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Optimum moisture content as related to grain size of cohesive soils

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ABSTRACT:

Grain size analysis of cohesive soils has been investigated to relate with the Proctor Standard compacted maximum dry density and optimum moisture content. It omits the usual procedure of compaction test and requires only the determination of grain size distribution and then sum up the per cent finer (cummulative) below 200 mesh U.S. Standard sieve (0.074 mm) and the size below 0.002 mm (per cent clay). This sum is denoted by S_m and with the help of S_m the optimum moisture content (m_o) and thereafter maximum dry density (γ_{max}) can be found out. This procedure shortens the time for the determination of the variables of compaction test of cohesive soils alongwith the additional determination of their classifications.

On the basis of experimental data collected at the Irrigation Research Institute Lahore (PAKISTAN), some mathematical relationships have been developed to compute the values of optimum moisture content and maximum dry density based on grain size analysis of cohesive soils.

INTRODUCTION:

Earth or soil, in the engineering terminology is defined as "any unconsolidated material made up of discrete solid particles with gases or liquids between". Soils include a wide variety of materials such as the gravel, sand, silt and clay mixtures deposited by glaciers, the alluvial sands, silts and clays of flood plains of the rivers, the soft marine clays and beach sands of the coast. Soils can be well-defined mixtures of a few specific minerals of almost any thing.

Problems in soil engineering can seldom be solved by blind reliance on either emperical data gathered on past projects or on the most sophisticated computerized analysis. Each situation is unique; it requires careful investigation and thorough scientific analysis combined with engineering judgement based on varied experience.

As mentioned above, soil is an unconsolidated material made up of discrete solid particles, the range in the sizes of soil particles or grains is almost limitless; the largest grains are by definition those that can be moved by hand, while the finest grains are so small that they can not be identified by an ordinary microscope. The particles produced by mechanical weatering are rarely smaller than 0.001 mm in diameter and are usually much larger, for nature's grinding processes are not very efficient and the small grains often escape further punishment by slipping through the voids between the larger grains. The product of chemical weathering including iron oxides and clay minerals, are tiny crystals which occasionally are larger than 0.002 mm in diameter but are usually very much finer.

Two methods are commonly used to determine the grain sizes present in a soil. Calibrated sieves or screens having openings as large as 4 inches and as small as 0.074 mm (U.S. Standard sieve No. 200) are used for seperating the coarser grains. Sieves with smaller openings are available but

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are impractical for fine grained soil classification. The portion finer than 0.074 mm can be measured by sedimentation. This is based on the principle that the smaller the size of a particle, the more slowly it settles through water. The time of settling is inversely proportional to the square of the diameter of the soil grain (Stroke's Law).

Because of the wide range in diameters between the coarsest and finest soil particles, and in order that the contents of the fine sizes may become more visual, the diameters of the soil particles are plotted on a logarithmic scale against per cent finer by weight on the simple scale, thus obtaining a more compact curve. Fig. 1 shows a plot of a soil having 19 per cent clay content, 21 per cent silt and 60 per cent sand.

COMPACTION TEST:

Densification, or reduction in the void ratio, occurs in a number of ways: re-orientation of the particles; fracture of the grains or the bonds between them, followed by re-orientation; and bending or distortion of the particles and their adsorbed layers. Energy consumed in this process is supplied by the compactive effort of the compaction device. In cohesive soils the densification is primarily accomplished by distortion and re-orientation, both of which are resisted by the interparticle attractive forces of cohesion. As the water content of the soil is increased, the cohesion is decreased, the resistance become less, and the effort becomes more effective. From the usual procedure of compaction by proctor standard, the two variables of compaction test viz. the optimum moisture content and maximum dry density are determined.[1]

The values of optimum moisture content and maximum dry density vary from soil to soil depending upon the presence of sand, silt and clay content in them. Each particle fraction makes its own contribution to this property of soil. In the present paper we will consider only the cohesive soils having their liquid limit above 20 per cent for the study purpose. The values of optimum moisture content and maximum dry density of different types of soils alongwith their respective grain size analysis were determined in this Institute. Analysing this data, it has been concluded that cummulative per cent passing 200 mesh sieve and the per cent finer than 0.002 mm (clay content) play domenant role towards the optimum moisture content required for compacting cohesive soils to their maximum density.

