF HEAD
2%
4%
6%
8%
10%
12%

DIAGRAM III

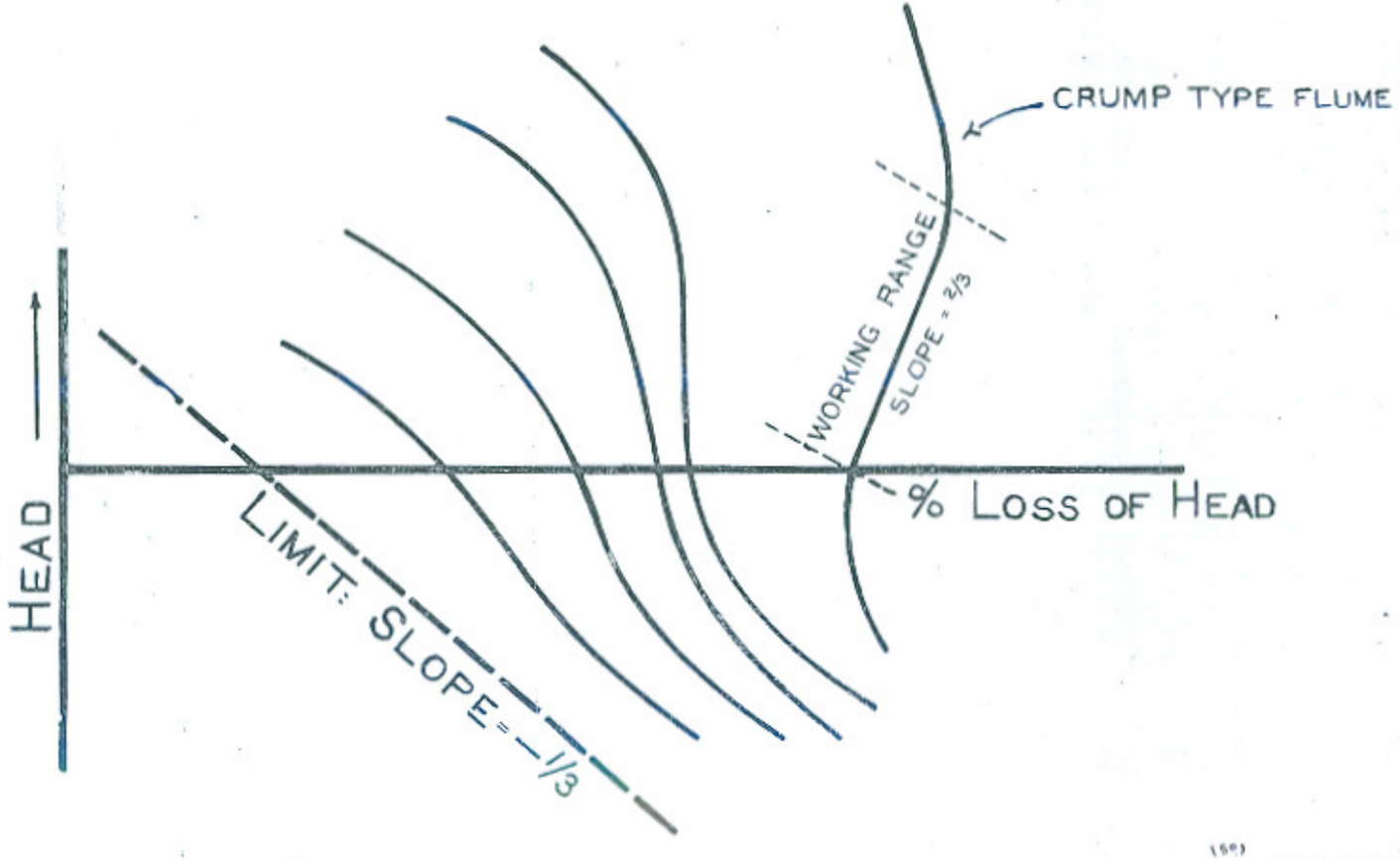
ILLUSTRATING REASON FOR EXISTANCE OF MINIMUM ENERGY AT THE DOWN STREAM EDGE OF WEIR CREST

