CORRECTION SLIP FOR PAPER No. 236

PUNJAB ENGINEERING CONGRESS.

Page 181. 10th line from bottom.
for "symetrical" *read* "asymetrical"

Page 183. Last three lines.
for "If the increase in depth .... greater than A_1,"
read "A_1/A_2 decreases with increase in F.N."

Page 184. Equation (2) first line.
for "$(B_2\ d_2)$" *read* "$(B_3\ d_2)^2$"

Page 190. Line 22.
for "produce" *read* "produced."

PAPER No. 236

"FLOW IN EXPANSIONS IN OPEN CHANNELS."

BY

A. R. Thomas, B. Sc., (Eng.) I. S. E., Executive Engineer, Central Irrigation & Hydrodynamic Research Station, Poona (India).

The flow of water in expansions in open channels is a matter of frequent concern to irrigation engineers, as so often the flow decides to misbehave and follow only one side of an expansion, with the formation of a large return eddy on the other, as in Fig.1 (*a*), or form a central jet, either of which having disastrous results in the form of scour of the banks and bed of the channel downstream.

The reason for the formation of a return eddy is not immediately apparent, though a clue was given by Prandtl* when he showed that a condition for stagnation of fluid particles at the boundary was the reduction of kinetic energy, by friction loss, to a stage when an adverse pressure gradient, could not be overcome. Use of this interpretation has been made in aerodynamics, for example in the Handley-Page Slot, but so far as the Author is aware no rational analysis of flow in expansions in open channels has been published.

It has been the custom to speak of expansions simply in terms of the angle of divergence, *e. g.*, 1 in 8, though it is generally recognised that expansions of equal angles may not behave in a similar manner in different circumstances. In this Paper the Author attempts to analyse the flow in expansions qualitatively with the aid of some of the basic principles of fluid mechanics, indicating the effects of various factors and mention is made of methods of improving the flow conditions as found by experiments at the Central Irrigation and Hydrodynamic Research Station.

Basic Principles.

These are :—

(*i*) Newton's 2nd law, by which an acceleration takes place proportional to and in the direction of a positive pressure gradient. This may be applied by resolving forces acting on any "block" of water, *i. e.*, water enclosed within

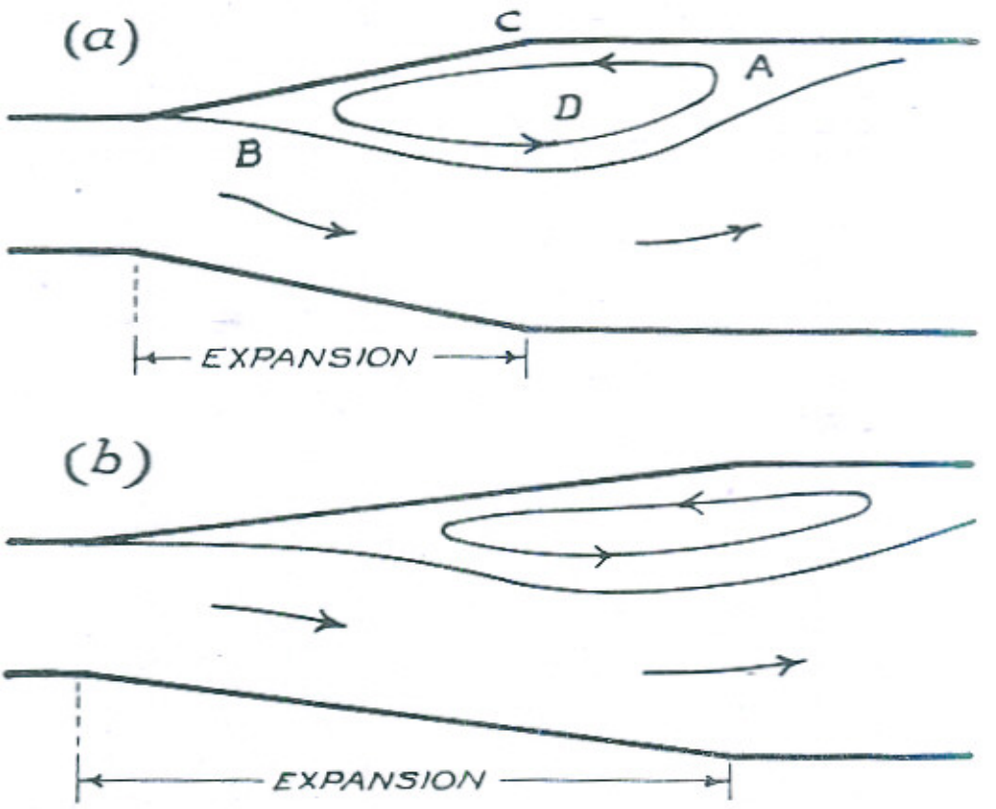
* "Prandtl-Tietijens 'Applied Hydro and Aero-Mechanics' 1934, pp. 64 and 81."

an imaginary boundary; and equating the resultant to the net rate of increase of momentum in the block.

(*ii*) Bernoulli's Theorem—the energy of flowing water may be in the form of potential pressure or velocity energy, so that with a given total energy the increase of one reduces the others.

(*iii*) Turbulence has the effect of spreading or equalising velocity, due to interchange of momentum caused by the "mixing" of the fluid.

FIG. 1.
EXPANSIONS IN PLAN



Conditions under which return flow develops.

Consider the most simple case of expansion...with vertical sides and horizontal bed. In figure 1 (*a*) is shown a sharp expansion which the flow does not follow.

It is clear that pressure distribution favourable to the development of return flow exists as, owing to the conversion of velocity head into pressure head, the W.L. and therefore the pressure, at *A* is higher than it is at *B*, leading (by Newton's 2nd Law) to a decrease in momentum in the

direction BA or an increase in momentum in the direction AB.

Let us first assume that a return eddy has developed and examine the factors tending to reduce or increase it. The forces acting on it are :—

(*i*) Pressure gradient from A to B.

(*ii*) Shear force on the fluid at D adjacent to the high velocity jet, caused by turbulent intermixing and by viscous drag (mainly intermixing except when R. N. is low).

(*iii*) Frictional drag of bed and sides.

The result depends on the degree of intermixing. If it extends past D to the fluid against the boundary at C, as it may if the eddy is narrow, the addition of forward momentum there may overcome the adverse pressure gradient, so that the whole eddy fluid will move forward and the eddy will disappear.

If, on the other hand, the intermixing is sufficient to outbalance the adverse pressure gradient at D, but not at C, there will be forward motion at D and return flow at C, and the eddy will persist.

If the intermixing is not sufficient to outbalance the adverse pressure gradient even at D, the width of eddy will increase, contracting the main stream until the transmitted momentum is sufficient to outbalance the pressure gradient to the required degree.

The effect of frictional drag is likely to be relatively small. It tends to slow up the eddy, but also the more the main stream is contracted the greater the frictional loss therein, tending to reduce the pressure gradient.

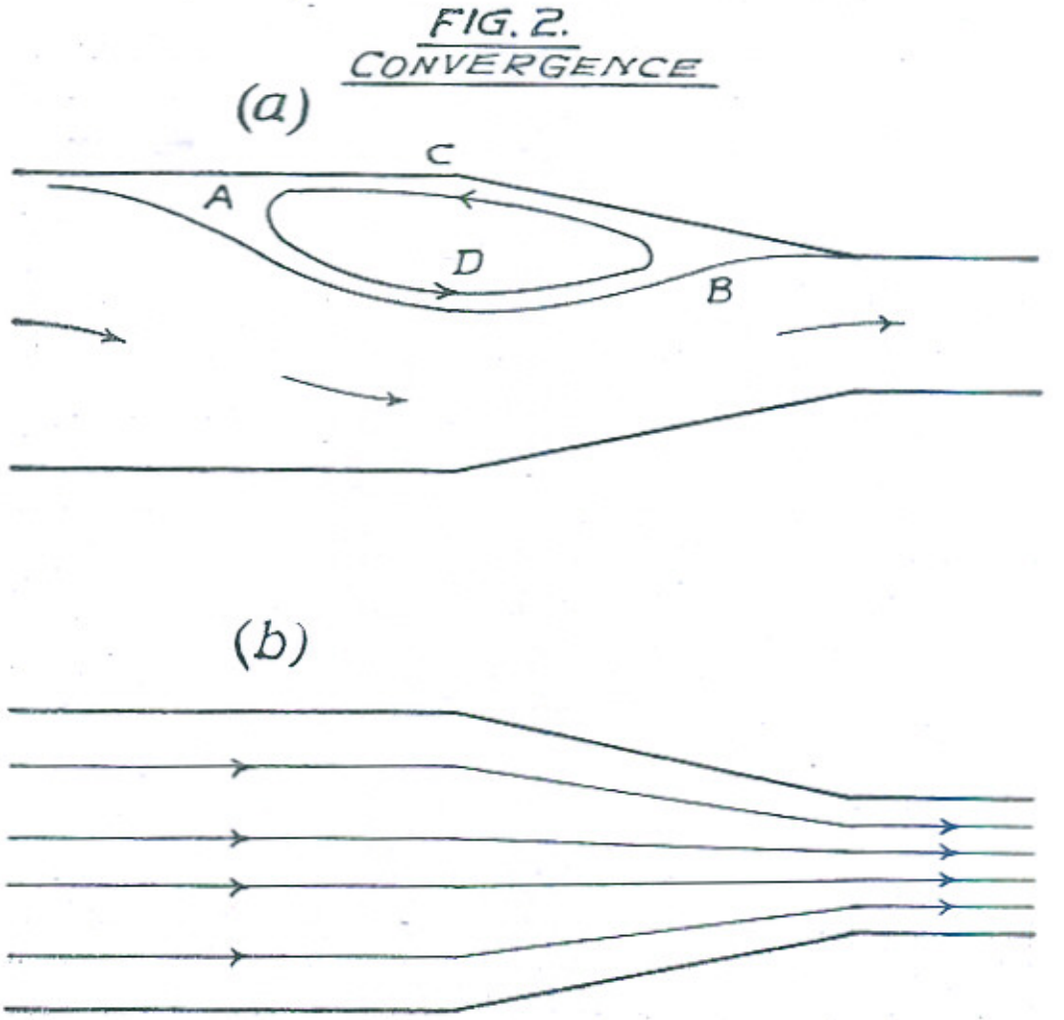
It would thus appear that, given initially a condition of symmetrical flow (as must always occur in practice, however small) a return eddy may develop depending mainly on the pressure gradient, velocity of main stream and turbulence, increasing or decreasing in size until a balance is reached. The primary condition is the existence of an adverse pressure gradient.