Two types of soils are shown in the Fig. 1. Thick line represents a soil of medium plasticity whereas dotted line highly plastic one. After establishing the above phenomenon, it is desired to establish the quantitative analysis, that how the optimum moisture content and maximum dry density of cohesive soils can be related to their grain size data.

With the help of extensive data on grain size analysis and Proctor compaction test data, the sum of per cent finer than 200 mesh sieve and per cent finer than 0.002 mm (S_m) and optimum moisture content (m_o) were determined for all types of cohesive soils. A graph between m_o and S_m was plotted on a natural scale. The trend of the plot suggests a circle of which the equation has been evolved as (Fig. 2):

$$S^2_m + m^2_o - 52.06\, S_m - 1852.92\, m_o + 22329.94 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which:

m_o = optimum moisture content (%)
S_m = sum of per cent (cummulative) finer by weight than 200 mesh sieve and 0.002 mm

The preceding equation of the circle has been further simplified as:

Centre of the circle: (26.03,926.46)
Radius of the circle: (914.7 %)

The polar co-ordinates of the point (S_m, m_o) lying on the foregoing circle are given by:

$$S_m - 26.03 = r \cos\Theta \quad \ldots\ldots (2)$$

$$926.46 - m_o = r \sin\Theta \quad \ldots\ldots (3)$$

in which Θ = the angle of inclination of the radius with S_m-axis and r = 914.7%.

Substituting the value of S_m (computed from grain size analysis) in Eq. 2, Θ is known which on further substitution in Eq. 3, yields the value of m_o.

The second parameter of Proctor Compaction Test viz. maximum dry density can be computed from the following relationship.[2]

$$m_o = 376 \exp[-0.0287 \gamma_{max}] \quad \ldots\ldots (4)$$

or

$$\gamma_{max} = \frac{-1}{0.0287} \ln\left[\frac{m_o}{376}\right] \quad \ldots\ldots (5)$$

CONCLUSION:

With the help of grain size analysis, adding the per cent finer (cummulative) on the U.S. Standard sieve No. 200 and from sedimentation process, the per cent finer than 0.002 mm, it is possible to determine optimum moisture content and maximum dry density of cohesive soils by using the mathematical relationships developed in this paper.

SOLVED EXAMPLE:

Involving the following data as obtained from grain size analysis (dotted curve in fig. 1), determine the optimum moisture content and the maximum dry density of a given cohesive soil:

finer than 200 mesh U.S.S sieve	: 80 %
finer than 0.002mm size (clay content)	: 40 %

Solution:

Adding per cent finer as given above, $S_m = 120$. Substituting the value of S_m in Eq. 2 we get:

$120 - 26.03 = 914.7 \cos\Theta$

or

$\Theta = 84.10^o$

Now further substitution of Θ in Eq. 3 we get:

$926.46 - m_o = 914.7 \sin 84.10^o$

or

$m_o = 16.60\ \%$

Again substituting the value of m_o in Eq. 5 we get:

$$\gamma_{max} = \frac{-1}{0.0287} \ln \frac{16.6}{376} = 108.72 \text{ lbs/cu ft} = 1739.52 \text{ KG/M}^3$$

For confirmation, the same soil sample was put under Proctor Compaction Test. The values of m_o

and γ_{max} computed above with the present method to confirm its authenticity with slight variation of -1.48 % and +0.48 % from determined values of m_o and γ_{max} respectively.

Basing extensive data processing, the per cent variations in the computed values of m_o and γ_{max} have been noted to be of the order of - 4.5 % and - 2.5% respectively (table 1).