NO SCALE



Sd/- A.M.R. MONTAGU
EXECUTIVE ENGINEER
OKARA DIVISION - L. B. D. CANALS

DIAGRAM IV (NOT TO SCALE)
ILLUSTRATING CURVE ON LOGARITHMIC BASIS CONNECTING HEAD-ACTING AND %LOSS OF HEAD



(50)

A.M.R. MONTAGU
EXECUTIVE ENGINEER
OHARA DIVISION L.B.D. CANAL.

DISCUSSION.

Mr. Bedford, in introducing the paper, said that it might be as well to explain in a few words the reasons that led to the experiments and the little paper. He said that before he took over the Upper Bari Doab Circle there was a scheme for measuring the absorption losses in the Main Canal. The Main Canal had several rapids which at a small expenditure of money could be converted into a long crested weir.

It, however, seemed to him to be essential to know with accuracy the co-efficient that could be applied to the formula $D=CLH^{3/2}$ before they could hope to obtain any results from the bigger problem of absorption losses in the Main Line.

It was then with this object that the experiments placed before the Congress were carried out. The results, however, immediately showed that it was not possible to apply any co-efficients obtained from the experiments directly to the formulae for weirs in the Main Canal with a maximum discharge of 6,750 cusecs. He said that while he was studying the experiments in Dalhousie Mr. Montagu happened to drop in, and on his showing him the results of the experiments Mr. Montagu very kindly suggested the line of attack, and they spent the next few days in producing the small paper.

Mr. Montagu as joint author of the paper first thanked Mr. Bedford for so kindly allowing him to associate himself with the paper. It was an extremely small paper. He deduced no law, but merely endeavoured to show from the data supplied by Mr. Bedford that it was possible to produce one curve for all the varying co-efficients for varying bedwidths and depths of crest. Hitherto it had been customary to vary the co-efficient itself; he had given his reasons for not following this process. He pointed out that the curve did not apply to all weirs but merely indicated a line of attack. If other things could be deduced from the method of treatment, so much the better, but he had not attempted to do so.

Mr. Inglis remarked that there was very much in the paper with which he was in entire agreement. He agreed that if the velocity is assumed to be uniform at all points in the section then the minimum energy must be at the downstream end of the crest, which in that case is the control section in the sense that any modification of this section would have an effect on the discharge. If, however, the velocity is taken to vary from point to point in the stream—due to curvature caused by friction and other losses—which is actually the case—then there is an enforced control section somewhere inside the throat.

Figure 1 shows an exaggerated plan showing the effective throat width B, which in practice is the control section; the portion below B acting merely as an expansion.