If the expansion is very sharp, or influences are present to maintain the stream only in the centre (such as excessive bowing of the bed), "jetting" will occur, a central jet passing between two large eddies.

If however, the expansion of the sides is more gradual, as shown in Fig. 1 (*b*), there is less tendency for the return flow to form, as the high velocity stream is of greater length, with greater frictional drag, and the return flow is of elongated shape with greater frictional and internal losses, both factors, (particularly the latter), reducing the effect of the adverse pressure gradient. There is also a greater length over which shear stress is transmitted from the main stream into the return flow, tending to equalise velocity. Thus the more gradual the expansion the less favourable are the conditions for a return eddy.

In converging flow, the pressure gradient is unfavourable to the formation of return flow. This may be seen by assuming return flow to exist and seeing whether it will tend to persist or disappear. In Fig. 2 this state of flow is shown in (*a*).

FIG. 2.
CONVERGENCE



The pressure at A is higher than at B owing to the conversion of pressure head into velocity head, and the force transmitted by shear stress or intermixing, from the main stream is also in the direction A to B. Under such conditions the return flow must disappear and steady converging flow would take its place as shown in (*b*). Thus, while a divergence creates conditions favourable to return flow, a convergence, on the other hand, is always a stabilising influence.

It is now possible to state the circumstances in which return flow can exist, *viz.*, alongside an expanding stream, the downstream *W. L.* of which is higher than the upstream due to conversion of velocity energy into pressure energy, or upstream or downstream of the impact of a jet on a stationary body, or upstream of a standing wave, or alongside a rising jet of hypercritical flow, as these are the only instances of a rise in pressure in a downstream direction. Examples of the first two are shown in Fig. 5 (*b*).

The effect of curvature of flow is also to be borne in mind, as this may raise or lower pressures. In Fig. 1, for example, convex curvature at B increases pressure, but this has little effect on the pressure gradient from A to B as it is balanced by the increase of pressure due to convex curvature at A.

The Influence of Froude No. and channel dimensions.

An increase in *F. N.* (Froude No. V^2/gd) in the channel upstream of the expansion, keeping other dimensions the same (hence with an increase in discharge, Q) leads to a greater recovery of pressure head, because the latter

$$= \frac{V_1^2 - V_2^2}{2g} = V_1^2/2g \quad \{1 - (A_1/A_2)^2\} \ldots (1)$$

where V_1 and A_1 are the velocity and C. S. area upstream and V_2 and A_2 are the same downstream of the expansion. If the increase in depth due to recovery of pressure head is considerable compared with the depth of flow, A_2 will be appreciably greater than A_1, so the dif-

ference in pressure increases in greater ratio than V^2. The velocity of the return eddy is governed both by the pressure difference and by the velocity of the main stream with which the eddy is in contact, while the boundary (*i.e.*, bed and sides) resistance to the return flow may be proportional to the square or lower power of V according to the roughness of the boundary. The higher F. N., therefore, due to the greater pressure difference, enables a larger return eddy to be maintained, particularly where the F. N. is high or (to less extent) the boundary smooth. In other words, the higher the F. N. the greater the tendency to return flow, this being most marked when the F. N. approaches the value 1.

An increase in depth with constant F. N. (*e.g.* vertical exaggeration in models) results in greater recovery of pressure head, as this is proportional to depth, and hence a larger return eddy. To offset this result a more gradual expansion is required *i. e.*, longitudinal exaggeration, which incidentally confirms a conclusion reached in a different way by Mr. Inglis in connection with canal fall experiments.

An increase in depth with constant Q, (and hence reduced F. N.) on the other hand, reduces the recovery of pressure head, because from (1) recovery

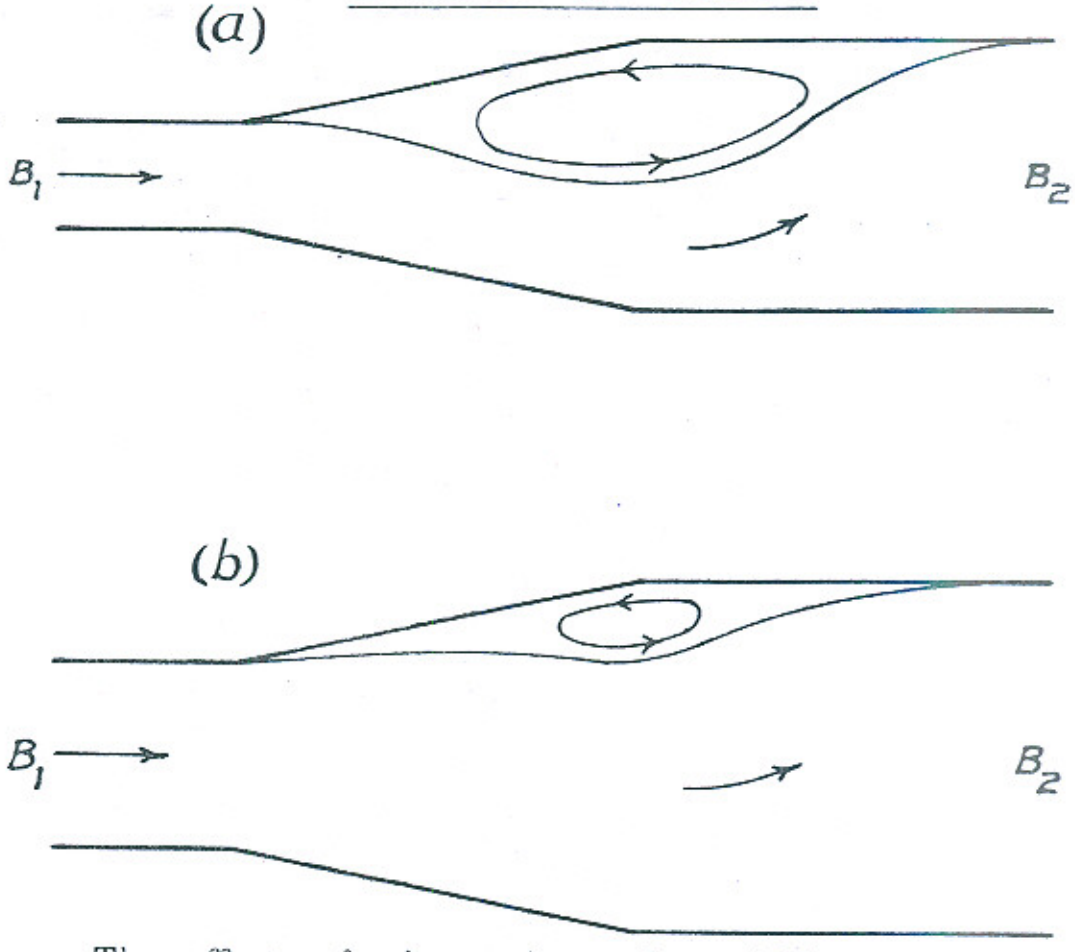
$$= \frac{Q^2}{2gB_1^2d_1^2}\left(1-\frac{(B_1d_1)^2}{(B_2d_2)}\right)$$

$$= \frac{Q^2}{2gB_1^2B_2^2}\left(\frac{B_2^2d_2^2-B_1^2d_1^2}{d_1^2d_2^2}\right)\text{.....(2)}$$

(Where B_1 and B_2 and d_1 and d_2 are the widths and depths upstream and downstream of the expansion) which, as will be seen, varies inversely approximately as (depth)^2. With a given discharge, therefore, the greater the depth the more stable the flow. The slight increase in adverse

pressure gradient, due to reduced frictional losses, is probably negligible in short expansions.