The above relationships can also be used for computing m_o and γ_{max} of non-plastic soils except sands and loamy sands after imposing the following limitations on the available data:

1. Clay content should not be less than 10%
2. Per cent Passing 200 mesh should not be less than 30%
3. If clay content is less than 10% then drop between per cent 0.02mm and 0.002mm should not be less than 20
4. S_m should not be less than 50 when clay content less than 10
5. If sand content (0.06mm) is above 70%, this formula does not hold good
6. If sand is more than 50% but less than 70% and clay content less than 10%, then the limitation at (3) may be waived of and the computed value be acceptable
7. If drop between per cent passing 200 mesh and that finer than 0.02mm is more than 60%, this formula does not hold good
8. If percent passing 200 mesh sieve is more than 80% and clay content less than 10% then optimum moisture content is computed by involving S_m with and without adding per cent 0.02mm size. The two values of m_o thus obtained be averaged to get the desired value of m_o but for computing maximum dry density add for S_m, per cent passing 200 mesh sieve, 0.02 mm and 0.002 mm provided drop between 0.02 mm and 0.002 mm is not less than 20.
9. If ratio between per cent sand and clay in sandy clay or sandy clay loams is 1-2 then add about 4-8 lbs/cu ft in the computed value of maximum dry density and optimum moisture content be amended with the help of Eq. 4

REFERENCES:

1. Soil Mechanics — Jumikis. A.R. published by D. Van Nostrand Company Inc. Princeton New Jersey, 1968
2. Optimum Moisture Content for Compacting soils: One-Point Method — I.H. Hamdani published in Journal of Irrigation & Drainage Engineering, Proceedings Vol. 109 No. 2 June, 1983 · American Society of Civil Engineers U.S.A.

TABLE-1

COMPARATIVE TEST DATA OF PROCTOR COMPACTION AS DETERMINED FROM ACTUAL TEST AND COMPUTED FROM THE FORMULA

Sample No.	% pass 200 mesh	% 0.02 size mm	% 0.002 size mm	S_m	Proctor Compaction test		Computed from formula		% variations	
					Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	m_o	γ_{max}
1.	83.8	41.9	14.5	98.3	14.4	111.7	14.9	112.5	+3.47	+0.72
2.	83.1	49.5	20.5	103.8	15.0	110.8	15.1	112.1	+0.67	+1.17
3.	82.3	52.8	21.9	104.2	14.8	110.8	15.1	112.1	+2.03	+1.17
4.	88.8	75.4	35.8	124.6	17.6	107.4	17.2	107.5	-1.15	+0.09
5.	95.4	92.7	55.1	150.5	21.4	101.8	20.7	101.1	-3.27	-0.69
6.	87.4	31.2	41.6	129.0	18.1	105.2	17.9	106.1	-1.10	+0.86
7.	93.6	85.5	46.2	139.8	19.2	102.8	18.9	104.3	-1.56	+1.46
8.	57.1	28.0	7.2	64.3	12.4	117.8	12.6	118.4	+1.16	+0.51
9.	56.6	29.9	9.2	65.8	12.5	119.3	12.8	117.9	+2.40	-1.17
10.	53.1	31.2	12.2	65.3	12.4	119.0	12.6	118.3	+1.61	-0.59
11.	55.0	39.0	17.5	72.5	13.0	118.4	12.9	117.4	-0.77	-0.84
12.	58.8	47.0	23.2	82.0	13.5	116.4	13.7	115.5	+1.48	-0.77
13.	55.8	46.2	17.5	73.3	13.4	117.4	13.0	117.3	-2.99	-0.09
14.	54.4	41.7	24.1	78.5	13.2	116.6	13.3	116.6	+0.76	±0.00
15.	62.7	33.5	11.5	74.2	12.8	115.4	13.0	117.2	+1.56	+1.52
16.	63.8	34.0	13.0	76.8	12.9	116.2	13.2	116.8	+2.33	+0.52
17.	61.7	21.2	15.7	77.4	12.8	117.6	13.2	116.6	+3.13	-0.85
18.	67.3	49.5	23.2	90.5	13.8	114.4	14.0	114.6	+1.45	+0.17
19.	66.3	57.5	34.2	100.5	15.2	111.2	14.8	112.7	-2.63	+1.35
20.	66.2	60.2	31.5	97.7	14.0	111.7	14.6	113.3	-2.67	+1.43