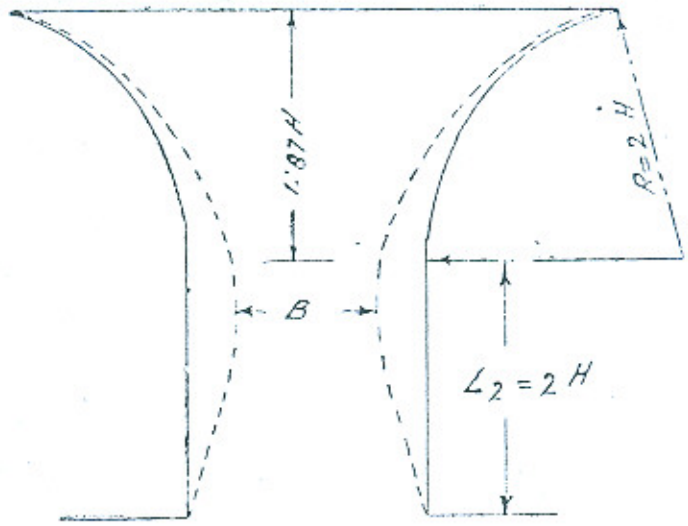


FIG: 1

This is the reason why in practice it is the co-efficient that varies while the exponent remains constant within the working range. It also explains why the range of modularity increases as the width decreases for any particular depth of water—as clearly shown in these experiments when they are plotted on log paper. There was, he thought, no question that if we had two flumes which were dynamically similar, the loss would be a function of the hydraulic mean depth, and would bear an inverse ratio of the $^3/_2$ power of their hydraulic mean depths. It was unreasonable to suppose, however, that a small flume flowing at full supply with a crest length twice the full supply depth and a curved entrance (with R = 2 H), would give the same discharge as a big flume of the same design flowing at $\frac{1}{4}$th full supply, with the same hydraulic mean depth. The data presented was, in fact, rather misleading in that the length of the flume was a constant and that this length was very great relative to the depth of water. His opinion was that the *full supply coefficient* could best be worked out on the basis of hydraulic mean depth, but that as we could not vary curvatures with depths, the effect of variations in friction, shock and eddies could not be eliminated—in other words working entirely on Hydraulic Mean Depth would lead to error. On the other hand flumes could be designed in which the discharge varied as $H^{3}/_{2}$ within the working range with a negligible error, and he thought that such a flume was what we required in practice. In this connection his experience was that Mr. Crump's flumes fulfilled all the conditions desired *provided the width relative to the depth was not too great.* In Technical Paper No. 15 of the Bombay Public

Works Department it was suggested that $B=D^{1.5}$ was about the correct ratio.

Further experiments with flumes of varying lengths and radii were now required to clear up the question.

Mr. Inglis considered that the Paper had not merely led to the theoretical side of the question being cleared up but had also shown that there was a modular range depending on the bed width-depth ratio and that with some further experiments, preferably on Mr. Crump's standard design, it would be possible to predict the co-efficient with a high degree of accuracy.

Mr. Burkitt remarked that he had worked out a few of Mr. Montagu's and Mr. Bedford's examples and had found that the loss of head could be treated simply as friction head with N=·013 in Manning's formuls.

Mr. Lacey said that the authors were to be congratulated on presenting within the limits of a very modest paper much valuable information. The method of dealing with the loss of energy between the gauge site and the downstream edge of the crest was rational and ingenious. It was stated in the paper that various quantities were plotted against one another and that finally the authors hit on the plan of plotting the hydraulic mean depth against the percentage loss of head, both to a natural scale. The curve in question was printed as a plate accompanying the paper and appeared to average the points fairly successfully. Of this curve the authors remarked that the points plotted indicated an undeniable relation, and with this statement the speaker was in entire agreement. Had the authors tried a 'log' plotting, a course of action which should never be omitted, they would have obtained the following relationship which the speaker had discovered namely:—

$H_L = \cdot 03\ H/R^{2/3}$, and when the channel is of infinite breadth

$H_L = \cdot 0393\ H^{1/3}$.

This curve it would be found fitted the points with considerable accuracy.

Mr. Lacey concluded by saying that in a case like this when an exponential curve should be sought, the use of 'log' paper was the more imperative; of course in this instance, as the authors had pointed out, the loss was composite and due to three sources; nevertheless, a log plot might show which source was predominant, and also indicate on what lines any further attempts at analysis of the data should proceed.

Mr. Ishar Das said that after reading the paper, his first thought was that the hydraulic mean depth, which for ages was not so important in determining velocities and discharges, had again come into prominence.

He said that knowing the head available we could plot M from the curve and design the dimensions of a flume accordingly. He was

of opinion that it was desirable to abandon the idea of making H a function to determine throat widths, approaches etc., for designing flumes and to launch an effort to make a flume which would give directly on a gauge the loss of head. This was, however, a matter for research.

Coming to the critical side of his study of this interesting paper, he offered the following remarks :—

1. The conclusions of the authors were based on the curve they had drawn and given at the end of the paper.

2. A parabolic curve when plotted on logarithmic paper gave a straight line the equation of this being in the form

$$y=mX^n.$$

the tangent of the slope of the line was equal to the exponent n and the intercept on the y axis was equal to m.