FIG. 3.
EFFECT OF WIDTH



The effect of increasing the width of channel may be seen from Fig. 3. If the F. N. is equal in (*a*) and (*b*), the recovery of pressure head is greater in (*a*) owing to the greater expansion ratio B_2/B_1. The conditions are thus more favourable to the formation of a return eddy in (*a*). If Q is equal in each, and therefore F.N. lower in (*b*), the conditions will be still more favourable for a return eddy in (*a*) than in (*b*). In other words, for a given angle of divergence, the wider the channel the more stable is the flow.

For the same reason, with constant angle of divergence, the greater the expansion ratio (and hence the longer the expansion) the greater the tendency to return flow.

To summarise :—

For a given Q, increase in either depth or width of channel stabilises flow. For a given F.N., increase in depth assists the formation of a return eddy, while increase in width stabilises flow.

Increase in Q, F.N., angle of divergence or expansion ratio, assist the formation of a return eddy, other dimensions remaining constant in each case. Reynolds No. (VR/v) also has an effect on expanding flow in that it is a measure of turbulence—the greater the R.N. the more stable the flow.

The influence of upstream conditions may just be touched upon. If the expansion immediately follows a contraction, the flow will follow the expansion more easily than if the contracted channel is parallel for some length, as in the former case there is concentration of flow at the sides which easily expands, whereas, in the latter, the flow adjusts itself to the contracted section, the concentration expanding inwards from the sides, thus equalising the flow, upstream of the expansion.

Also, one-sided flow at the entrance to the expansion tends to make one-sided flow in the expansion.

Practical Methods of inducing flow to follow expansion.

Mr. A .M. R. Montagu in a general discussion on expansions* has stressed the value of a rising floor and of boundary roughness. A rising floor is effective as it forms a convergence in the bed as an antidote for the divergence of the sides ; it requires a lower bed upstream of the expansion than downstream. Boundary roughness to have much effect would appear to have to be very considerable, as for example provided by the stabiliser blocks mentioned below.

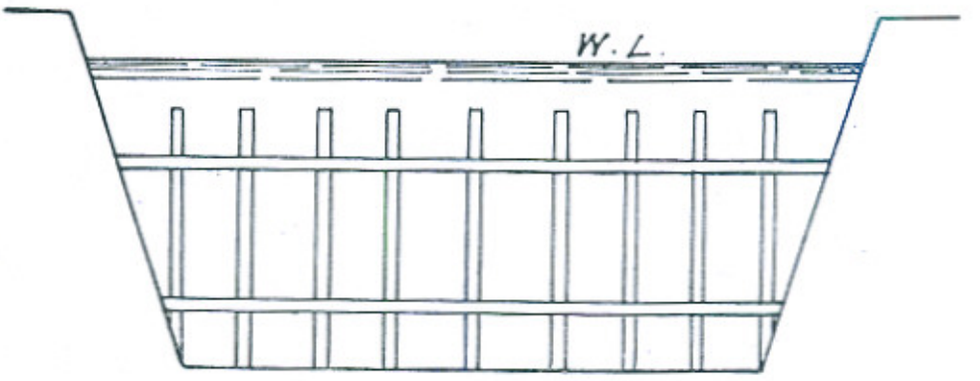
Four methods have been tested at Poona :

(*i*) grid,
(*ii*) baffle stabiliser,
(*iii*) stabiliser blocks,
(*iv*) vanes.

*Central Board of Irrigation, Publication No. 10, 1935.

The grid (Fig. 4) which is fixed vertically in the expansion near the upstream end, generates numerous small eddies which have the effect of transferring momentum and equalising velocity from side to side. Under such conditions tendency to return flow is necessarily reduced.

FIG. 4. GRID

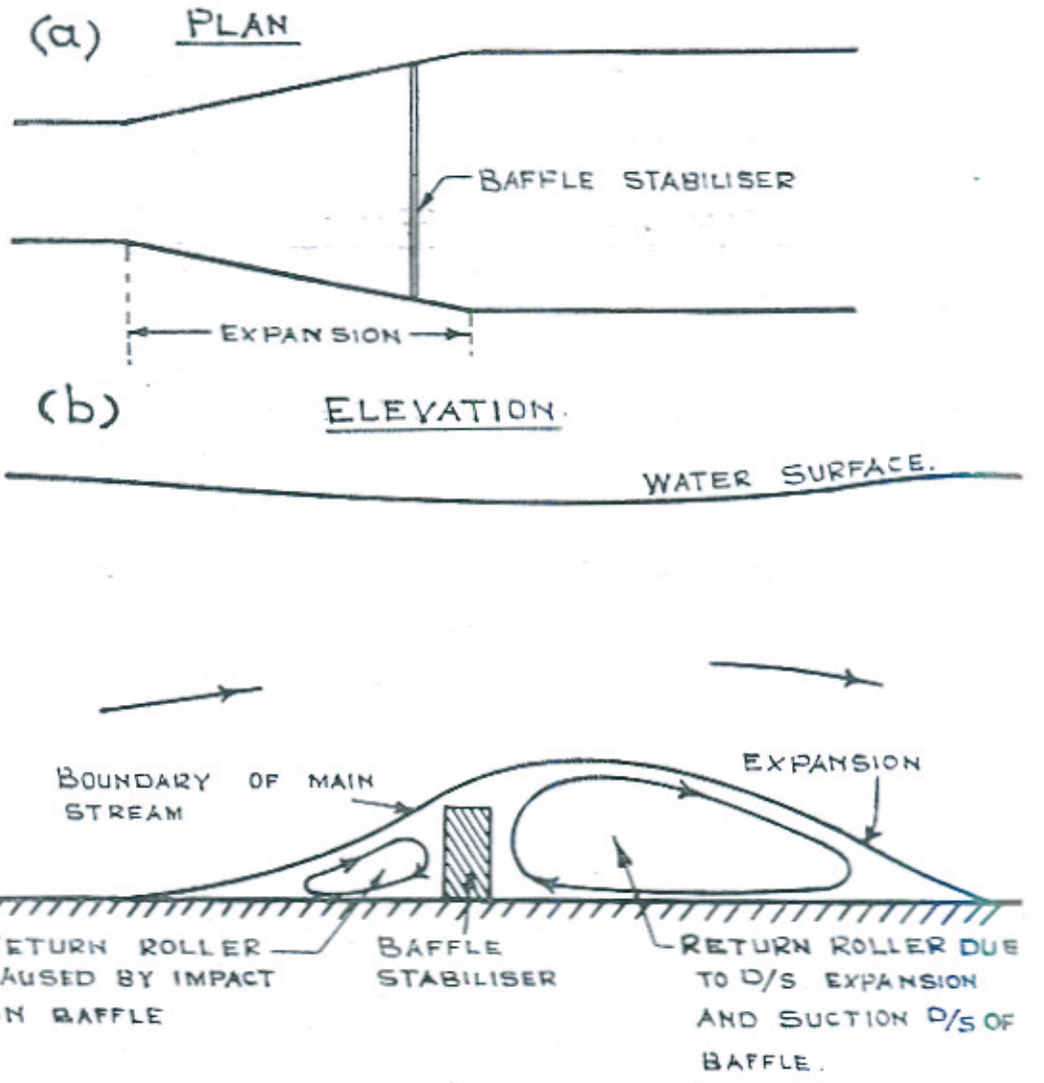


The baffle stabiliser (Fig. 5) consists of a raised step across the bed in the expansion, usually near the downstream end. Owing to the impact of the stream on the baffle, (increased by the flow rising up over the baffle) pressure is created in the upstream direction, which varies as V^2, and causes afflux, *i. e.* increases energy slopes. If the flow follows one side of the expansion only the upstream pressure caused by the baffle is greater than it is if the flow is evenly distributed over the whole width, owing to the V^2 relation. The baffle stabiliser therefore induces the flow to follow the expansion. Also a baffle, like a rising floor, causes converging flow upstream which has a stabilising influence. The diverging flow downstream of the baffle has the opposite effect, and this

is why a baffle is most effective when placed near the downstream end of an expansion.