Sample No.	% pass 200 mesh	% 0.02 size mm	% 0.002 size mm	S_m	Proctor Compaction test		Computed from formula		% variations	
					Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	m_o	γ_{max}
21.	82.4	46.3	17.8	100.2	15.0	111.4	14.8	112.8	-1.33	+1.24
22.	84.7	57.5	26.8	111.5	15.4	110.7	15.8	110.5	+2.60	-0.18
23.	84.6	60.3	29.0	122.3	16.0	109.4	16.0	110.1	±0.00	+0.64
24.	89.0	63.3	33.3	122.2	16.8	108.8	16.8	108.2	±0.00	-0.55
25.	84.7	58.3	28.5	113.2	16.3	110.2	15.9	110.2	-2.45	±0.00
26.	85.5	67.2	35.5	121.0	16.8	108.8	16.7	108.5	-0.60	-0.28
27.	86.1	60.5	25.5	111.6	16.0	110.4	15.8	110.5	-1.25	+0.09
28.	88.6	65.7	32.7	121.3	16.4	109.8	16.7	108.4	+1.83	-1.28
29.	89.2	48.5	14.5	103.7	15.4	110.1	15.1	112.0	-1.94	+1.73
30.	86.9	74.5	36.5	123.4	17.3	109.4	17.0	108.0	-1.73	-1.28
31.	85.4	47.5	26.2	111.6	16.0	109.8	15.8	110.5	-1.25	+0.64
32.	79.1	63.5	31.5	110.6	16.2	111.5	15.7	110.7	-3.09	-0.72
33.	86.9	75.0	41.6	128.5	18.0	104.7	17.5	106.8	-2.78	+2.01
34.	82.7	71.3	39.5	122.2	17.4	106.8	16.8	108.2	-3.45	+1.31
35.	78.5	67.5	37.4	115.9	16.6	108.8	16.2	109.6	-2.41	+0.74
36.	74.2	63.8	35.4	109.6	16.0	110.8	15.6	110.9	-2.50	+0.09
37.	70.0	60.0	33.3	103.3	14.9	112.6	15.0	112.2	+0.67	-0.36
38.	65.8	56.3	31.2	97.0	14.6	114.5	14.5	113.4	-0.68	-0.96
39.	61.6	52.5	29.1	90.7	14.2	114.0	14.0	114.5	-1.41	+0.44
40.	57.3	48.8	27.0	84.4	13.9	116.0	13.6	115.6	-2.16	+0.34
41.	53.1	45.0	25.0	78.1	13.5	116.8	13.2	116.6	-.2.22	-0.17
42.	48.9	41.3	22.9	71.8	13.5	116.9	12.9	117.5	-.4.44	+0.51
43.	44.5	37.5	20.8	65.5	12.4	119.3	12.6	118.3	+1.61	-0.84
44.	40.4	33.8	18.7	59.2	12.5	119.4	12.4	119.0	-0.80	-.0.34
45.	36.2	30.0	16.6	52.9	12.7	118.5	12.2	119.6	-3.94	+0.93
46.	93.2	84.5	57.2	150.4	20.0	101.4	20.3	101.8	+1.50	+0.39
47.	96.2	71.5	20.7	116.9	16.5	108.0	16.3	109.3	-1.21	+1.21
48.	92.1	40.5	13.0	105.1	14.8	111.8	15.2	111.8	+2.70	±0.00
49.	94.1	63.5	12.5	106.6	15.1	110.6	15.3	111.5	+1.32	+0.81
50.	82.7	60.5	26.0	108.7	15.5	111.1	15.5	111.1	±0.00	±0.00