3. He presented a plotting on logarithmic paper of the data of the authors' curve. Three lines have been drawn, one corresponding to flume widths of 6·25, the second to widths of 3·5 and the third to the remaining four widths ranging from 2·2 to 0·5.

4. All these lines were parallel indicating that the curves represented by the straight lines were of the same family.

5. In each of the lines there was a kink between values of M of ·4 and ·5.

6. The upper part of the lines if produced would meet the y ordinate at n= 13, showing that at that value of m the loss of head was nil and that still further up there would be a gain. This would not be correct. Probably if there were more results plotted for higher values of M there would be more kinks. The position of points indicated by triangles (*i.e.*, for widths of 0·5) was an indication that this would be the case.

7. Generally in our designs values of M ranged from about 1 to 4, therefore more experiments were needed to cover this range and establish a law.

8. The influence of silt on the flow had also to be considered. He firmly believed that silt accelerated the flow up to a certain limit. By this he meant that the same gauge would give different discharges for clear and silted water. Accepting this, the loss of head in the same would be different for water of different silt charge.

9. He thought it possible that the kinks might be due to faulty design. He was hopeful that a design might be evolved which would eliminate these kinks and thus establish a straightforward law.

10. He wished to thank the authors for presenting this interesting and suggestive paper which opened up a fresh line of research.

Mr. Mithal presented a graph showing that points for different widths of flume lay on different curves, which appeared to be of the same family. The curves showed that the loss of energy on a flume increased as B decreased. The authors should therefore have not drawn a mean curve, applicable to all values of B, under such circumstances.

He also criticised the values of B=2·2, which he considered to be erroneous: He instanced the two values :—

M=·715, % L=5.03

M=·442, % L=5.04.

and enquired whether it meant that the loss between these two values of M was to be taken as 5 % only.

He enquired if account was taken of the head loss in the channel between the gauging section and the flume.

Mr. K. R. Sharma said that the correct formula of discharge for a rectangular notch was

$$Q=\sqrt{3c}\sqrt{2g}\,B\,H^{3/2}$$

where C is a constant which takes care of the practical errors in the formula.

The theoretical value of the constant in the above formula neglects friction and assumes a depth of $\frac{2}{3}$ H on the crest.

He objected to the assumption that the average depth on the crest is 2/3rd H. He instanced the meter flume at R. D. 10,000 Northern Branch, Lower Jhelum Canal of which the discharge co-efficient worked out to 3·05. He had calculated the change in level from the gauge site to the beginning of the crest to be 0·490 feet, whereas in practice it was found to be 0·60 feet. He attributed this loss to afflux.

He quoted other cases wherein the co-efficient of discharge was found to increase with the rise in gauge owing to reduction of afflux. He concluded that the hole in the experimental flume was very near to the contraction.

The meter flume at R. D. 10,000 of Northern Branch Lower Jhelum Canal was calibrated in the month of January 1929 when canal water was clear. It had been found that when muddy water comes there was excess in the canal with the same meter gauge. The gauges downstream and the discharge in the canal showed an excess of 2 to 3 per cent. This factor effecting the co-efficient of discharge could correctly be determined in tank measurements. Some light could be thrown by the authors on this point if it was observed by them.

There were three points which were not clear to him in the working of this meter and he requested the authors of the paper to throw light on them.

(1) Loss of head in afflux upstream and entry was a loss in the structure of the flume. Should the gauge measure the loss of head in afflux on entry? How could it be allowed for in the co-efficient of discharge?

(2) Would it not be possible to do away with the unnecessary loss of head in approach to the crest.

(3) Although the ideal conditions of flow were not available at the crest and the depths at the end of the crest were considerably less than 2/3rd but still the meter was working with a good co-efficient as much as 3·05.

Mr. Colyer briefly reviewed the growth of the free fall formula and stated that the control section was assumed to be at the point of contra-flexure of the water surface, *i.e.*, at the point where parallel flow occurred.