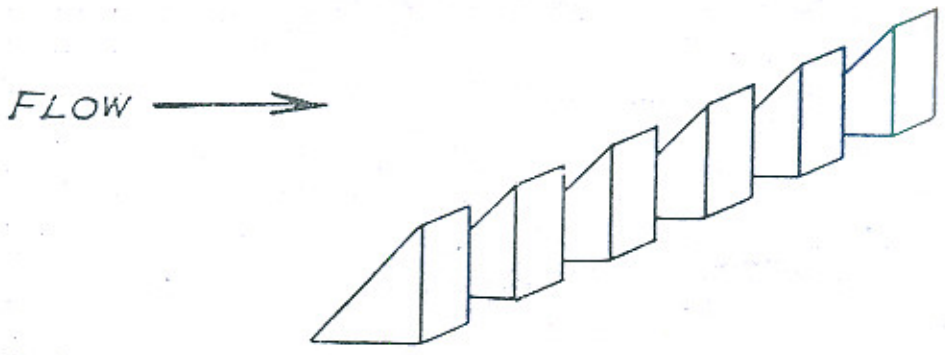
FIGURE:- 5.

ACTION OF A BAFFLE STABILISER.



Stabiliser blocks, (Fig. 6) fixed to the pavement in the

FIG. 6. ROW OF STABILISER BLOCKS.



expansion, also exert pressure upstream and in addition, like the grid, create turbulence, but more localised and less effective.

All these methods result in some energy loss. In addition, the grid has the disadvantage that it catches debris; the baffle stabiliser may hold up sand upstream; but the blocks, if given sloping upstream faces, have neither of these defects. Blocks also have the advantage of reducing bed scour downstream as has been found, for example, in the case of canal falls.

The best arrangements of blocks, found so far in divergence experiments at the Central Irrigation & Hydrodynamic Research Station, are (as described in the 1938-39 Annual Report*) blocks $d/4$ high, where d = depth of flow, arranged either in one row just upstream of the wide end of the expansion, or in two rows ; one at each end. Higher blocks cause surface waves (depending also on F. N.) and increase afflux. If blocks are placed in rows too close together, they mask each other and their efficiency is reduced.

If a baffle stabiliser is made too high, the bed flow may be deflected spirally side-ways at the baffle, rising to the surface near the sides. This results in a high velocity jet in the centre.

* Now in the Press.

Vanes form a physical obstruction to the formation of a return eddy. Their disadvantages are :--

(*a*) cost,

(*b*) liability to catch debris at upstream ends,

(*c*) fixed spacing which may not suit a large range of stage, *i e.*, the most suitable spacing for high stage is usually different from that for lower stages.

For these reasons it is better to do without them, if possible, but even when the expansion is so sharp that no other device succeeds, the provision of vanes is likely to be effective. An example of the successful use of multiple vanes is the Tando-Mastickhan Fall in Sind*. A single central vane has been used with success elsewhere in Sind, particularly where the upstream approach is one-sided, and has also been found effective in experiments with Standing Wave Flumes at Poona. In the latter, the nose of the vane is placed in the hypercritical flow, so that equal discharge each side is ensured.

The location of a standing wave in an expansion is one of the most effective means of spreading the flow. Owing to the turbulence produce, velocities are equalised, so that expansion can take place downstream of the wave, despite any instability caused by the sudden expansion in the hydraulic jump.

Warped sides are sometimes necessary to prevent small regions of return flow (by avoiding sharp returns in the sides) when the expansion leads from a channel with vertical sides to one with sloping sides or natural berms. The expansion at the surface being sharper than at the bed, would (from I) be expected to result in a central surface jet with bed flow expanding and rising at the sides, which result would be assisted by the action of a baffle stabiliser. In such cases blocks may be more effective owing to the turbulence they produce.

A bowed bed reduces the tendency for the flow to hug one side, as the energy slope is flatter where the

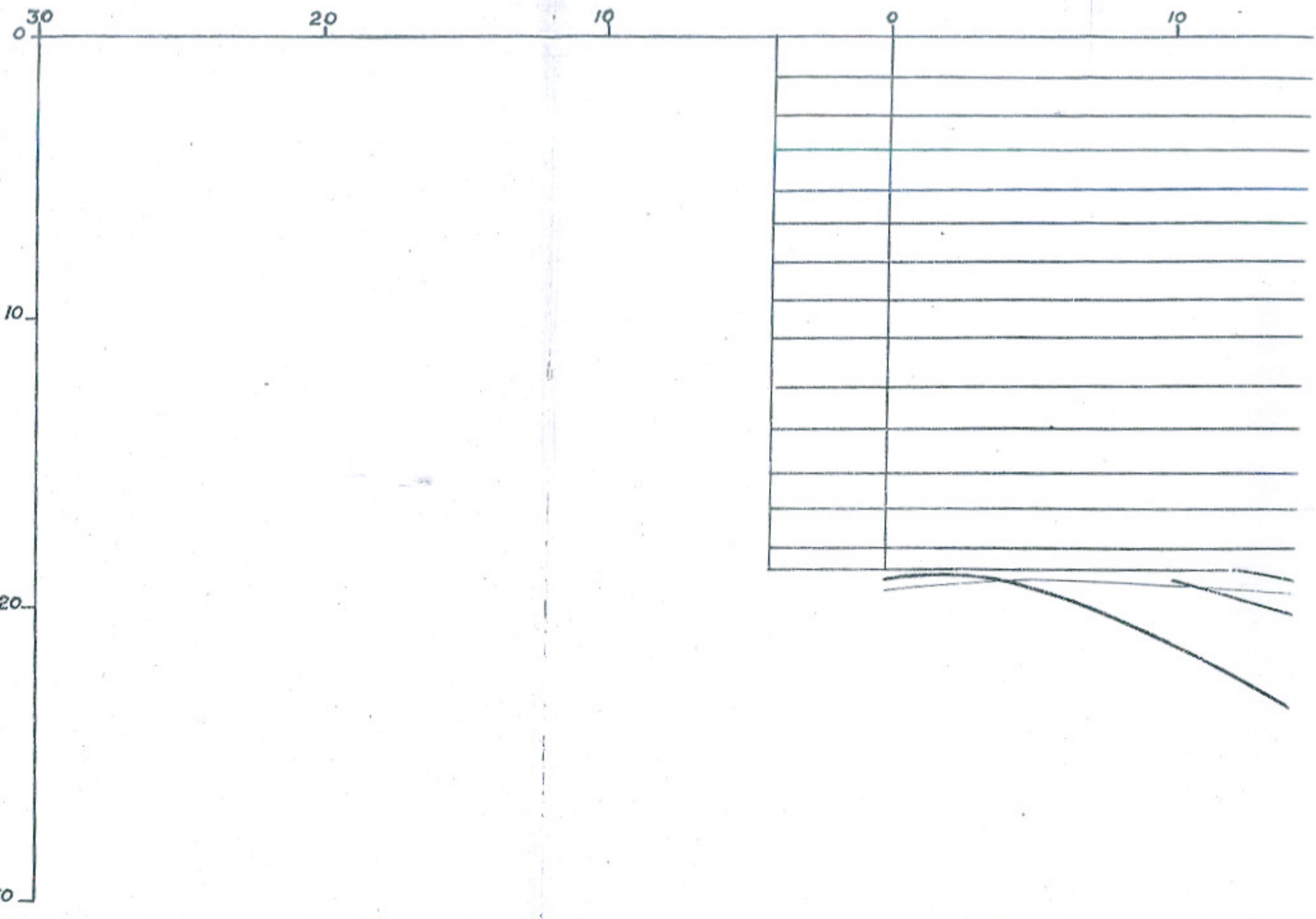
* Bombay P.W.D. Technical Paper No. 44 "Dissipation of Energy below Falls," by Inglis and Joglekar.