50.	82.7	60.5	26.0	108.7	15.5	111.1	15.5	111.1	±0.00 ±0.00

Sample No.	% pass 200 mesh	% 0.02 size mm	% 0.002 size mm	S_m	Proctor Compaction test		Computed from formula		% variations	
					Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	m_o	γ_{max}
51.	97.6	74.0	22.7	120.3	16.2	108.8	16.6	108.7	+2.47	-0.09
52.	98.9	90.2	52.5	151.4	20.4	102.4	20.4	101.5	±0.00	-0.88
53.	99.2	95.7	48.7	147.9	20.4	101.5	19.9	102.4	-2.45	+0.89
54.	99.7	91.5	36.0	135.7	18.6	104.6	18.4	105.2	-1.08	+0.57
55.	99.6	93.7	43.5	143.1	19.6	105.6	19.3	103.5	-1.53	-1.99
56.	96.4	86.2	48.5	144.9	20.1	103.4	19.5	103.1	-2.99	-0.29
57.	87.7	81.7	45.2	132.9	18.5	103.3	18.0	105.8	-2.70	+2.42
58.	98.5	92.7	40.7	139.2	19.6	103.6	18.8	104.4	-4.08	+0.77
59.	99.6	88.7	39.7	139.3	19.5	102.5	18.8	104.4	-3.59	+1.85
60.	98.9	93.7	55.2	154.1	21.1	99.7	20.8	100.9	-1.42	+1.20
61.	97.6	89.7	46.6	144.2	19.1	101.4	19.4	103.2	+1.57	+1.78
62.	99.7	88.2	44.2	143.9	20.0	102.1	19.4	103.3	-3.00	+1.18
63.	99.6	99.2	50.0	149.6	20.4	101.6	20.1	102.0	-1.47	+0.39
64.	55.1	40.7	11.0	66.1	11.6	120.4	11.6	118.2	±0.00	-1.86
65.	68.5	56.5	18.5	87.0	14.4	113.2	13.8	115.2	-4.35	+1.74
66.	34.2	28.7	19.0	53.2	11.0	119.4	11.2	119.6	+1.79	+0.17
67.	74.1	52.0	22.5	96.6	14.0	112.5	14.5	113.5	+3.57	+0.88
68.	32.0	28.7	17.0	49.0	11.8	121.6	12.0	119.9	+1.67	-1.42
69.	57.7	35.7	19.5	77.2	13.6	115.2	13.2	116.7	-3.03	+1.29
70.	87.5	76.2	37.2	124.7	16.5	107.0	17.1	107.7	+3.58	+0.65
71.	88.8	80.0	36.2	125.0	17.5	106.8	17.1	107.6	-2.34	+0.74
72.	54.4	50.0	19.7	74.1	13.5	115.3	13.0	117.2	-3.85	+1.62
73.	81.9	79.7	22.5	104.4	15.4	110.0	15.1	112.0	-1.99	+1.79
74.	61.9	13.0	10.5	72.4	12.6	116.2	12.9	117.4	+2.33	+1.02
75.	32.1	25.2	15.2	47.3	12.3	117.1	12.0	120.0	-2.44	+2.42
76.	31.5	28.5	17.7	49.2	11.9	118.1	12.1	119.9	+1.65	+1.50
77.	32.6	25.4	18.0	50.6	11.8	118.0	12.1	119.8	+2.48	+1.50
78.	83.8	80.0	31.2	114.3	16.6	107.6	16.0	109.9	-3.75	+2.09
79.	86.2	62.5	20.5	106.7	15.0	109.0	15.3	111.5	+1.96	+2.24
80.	88.8	81.5	34.5	123.3	16.4	106.8	16.9	108.0	+3.05	+1.11
81.	94.6	60.5	14.0	108.6	15.5	109.6	15.4	111.1	-0.65	+1.36