The depth at this point was 2/3 H. He failed to follow the authors in working back from the discharge to obtain H the depth over the crest, and disputed that the head acting should be measured at the downstream crest of the weir.

Certain other questions arose in his mind on reading the paper. For instance in paragraph 15 (*i*) he failed to see why the co-efficient was independent of the discharge which of itself was a function of M the hydraulic mean depth.

Mr. Colyer then quoted a number of observations of his own, used to obtain the co-efficient for various weirs, which he found, were in close agreement with the percentage losses indicated by the author's curves.

Mr. Khosla remarked that Messrs. Bedford and Montagu had made an important contribution in introducing hydraulic mean radius (M) in the discharge calculations over long crested weirs.

He said that he had drawn attention in 1929 in connection with Mr. Montagu's paper to the effect of bed-width depth ratio on the co-efficient of discharge. That effect was fully covered by the H. M. R. (M) now introduced by Mr. Montagu.

He said that the data supplied by Messrs. Bedford and Montagu did not however cover a sufficiently wide range. The H. M. R. of the experiments did not go beyond 0·9.

In regard to the curve of percentage loss of head, he remarked that there seemed to be an indication that the results for the various widths formed separate curves by themselves. Thus, for instance, all points for 6·25 were to the extreme left of the mean curve, and all points for 0·5 width were to the extreme right. For intermediate widths the points lay between the two extremes and generally followed the order of flume widths. Taking a concrete case:

For B=6·25 m =0·173 loss % = 8·73

For B =0·5 m =0·179 loss % =11·7

or the percentage loss for the same value of m was less for a wider flume than for a narrower flume. Mr. Montagu's deduction No. (*i*) was therefore open to question. His deduction No. (*ii*) was not quite clear to him. He requested Mr. Montagu to please explain further.

Mr. Khosla said that Mr. Montagu's investigation was an advance in the right direction, and perhaps Mr. Montagu would pursue it further

to extend the range, and also to determine if the effect of B really persisted throughout or not and in either case to find out a simple formula for ready application.

Mr. Musto said that he thoroughly agreed with the authors' conclusions as given in paragraph 8 of their paper. From this arose the difficulty of selecting co-efficients when designing new works. Referring to paragraph 15 (*iv*) he suggested that this increase of percentage loss as M falls might rather indicate the great effect of side friction. He instanced the case of two channels each 20 feet deep having bed widths of 10,000 feet and 100 feet respectively. The values of M for these would be 20·0 feet and 14·3 feet respectively.

Either there was something wrong with the formula for M. and too much weight was given to the depth (side slopes) in the denominator, or it was evident that side friction or restriction played a far greater part than bed friction, on all problems of flow. He suggested that this should be verified by experiment ; for example, on test channels of equal depth, one having 10 times the width of the other to ascertain whether the discharges of each varied directly with their H. M. D.

He further suggested testing the effect of sloping the sides of the channel. He felt that the losses in a vertical sided channel were much greater than in a channel with sloping sides, due to constriction at surface and prevention of spreading from high velocity in the centre of streams, and to spiral action in the bottom angles. It would be an easy matter to carry out tests on two such channels having practically the same value of M.

Mr. Montagu in replying to the discussion, opened with the remark that most of his critics apparently accepted the logical nature of the method of attack suggested in the paper and then proceeded to explore the implications. It was with a view to avoiding this very thing that the authors had confined themsleves to indicating the logical reasons for their method and depicting the results in the form of one simple curve.

2. The Authors now felt bound to follow up the lead given by their critics, and their general reply, which followed, covered most of the points raised.

If a " Long crested " weir had infinite length, no critical section would form ; in fact if the weir surface were horizontal, no flow could take place.

If the weir were merely very long, the surface of the water would be nearly horizontal (actual slope being determined by friction), and the depth would, of course, be perilously near the critical depth until the downstream crest of the weir was approached.

3. (Mr. Montagu produced diagram III.)