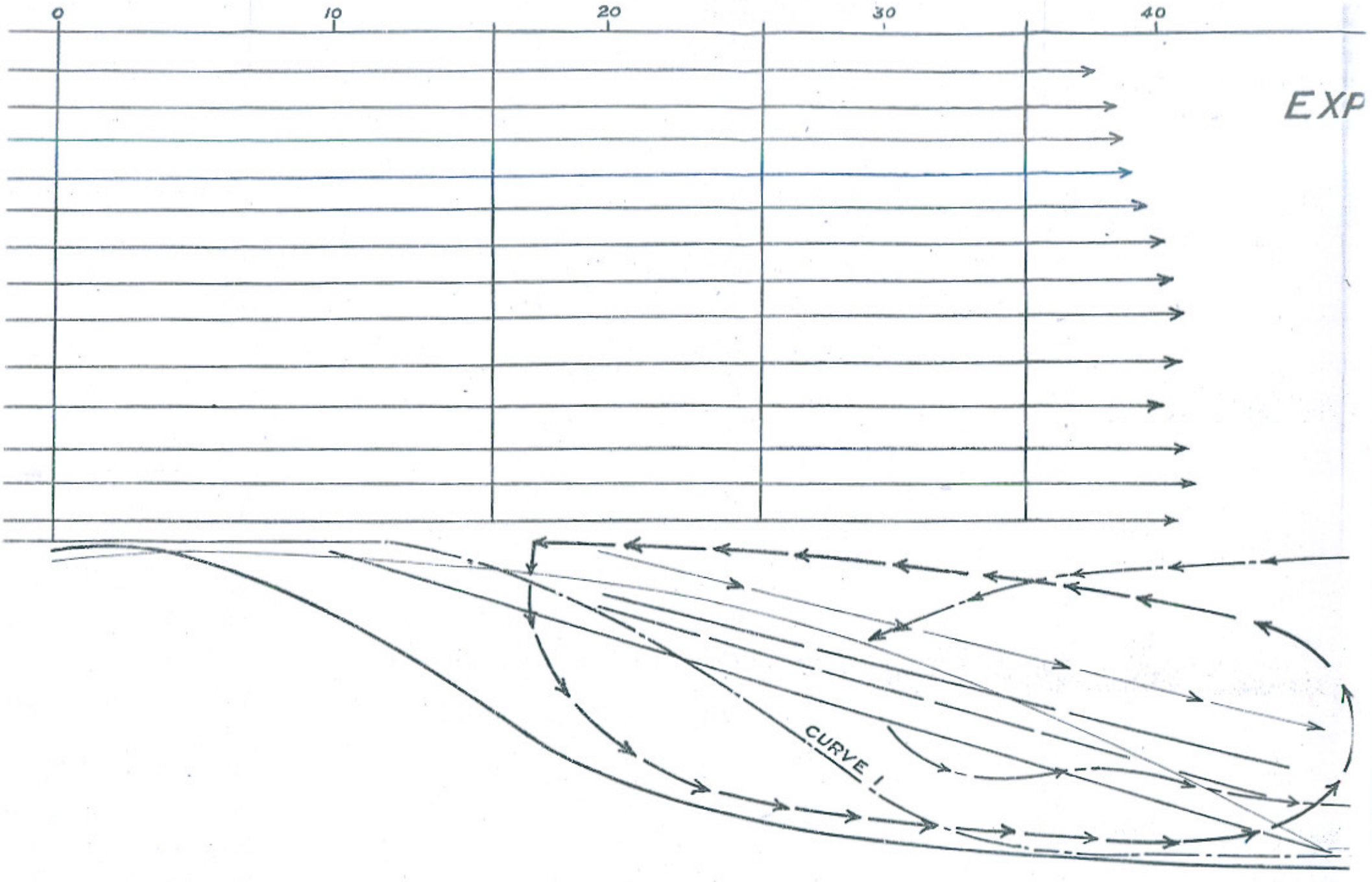
channel is deeper. As previously stated, however, it may if excessive result in central jetting, for the same reason.

Conclusion.

The above analysis of the effects of certain factors on flow in expansion is necessarily qualitative, but the Author intends, with the aid of model experiments and more detailed analysis, to derive quantitative criteria for the satisfactory performance of expansions and present them in a form suitable for general application.

In the meanwhile it is hoped that this Paper will be of some assistance to engineers who may be confronted with the problem.

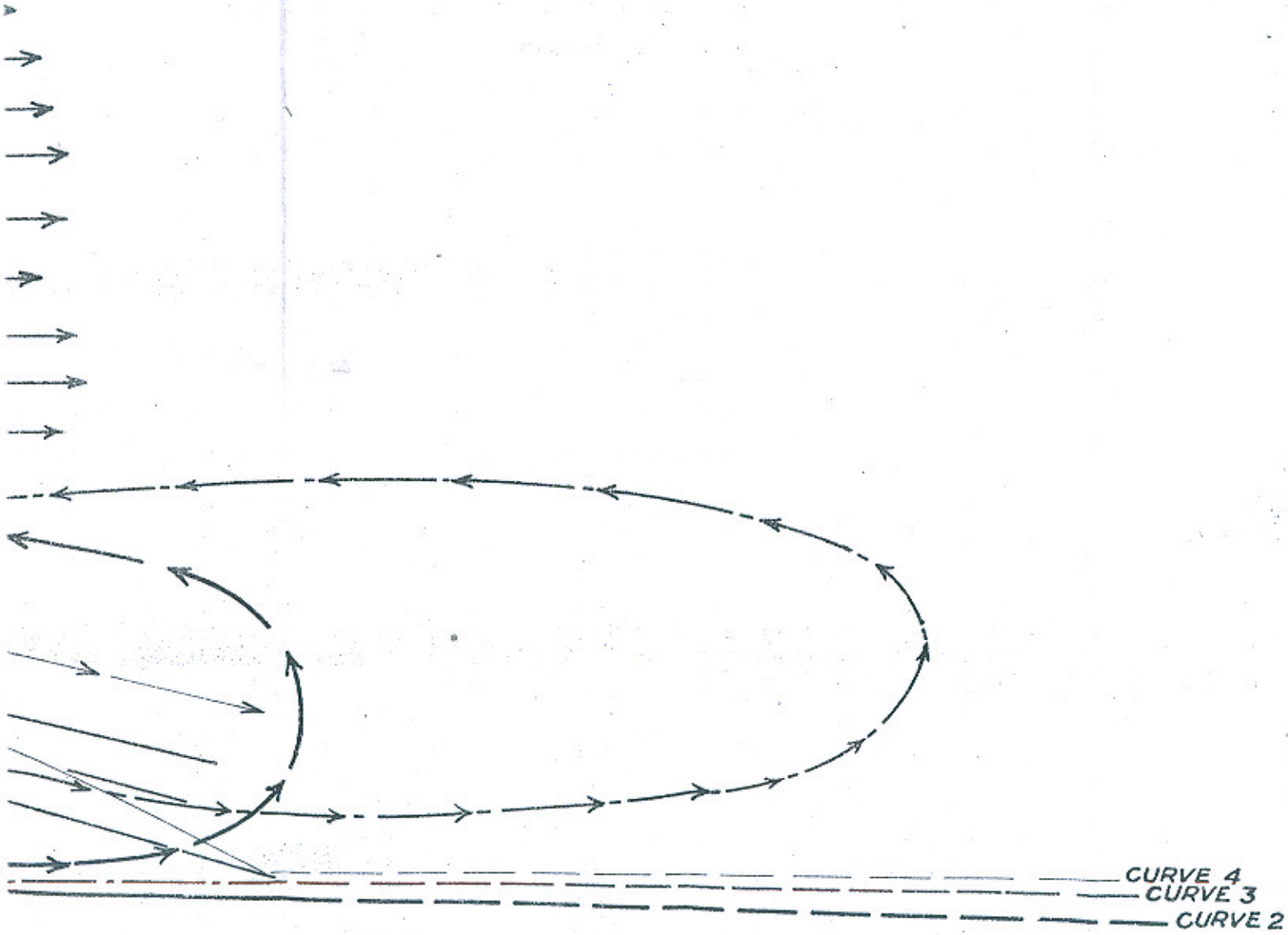

30
20
10
0
10
0
10
20
30


0
10
20
30
40
EXP
CURVE I

40 50 60 70

FIG. 7
PAPER NO. 236

EXPERIMENTS ON EXPANSION



PUNJAB ENGINEERING CONGRESS
1940

DISCUSSION

In his introductory remarks the **Author** said that the Paper comprised two parts: the first, his interpretation of the mechanics of flow in an expansion, the second, the application of the theory to some practical methods of inducing flow to follow expansions. A description of experiments with these methods would be found in the Annual Reports of the Central Irrigation and Hydrodynamic Research Station Poona.

The conclusions reached would be proved or disproved by results obtained in practice, and the Author hoped that the discussion would bring forth useful data based on actual experience.

Dr. Harbans Lal Uppal stated that this subject had been studied in the Irrigation Research Institute, Lahore. Experiments were started two years back in the Hydraulic Laboratory to determine the most suitable method of effecting expansion. Tests were made with different ratios of expansion under different discharges. In each case observations were made for :—

Velocity of flow.
Direction of flow.
Water surface profile.

The velocity observations were made by a universal pitot tube and the water surface profile by means of a point gauge. Photographs were taken to illustrate the current directions with special reference to the presence or the absence of return flow. For the purposes of this study lighted candles mounted on corks were placed on the water just upstream of the point where the expansion started and photographs were taken with a camera fitted with an automatic electric shutter which operated after every half a second. He considered it sufficient to refer only to one such experiment*. The expansion ratio adopted was 75%. This was achieved by four different methods shown in Fig. 7 :—

Concavo-Convex.
Concave.
Straight.
Convex.

* Harbans Lal Uppal and Nazir Ahmad.

In the last two cases the expansion was effected very gradually as shown in Fig. 7. The important indications obtained from these tests were that :—

When the expansion was sudden, *i.e.*, in case of 1 and 2, there was a considerable area in which return flow took place. Fig. 7 illustrates large back rollers forming close to the sides where the expansion started. However, the return flow disappeared as soon as the expansion was achieved by curve 4. The most satisfactory results were obtained by adopting very gradual expansion. In this case the distribution of flow was uniform and took place in the forward direction following the expansion. The conclusions obtained from these tests were further verified from observations made on large scale model *vide* Annual Report of the Punjab Irrigation Research Institute for the year ending April 1939.