Sample No.	% pass 200 mesh	% 0.02 size mm	% 0.002 size mm	S_m	Proctor Compaction test		Computed from formula		% variations	
					Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	Optimum moisture content (%) (m_o)	Max. dry density lbs/cft (γ_m)	m_o	γ_{max}
82.	82.1	53.5	14.5	96.3	13.9	115.6	14.4	113.6	+3.60	-1.76
83.	89.1	75.5	26.0	115.1	15.8	109.8	16.1	109.8	+1.90	±0.00
84.	97.9	79.5	20.7	118.6	16.4	109.8	16.5	109.0	+0.61	-0.71
85.	97.6	73.0	19.0	116.6	15.9	110.3	16.3	109.4	+2.52	-0.77
86.	99.5	92.0	35.2	134.7	18.2	105.9	18.2	105.4	±0.00	-0.44
87.	97.0	78.7	27.7	124.7	16.6	107.6	17.1	107.7	+3.01	+0.08
88.	84.9	62.0	16.5	101.4	15.2	112.5	14.9	112.6	-1.97	+0.05
89.	76.5	59.2	17.5	94.0	14.0	114.7	14.3	113.9	+2.14	-0.66
90.	97.0	74.5	21.0	118.0	16.5	110.2	16.4	109.1	-0.61	-0.95
91.	97.1	46.5	8.0	105.1	15.6	106.5	15.2	106.8	-2.56	+0.28
92.	99.7	97.2	35.7	135.4	19.0	104.0	18.3	105.3	-3.68	+1.23
93.	56.2	35.7	13.7	69.9	12.4	120.5	12.8	117.7	+3.23	-2.32
94.	80.4	36.6	11.2	91.6	13.9	112.9	14.1	114.4	+1.44	+1.33
95.	85.7	34.0	9.7	95.4	14.6	105.6	14.4	106.7	-1.37	+1.04
96.	69.8	35.2	14.2	84.0	13.6	114.3	13.6	115.7	±0.00	+1.22
97.	87.1	36.0	9.5	96.6	16.9	104.6	16.2	105.9	-4.14	+1.24
98.	94.9	37.2	8.7	103.6	16.8	103.2	17.0	104.0	+1.19	+0.78
99.	97.4	86.7	36.4	103.8	18.8	104.4	18.1	105.6	-3.72	+1.15
100.	88.9	80.0	29.5	118.4	15.9	108.4	16.4	109.1	+3.05	+0.64

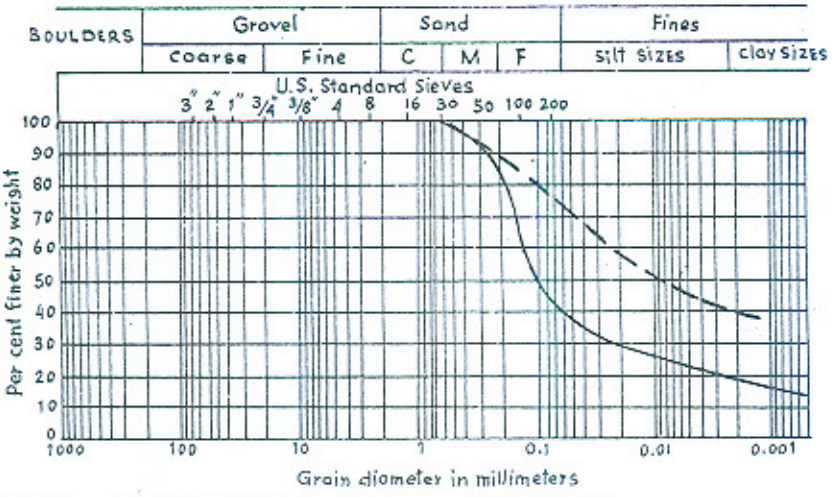


Figure-1 *Grain size chart and* M.I.T *grain size scale*

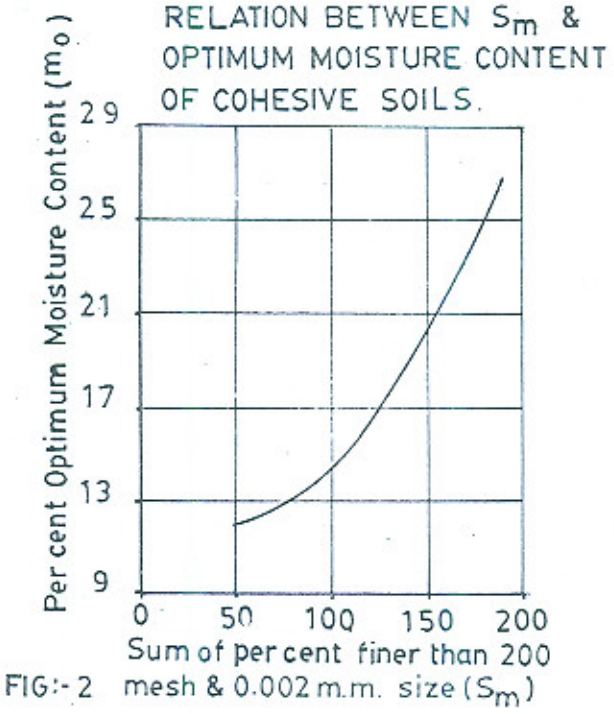


FIG:-2