As the weir became shorter in length the distance between normal flow upstream and the critical section became shorter and therefore curvature (and its effects) became more pronounced.

The curvatures of the weir approach and departure also became important, as the weir shortened. If the approaches were perfect then the length of weir crest could be reduced, theoretically to zero.

4. Referring to diagram III, the author went on to point out that over the flat weir crest the total energy line had a downward slope (due to friction). Over the glacis the total energy line still had a downward slope, but owing to the greater slope of the glacis, diverged therefrom. The minimum energy of flow therefore existed at the downstream crest. Hence this was the critical section. This point had been dealt with very fully in Part III of the author's 1929 paper.

5. It did not follow that the critical depth occurred here ; only the critical or minimum energy. As soon as curvature of the stream lines existed, Bernouilli's theorem *as ordinarily stated* no longer applied. A redistribution of the energies took place and resulted in a change in the relation of velocity, pressure and potential energies, which caused a change in depth.

But it was possible, in the author's opinion, to prove mathematically that the value of the critical, *i.e.*, minimum energy remained unaltered at the original figure.

6. This brought them directly to the first point made by Mr. Inglis. This might be restated as follows :—

" If the curvature of the boundary was such that the water could not follow it, then the water constructed its own approach curves, and virtually a new boundary." If Mr Inglis agreed to this the author freely admitted that a " forced " control section would immediately come into existance which probably would differ from that postulated by the author. If such really did take place, then one of the prime conditions of the problem no longer existed, and the actual downstream crest of the weir no longer formed the control section. But this did not vitiate the author's argument. It merely instanced a case in which circumstances prevented the *virtual* crest being known.

7. Although Mr. Inglis' sketch is in plan only, the departure of the stream lines from the **bed** surface of the weir might be postulated also (*viz.* in section).

In their present state of knowledge of the effects of curvature, a weir so constructed was not a reliable meter, because neither H nor B could be measured or otherwise determined.

Fortunately for the author, both Mr. Inglis (Bombay) and Mr. Lacey (U. P.) had insisted upon the author sitting down and plotting the results on double logarithmic paper and had been good enough to discuss the results so obtained. Had the Author been left to himself, he had no hesitation in stating that what he would now place before them would have taken far longer to arrive at.

Here the author produced diagram IV.

8. If the percentage loss of head was plotted horizontally against the head acting on double logarithmic paper, for each of the six series of experiments a family of curves resulted, the general nature of which was depicted in the diagram. The following deductions could be drawn at once :—

(*i*) The loss effects of curvature and friction caused a "kink" in the logarithmic curves, the position, duration and slope of which varies with the value of B (width of throat).

(*ii*) Clearly, for the *length* of crest adopted, a value of B existed for which a constant per cent. deduction from the corrected gauge reading could be substituted for the constant co-efficient advocated by Mr. Inglis, for all ranges of H within the working Semi-modular range.

(*iii*) Again, by selecting another value of B, the slope of the "kink" could be utilised such that the % deduction from the corrected G varied as the 2/3 power of H, in other words that the co-efficient k might be altered and would remain *constant* for the particular working semi-modular range desired. Clearly it was to this particular type that Mr. Crump's dimensions approximated and which Mr. Inglis had so strongly advocated.

(*iv*) Again, it was clear that the B : H ratio was of less importance than the ratio of B to length of crest and this again brought the curvature effect into prominence.

(*v*) If the B was at least very large, the logarithmic curve tended to sraighten out, and the kink to vanish : in the limit it became a straight line whose slope obviously was $\frac{1}{3}$.

9. This brought them to Mr. Burkitt's contribution. In the light of the preceeding remarks, obviously curvature *could not* be neglected. Mr. Burkitt had gone to the trouble of working out Manning's N to 0·013 and had stated that within the range of experiments, such a value would give approximately the same results. Now if Manning's "N" were a true co-efficient of rugosity, obviously it would hold over any "work." Mr. Burkitt had, then, indicated to the Author a method of separating out the curvature and friction losses. The Author had no doubt that a direct measurement of N in similar material in a "normal" channel would give a value of "N" distinctly lower, in which case the balance of the losses would be due to curvature.