Regarding the stabiliser blocks mentioned by the Author under the heading "Practical methods of inducing flow to follow expansion." Dr. Uppal stated that the Author considered the effective height of blocks to be $\frac{d}{4}$ (d = depth of water), whereas in his Paper No. 214 read before the Punjab Engineering Congress of 1938 he had arrived at the effective depth to be equal to $\frac{d}{4}$ to $\frac{d}{5}$. Again, he had stated in paper No. 214 that higher blocks increased the afflux over the Weir and the Author also states the same thing, *i.e.*, higher blocks cause surface wave and increase in afflux. He was glad that the Author's findings confirmed the results obtained in the Punjab Irrigation Research Institute.

Mr. Radha Krishan Khanna said that he had only one word to say on this excellent little paper, and that was that it contained more useful information it its short space than was generally to be found in Papers of this description. He congratulated the Author on the very good work that was being done at the Poona Research Station, and hoped that their example would be followed widely.

Mr. Inglis congratulated Mr. Thomas upon his interpretation of the mechanism of flow in expansions, which he had found very helpful in enabling him to visualise actual conditions.

He added that there seemed however to be a fourth factor, *viz.*, curvature of flow and cross gradients of pressure, which always accompanied it.

On page 186, the Author had said

"If the expansion immediately flows a contraction the flow will follow the expansion more easily than if the contracted channel is parallel for some length, as in the former case there is concentration of flow at the sides which easily expands."

It appears that the Author meant by "Concentration of flow at the sides" an increase of discharge near the sides. But on line 11 of page 184, the Author showed that so far from facilitating divergence, this would, by increasing the Froude Number at the sides, cause a tendency to return flow; because to quote his words "the higher the Froude Number the greater the tendency to return flow."

The explanation of this phenomenon was, however, clear when it was borne in mind that the depth of water entering a contraction is lower at the sides than in midstream which gives a gradient of pressure favourable to rapid expansion; whereas in a long-throated flume the depth at the sides becomes higher than in midstream and so conditions become less favourable to expansion.

Mr. K. A. Ghafoor said that he found guide vanes very effective in regulating flow in an expansion. The Author objected to them mainly on the grounds of cost and liability to catch debries at upstream ends. Cost could be reduced by making the vanes of thin reinforced brick work. To remove the defect of the upstream and catching debries, he suggested giving a gradual slope to the upstream and of the vane instead of keeping it vertical. The jungle coming in contact would then slide up the slope and pass on.

Dr. N. K. Bose said that Author had suggested differ-

ent methods of taking account of expansions and dealing with them by model experiments. One point which the Author had not made clear was the scale effect in such models. If was well-known that water with a certain velocity followed a definite curve with a fixed radius of curvature. If the velocity increased the water in contact with the curve would leave it and create turbulence behind it. This point was known as "ablosung" point and was very important in finding the resistance force by different obstacles in a flowing stream. The point of "ablosung" depended upon the velocity of flow and the radius of curvature of the boundary. In a model of the scale ratio of 1/N the velocities were reduced in the ratio of 1/H and the radius of curvature reduced in the ratio of 1/N. Hence the velocity with which a stream in a model would cling to the boundary with a certain radius of curvature would not behave in the same manner in the prototype. He asked how the Author would devise his experiment to get rid of this difficulty. He added that this was a real difficulty for interpreting prototype behaviour from model results, and in many cases, has proved a great stumbling block for the success of model experiments especially when three dimensional problems were dealt with. He further added that similar experiments were being made in the Punjab Irrigation Research Institute to find out the different scale effects.

Mr. Blench referred to the use of friction in spreading jets and the advantage of spreading flow above the standing wave when it would collapse on the floor like a jet hitting a board. The horizontal crest could be continued into the divergence for this purpose.

CORRESPONDENCE

Mr. C. C. Inglis sent the following written discussion :—

This interesting Paper by the Author deals with factors affecting expansion and so helps to clarify the question. The actual facts affecting expansions have been realised for some years.

In this connection, attention is drawn to Bombay

P. W. D. Technical Paper No. 44 by the writer and *Rao Sahib* D. V. Joglekar on "Dissipation of energy below falls",—published in 1933—, in which the minimum permissible divergences were given for normal channels for various values of B_s/d_s, where B_s=bed width of channels below falls and d_s=depth in channel downstream. The B_s/d_s ratio may, in the light of Lacey, be stated alternatively in terms of discharge, provided the silt factor 'f' is also taken into account. Table 1 is extracted from that Technical Paper and shows the minimum permissible expansions for various values of B_s/d_s and equivalent discharges :

TABLE 1

	Minimum permissible side divergence for normal channels.	B_s/d_s	Equivalent F. S. Q. of channels (cs.)
(1)	1 in 4	2·6	3
	1 in 5	3·4	30
	1 in 6	4·5	220
	1 in 7	5·9	700
	1 in 8	8·0	2400
	1 in 9	11·0	6000
	1 in 10	20·0	10500

["*Note.*—The permissible divergences would be less sharp in the case of shallow downstream channels flowing with high velocities, *i. e.*, in the case of channels with high silt factors."]

It may also be recalled that when model experiments were done at Khadakvasla with a 6·5 ft. fall in which Q =200 cusecs, throat width = 10 ft., expansion = 1 in 5 downstream, bed width=24 ft. (with 1 : 1 side slopes) ; surface width=32 ft.: depth of water=4 ft.; Froude Number=V^2/gd =0·25 ; and fluming ratio = 2, 0, satisfactory expansion results were obtained ; but when a geometrically similar model of the Tando Mastikhan Fall was tested with the same fluming ratio (Q = 3·75 cusecs; throat width=4·17 ft. downstream bed width = 10 ft ; downstream surface width = 11 ft. ; depth of water = 0·5 ft., Froude Number = 0·03. ; fluming ratio = 1·96), serious return flow occurred.

This was set right by buildings played vanes in extension of piers extending downstream as far as the baffle.

The alignment of these vanes, when produced, divided the bed width of the Canal at the end of expansion (from sides to midstream) at 1·26 R ; 1·09 R; 0·97 R; 0·88 R and 0·80 R, where $R = \frac{\text{Canal bed width}}{\text{Number of spans}}$

(*see* Technical Paper 44 referred to above).

This addition was so successful that it was considered worth adopting in later designs so as to reduce cost of construction, the downstream expansion length being thus reduced to half.

Mr. Gerald Lacey sent the following written discussion :—

The Author was to be congratulated on a valuable qualitative contribution to the subject of expansions in open channels and the results of his experiments designed to make quantitative analysis possible would be awaited with interest. The subject had been discussed at a recent meeting of the Research Committee of the Central Board of Irrigation and a resolution passed which made it clear that for an expansion to be free from return flow due regard must be paid to the discharge and the expansion ratio.

The subject so far from being academic was of the greatest practical importance. The writer had recently examined a large number of works on the Eastern Jumna Canal, and on the recently enlarged Mat Branch of the Ganges Canal, many of which exhibited the various phenomena which the Author had described. The writer would confine himself first to contractions caused by "tight" bridges, in which the bridges were not combined with falls.

On the Eastern Jumna Canal, constructed more than a century ago, the discharges had steadily increased, but in many cases the original bridges had been retained, and " tinkering " had been tried for many years to minimise the heavy action downstream and the formation of destructive return flow.

It was as well to examine the upstream side first. When the abutments were rectangular in plan, and stuck

well out into the main stream the immediate effect was to produce non-parallel flow within the bridge itself and to exaggerate jetting conditions downstream. When attention had been paid to this point, and the abutments were properly bell-mouthed, by means of quadrants, or parabolic, vertical walls, parallel flow within the bridge was obtained. The first requirement if it was intended to improve a tight bridge was to improve the entry conditions. As no upstream damage resulted from bad entries, the ill-effects downstream were frequently overlooked.

The expansions downstream on the Eastern Jumna Canal usually consisted of kankar block pitching laid at a slope of 1 to 1. The best form of expansion appeared to be that of a vertical wall as a continuation of the face of the abutment, warped to an appropriate side slope at the lower end, where it "faded out" into the earthen banks of the channel, and "retired" well into the bank by a moderately sharp quadrant. The latter was necessary as water passing along a smooth masonry expansion always gave trouble at the point where the masonry was replaced by earth. As the lower warped portion of the expansion consisted of thin, re-inforced brick pitching, it was necessary to protect the lower end from attack.

On the Eastern Jumna Canal many of the bridges were of single span, or two spans. On the Mat Branch the bridges were often combined with falls and consisted as a rule of three or more spans with the result that the flow, which was 'centred' upstream, was more uniformly distributed, in respect of the channel as a whole, when the water issued from it. There were examples of bridges with bell-mouthed entries upstream, and also rectangular entries, and the latter invariably led to irregular flow. It was also interesting to note that when the bridge was combined with a fall, the action downstream was usually due largely to "surging" and wave action, but that the water often followed the expansions better, even if they were acute, owing to the turbulent mixing to which the Author referred.