10. Mr. Lacey's remarks were much appreciated by the Author, particularly in regard to the use of logarithmic paper, which he heartily endorsed.

11. Most of Mr. Lacey's contribution had been answered n the preceding remarks. The Author's failure to produce a log plotting was due to the recognition of the composite nature of the source of loss and the desire not to obscure their primary object.

12. The formula quoted by Mr. Lacey, *viz.*, $H_1 = 0{\cdot}03\ H\ M^{2/3}$ (retains the original notation) was commended to canal officers engaged in the design of brick masonry weirs. It permitted a design of which the discharge would be accurate to within 1 per cent. or 2 per cent.—an advance on anything they had at the moment.

13. In reply to Mr. Ishar Das Mr. Montagu pointed out that as a result of the double log plot exhibited, a weir could now be designed, the % H_1 of which would be constant within the limits of the working range. The explanation of the "kinks" observed by Mr. Ishar Das had been given already.

14. Mr. Montagu took leave to point out that on a double log plot, the straight lines would never cut the Y axis at zero value unless produced to infinity.

15. In regard to silt and its effects on flowing water the Author referred Mr. Ishar Das to Mr. Buckly's (Jnr.) Paper No. 4425 at the Inst. Civil Engineers, London, in 1923. The only effect that would be experienced over a weir would be due to a change in co-efficient of rugosity and would be extremely small.

16. Mr. Mithal clearly had failed to grasp the essential factor in the paper, as also had Mr. Sharma. No parallelism of stream lines was postulated in any part of the paper. In fact, the change of losses due to friction being in the opposite sense to the change of losses due to curvature had been the key-note of the discussion. The original discharge formula $Q = K\ B\ H^{3/2}$ had been derived from a simplification of Bernouilli's original law and it was the errors arising out of this simplification which were, by implication, dealt with in the paper. The reason for assuming the wetted perimeter to vary with 2/3 H were given. The approximation was justified by the results.

17. Mr. Sharma's interesting remarks on the individual weirs on the Upper Jhelum Canal merely confirmed the Author's case that the accepted co-efficients were wrong.

18. The objections to crests at bed level were numerous. Hydraulically, they postulated high velocities, high losses and at present, bad curvatures.

19. The above had had further confirmation from Mr. Colyer who also shewed that the law deduced by Mr. Lacey from the authors' results agreed in practice for much larger discharges and very varying ratios of head, bed width and length of crest. But the Author pointed out to Mr. Colyer that the point of contra-flexure of the water surface did not indicate parallelism of flow, neither did it occur at the controlling section.

20. The word "independant" in para. 15 (*i*) was used in the sense of "not explicit."

Mr. Khosla very properly referred to his remarks on the speaker's paper No. 126 of 1929. The present paper was decidedly a continuation of that paper. The subject was without end. But the Author deprecated any attempt on Mr. Khosla's part to draw separate curves for separate cases. It was to get away from this that the joint Authors presented their contribution. The resulting discussion had cleared up Mr. Khosla's other enquiries.

21. Mr. Musto has carried the discussion on one point even further. The Authors expressly declined to differentiate the effects of the various sources of loss of energy. The discussion has compelled the speaker to shew how curvature and friction losses are essentially separate, but Mr. Musto wants to determine the relative effect of side and bed friction. For Mr. Musto's satisfaction the speaker can only state that his enquiries indicate that bed friction is, roughly, twice as effective as side friction but that the frictional loss in this respect is materially effected by curvature. The paper does not deal with this at all.

22. If what Mr. Lacey has termed the "isotacs" in a section of a stream are distorted, the frictional effect is more pronounced. It follows that a vertical sided channel is likely to require a slightly steeper slope than a channel whose sides are "natural" for the same discharge.

But all the flumes tested by the Authors and generally used in practice, had vertical sides, in order to eliminate any such consideration.