The practical problem arose of the action to be adopted when action downstream of a "tight" bridge, or fall, was excessive. The writer had come to the conclusion that if the spans of a given work were few, it was frequently cheaper to add a span on each side. and thus reduce the expansion ratio, rather than from motives of false economy, save on the bridge, and put in insufficient downstream expansions leading to heavy recurring charges. A properly designed expansion of the correct length, in other words, a gentle expansion, might prove as expensive as widening the work.

The Author had referred to the Froude Number, the expansion ratio, and the discharge all effecting the problem. Many problems in connection with expansion were associated with canals transporting silt of very much the same order and this made it possible to consider the problem in terms of discharge (effectively the scale) and the expansion ratio. Other things being equal the finer the silt, the lower the Froude Number, and the less risk of return flow.

From the observations that the writer had made it appeared possible to frame an empirical equation which would at least give an indication of the safe limits of expansion. The two principles were that the angle of divergence was an inverse function of the discharge, and also an inverse function of the expansion ratio.

Some of the expansion ratios which the writer had examined were greater than 2·0, and the equation which he proposed would be suitable for expansion ratios of 2·0 and less. When the expansion ratio was very small, the precise angle of expansion became of little moment.

The following equation appeared to give fairly concordant results :

Tangent angle expansion $=0{\cdot}4/Q^{\frac{1}{8}}$ and the table given below gave expansions for a number of discharges. The expansion referred to water surface.

Q	Tangent.	Offset.
8	·283	1 in 3·5
27	·231	1 in 4·5
64	·200	1 in 5·0
125	·179	1 in 5·5
196	·163	1 in 6·0
343	·151	1 in 6·6
512	·141	1 in 7·0
729	·133	1 in 7·5
1,000	·126	1 in 8 0
2,500	·109	1 in 9·25
5,000	·097	1 in 10 50
10,000	·086	1 in 11·50

When the discharges were very small, action downstream of falls, from the viewpoint of safety could be ignored, although the works were very unsightly and subordinates were tempted to indulge in "staking" and other ineffective means to restore the channel section and thus waste money.

The successful expansions on the Mat Branch were associated chiefly with combined bridges and low falls with discharges considerably over 1,000 cusecs. All had been bell-mouthed upstream, the expansion ratio was somewhat less than 2·0 and the expansions were 1 in 8.

On the Eastern Jumna Canal most of the works examined were tight bridges with discharges less than 1,000 and expansions of 1 in 7.

The writer did not claim that the table was anything but very approximate. The point to observe was that if an expansion of 1 in 10 were required, it might well be cheaper to reduce the expansion ratio. The reason for this, from the economic viewpoint, was that the *width* of a bridge did not increase with scale.

The writer was not in favour of excessive expansions, or in other words excessive contractions at masonry works such as bridges on large channels. The permanent works necessitated did not lend themselves to adaptation should the canal subsequently be called upon to take a greatly increased discharge. When works were first constructed, the expansion ratio should be kept as small as possible and approximating to unity. In small works contractions were more feasible.

Mr. N. N. Bhandari sent the following written discussion :—

The Author has presented a very interesting Paper, and the Congress should be grateful to him for this divergence of the routine sort of Papers to something more of the recent developments of fluid dynamics. However a few points in the Paper draw one's attention more prominently.

The formation of return eddies in hydraulic and aerodynamic structures always follows the separation of the boundary layer from its boundary, and the greater the divergence of the boundary, the earlier does the separation of the boundary layer occur.[1] Hence the necessity of a gradual divergence in flumes as mentioned on page 182. This phenomenon is illustrated very nicely by Prandtl and Tietjens[2], an excellent exposition of the control of which is given by E. G. Richardson[3].

It is now, more or less, an established fact that the formation of reverse eddies along the bed downstream of

[1] 'Modern Developments in Fluid Dynamics', by S. Goldstein, Oxford University Press, Vol I, p. 60.

[2] Applied Hydro and Aero-Dynamics, 1934, pp. 288—292.

[3] 'Manipulation of the Boundary Layer', Journal of the Royal Aeronautical Soc., Vol. XLII, No. 333, Sept. 1938, pp. 816—838.

a masonry structure is more of a preventive measure against erosion than having a destructive effect as the Author remarks on page 179.

On page 183, the Author's statement 'An increase in F. N. (Froude Number) in the channel upstream of the expansion, keeping other dimensions the same (hence with an increase in discharge, Q) leads.....', seems to be doubtful. An increase in discharge means an automatic corresponding increase in V and d or V_2 and d_2. So the Froude Number may not necessarily be increased with an increase in discharge. It can, however, be made to increase or decrease by varying the depth and the corresponding decrease or increase in velocity.

The observation made on page 184 'The higher F. N., therefore.....enables a larger return eddy to be maintained.....' tallies exactly with the results obtained by the writer. But the latter differs in that ' In other words, the higher the F. N. the greater the tendency to return flow, this being the most marked when the F. N. approaches the value 1.' When F. N. is 1, the flow is critical and the steady flow does not exist. It may, however, in some cases become a shooting flow and so there is no reverse eddy formed on the top of the water surface.

The purpose of any device put in for the prevention of erosion downstream of an hydraulic structure, is as already explained, to cause the flow along the bed to seperate from the boundary and consequently to form reverse eddies. Besides conveying the bed material from further downstream to just downstream of the structure, the eddies cause a considerable loss of energy due to the viscous, frictional interaction of the mutual surfaces of the eddies. Thus, the smaller the eddies the more is the dissipation of energy. Among the useful devices which have been found to function well in the above light are the triangular sill[4] and the staggered blocks[5], the former

[4] The Protection of Dams, Weirs and Sluices against Erosion by R. V. Burns and C. M. White, Proceedings of the Institution of Civil Engineers 1938, Paper No. 5127.

[5] Protection against scour below River and Canal Works, by H. L. Uppal and N. N. Bhandari, Proceedings of the Punjab Engineering Congress 1938, Vol. XXVI, Paper No. 214.

in creating back eddies downstream of an hydraulic structure and the latter in separating the flow from the boundary (*i.e.*, the downstream floor) and throwing the high velocity jet into the surface.

Replying to Dr. Uppal's discussion the **Author** said that he was glad to note that the results obtained at the Punjab Irrigation Research Institute were in agreement with his theory. The experimental results regarding the use of curved side walls were of much interest. A small degree of convex curvature at the downstream end was beneficial as a means of obtaining a more gradual expansion in a given length, the flow at the downstream end where the expansion was sharper being stabilised by curvature of flow. Regarding stabiliser blocks, he disclaimed any orginality in proposing their use—the object of the paper was not to introduce new methods of inducing flow to follow expansions but to analyse the mechanics of flow and show why certain devices were effective. Baffles, blocks and vanes had been used with success in India and elsewhere for some time.

He thanked Mr. Khanna for his remarks.

Replying to Mr. Inglis, he said that he had not included the effect of curvature of flow among the basic principles because it was covered by Newton's second law. He appreciated, however, that it was often easier to visualise curvature effect as such. The greater ability of the flow to expand if the velocity were uniformly distributed, as occurred immediately below a contraction, instead of being higher in midstream, as occurred in a parallel sided channel, could also be explained by the energy of flow, which in the former case was greater near the sides. As the following of the expansion by the flow was dependent on the supply of energy towards the sides, the uniform distribution clearly aided expansion. He thanked Mr. Inglis for his appreciative remarks.

Mr. Ghafoor suggested sloping upstream ends to vanes. This was certainly desirable and had been used, though it did not appear to be entirely effective in preventing the catching of debris.

The Author said he would reply to Dr. Bose's remarks in his reply to the written discussion.

Referring to Mr. Blench's point that a jet could be spread more easily above the standing wave, he said that his Paper dealt primarily with cases where the velocity was sub-critical, an expansion being accompanied by an increase in static head. If the velocity were hyper-critical, an expansion was accompanied by a reduction in static head so that the adverse pressure gradient which caused one-sided flow in the former case was absent. The limitation to sharpness of expansion was then pressure difference due to curvature of flow, with the result that a hyper-critical jet always tended to be central and stable but was accompanied in sharp expansions by a return eddy at each side. Expansion of the jet above the standing wave was normally desirable, short of causing the formation of such return eddies.

In reply to the written discussion the Author wrote that both Messrs. Inglis and Lacey had given information of the greatest value, based on practical experience, including the safe limits of expansion in channels of various sizes. It would be observed from these data that the safe angle of expansion varied inversely with the size of channel. Mr. Inglis had coupled this variation with the B/d ratio, which increases with increase in discharge in alluvial channels, and Mr. Lacey had related the angle of divergence directly with the discharge.

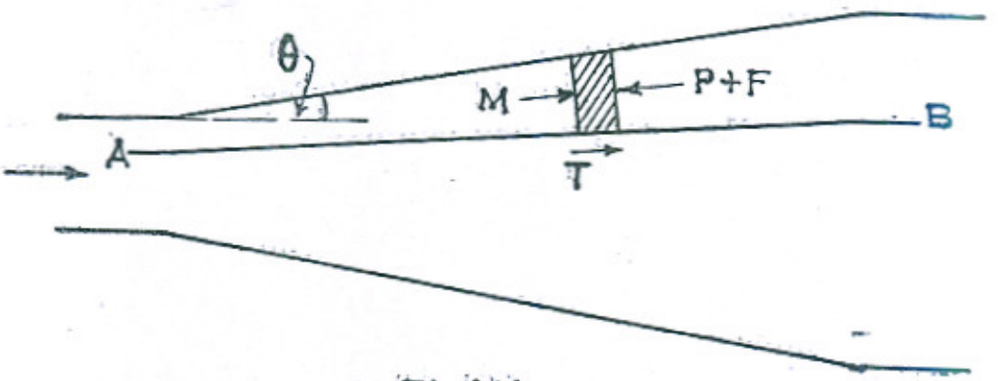
Though this result was not indicated in the qualitative deductions drawn in Paper, it would be seen to be not at variance with the description given of the forces tending to maintain the balance of a return eddy ; in fact the stabilising influence of the shear, transmitted from the main stream, reduced with increased B/d ratio and (in cases of smooth bed) with increased R. N., VR/ν.

An analysis of more quantitative character than that given in the Paper was, however, necessary to throw further light on the point and to indicate the relative importance of the various factors. An exact quantitative analysis would be complex, requiring equations for the velocity distribution in the expansion, but the following

simplified solution, in conjunction with practical data, indicated the main factors and the form which a more exact formula would take.

In an expansion symetrical about its centre line and with symetry of flow both u/s and d/s, balanced symetrical flow was possible in the expansion for a very brief period, however sharp the expansion. If the state of flow were also stable, small deviations would be automatically corrected and the balanced state would persist. If, on the other hand, the state of flow were unstable, any small deviation would be augmented and the balanced state would change into an unbalanced and one-sided state of flow, all flow lines being displaced towards one side. The stable state could be likened to a billiard cue hung from its top end, and the unstable state to the cue balance on its tip, when the slightest touch would make it over-balance.

FIG:- 8.



PLAN.

In Fig. 8, the line AB represented a flow line when the state of flow was balanced and symetrical. The forces acting on the small shaded 'block' of water between the flow line and the side wall were :—

P due to pressure difference

F due to boundary drag, and

T the shear force transmitted from the main body of the fluid.

These forces together resulted in a rate of change of momentum in the block, amounting to M.

If a small deflection occurred in the flow line, increasing the width of the block, but not the rate of discharge passing through it, P and T would increase and F. and M decrease. If there were a nett increase in the direction of flow, the flow line would be brought back to the balanced position, if a nett decrease, the deflection would increase and the flow pattern entirely change to a one-sided one.

If $\times = P + F + T + M$ (taking all forces acting d/s to be + ve), the condition of stability was clearly given by $\frac{\partial \times}{\partial y_1} = +$ ve and the condition of instability by $\frac{\partial \times}{\partial y_1} = -$ ve, where y_1 was the width of block, *i. e.*, distance of flow line from side-wall. The change from stability to instability, *i. e.*, the limit of expansion for symetrical flow, was given by $\frac{\partial \times}{\partial y_1} = 0$, *i e.*,

$$\frac{\partial P}{\partial y_1} + \frac{\partial F}{\partial y_1} + \frac{\partial T}{\partial y_1} + \frac{\partial M}{\partial y_1} = 0 \qquad \ldots (3)$$

Evaluation of $\frac{\partial P}{\partial y_1}$ and $\frac{\partial M}{\partial y_1}$ was possible by means of Bernoulli's theorem and Newton's 2nd law, respectively, and $\frac{\partial F}{\partial y_1}$ by a resistance formula of the Blasius type. $\frac{\partial T}{\partial y}$ was essentially dependent on the velocity distribution but, an indication of its form was given by analogy with the increase in shear stress on restriction in width of a uniform channel.

Substitution in (3) of the values so obtained, and simplification by omitting relatively small quantities, gave the following relationship for the limiting conditions for stable flow in an expansion with straight vertical sides, smooth level bed, expansion ratio of 2 or more and sub-critical flow throughout :—

$$\tan\theta = \frac{C\left(\frac{R}{B}\right)^{n} + 0{\cdot}018\left(\frac{R}{B}\right)^{-1}}{(R.N.)^{1/5}\left\{\frac{\alpha}{1-\alpha(F.N.)} + \beta\left(2 - \frac{r}{r-1} + \alpha(F.N.)\right)\right\}} \quad ..(4)$$

where C was an unknown constant and n an index having a value of the order of 1/5, both assumed to be constant over the range of stable expansion used in practice,

B = width of channel.
R = H. M. depth of channel.
F. N. $= V^2/gd$
R. N. $= VR/\nu$
θ = angle of divergence of sides (see Fig. 8)

α = energy factor $\frac{1}{V^2 Q} \int_0^A v^3 da.$

β = momentum factor $\frac{1}{VQ} \int_0^A v^2 da,$

all referring to the d/s end of expansion, where due to the greater B/d ratio the flow was least stable, and

r = expansion ratio = B/B_1

As a further simplification, referring to expansions of expansion ratio = 2 in alluvial channels where the Lacey f = 1·0 or thereabouts and assuming $\alpha = 1{\cdot}2$ and $\beta = 1{\cdot}0$ and $\nu = 10^{-5}$ so that R. N. $= 10^5$ Q/B :—

$$\tan\theta = \frac{C\left(\frac{R}{B}\right)^{n} + 0{\cdot}018\left(\frac{R}{B}\right)^{-1}}{13\left(\frac{Q}{B}\right)^{1/5}} \quad .....(5)$$

In practice some degree of asymmetry was tolerated, so C and n could not be satisfactorily evaluated from the Inglis and Lacey data.

(4) did, however, give useful qualitative information. The main factors besides θ were shown to be the expansion

ratio r, and the R.N. (or Q/B) and B/R (or B/d) at the d/s end. With greater r or Q/B, expansion had to be more gradual. Also, within a certain range, with greater B/d, expansion had to be more gradual, but beyond this range the effect of boundary drag predominated and allowed sharper expansion. The F. N. (V^2/gd) was unimportant when small (say less than 0·05), but with higher values of F.N. a more gradual expansion was required.

With rough bed, the term including R. N. would be replaced by one including relative roughness, but the effect would be small unless the roughness were very considerable, such as provided by stabliser blocks.

The effect of curvature of the flow lines at the u/s and d/s ends of the expansion was to reduce the adverse pressure gradient in the centre and increase it at the sides, leading to concentration of flow in the centre and, in extreme cases, to return flow at the sides. As regards symetry of flow, however, it was a stabilising influence. The effect appeared to be small unless the expansion were very sharp, and therefore it had not been included in the derivation of (4).

In consequence of the above analysis, the Author qualified his statement on pages 184 and 186 that increase in depth with constant F.N. resulted in a larger return eddy. This was true only when B/d was very great or when there were stabiliser blocks of fixed height.

Replying now to the oral discussion of Dr. Bose, the Author said that his analysis showed that the flow conditions in an expansion were accurately reproduced in a geometrically similar model with equal F.N. when the boundaries were effectively rough, but that with smooth boundaries the expansion in the model should normally be made sharper to offset the effect of the reduced R.N. In a vertically exaggerated model a still sharper expansion would be necessary owing to the reduced B/d ratio.

Replying to Mr. N. N. Bhandari, the Author stated that his remarks on page 179 on the destructive effects

of eddies referred to eddies with vertical axes forming along the side of an expansion. Such eddies caused damage to the banks downstream of the expansion. The eddies on the bed, with horizontal axis, mentioned by Mr. Bhandari were, as he said, beneficial as they caused bed material to pile up and so prevented undermining of the pavement. Such eddies could be made to form and their size and location controlled by a deflector constructed at the end of the pavement*.

It was not suggested that an increase in Q was necessarily accompanied by an increase in F.N. The latter depended on other factors, but an increase in F.N. keeping either V_1 and d_1, or V_2 and d_2, constant connoted an increase in Q. This was the meaning of the statement on page 183 "hence with an increase in discharge".

Regarding tendency to return flow when the F.N. approaches the value 1, the Author was dealing with flow with a fixed boundary, such as the side wall of an expansion, and not with the free water surface, the shape of which could change with the type of flow.

In conclusion, the Author thanked all the contributors to the discussion, particularly those who had given information of practical application.

*C. C. Inglis and D. V. Joglekar "Dissipation of Energy below Falls" ; Bombay P. W. D. Technical Paper No. 44, 1933.