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FLOW LINES THROUGH EMBANKMENT WITH HETROGENEOUS & STRATIFIED MEDIUM

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FLOW LINES THROUGH EMBANKMENT WITH HETROGENEOUS AND STRATIFIED MEDIUM.

Introduction :—

Flow through a saturated incompressible medium is stream lined and follows a particular course depending upon the geometry of the medium, nature of stratification, water levels etc.

The discharge, Q through a stream tube is expressed by a relation first expressed by Darcy as

$$Q = KA\frac{h_2 - h_1}{1} \text{ or } KA\frac{dh}{d1} \quad \ldots \ldots \ldots \ldots \text{(i)}$$

where dh represent the head loss through a length of flow d1, A being area of cross section and K the transmission constant.

For a three dimensional case of flow, the velocities in three directions x, y and z can be expressed as :

$$U = K\frac{dh}{dx},\ V = K\frac{dh}{dy} \text{ and } W = K\frac{dh}{dz}.$$

If these values are substituted in Laplace equation of continuity :

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{(ii)}$$

we get

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} + \frac{d^2h}{dz^2} = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \text{(iii)}$$

For a two dimensional case it is reduced to

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{(iv)}$$

This equation represents two familes of curves which are at right angle to one another and are known as stream lines and equipotential lines, constituting the flow net.

During recent years extensive use of this equation has been made in one form or another to solve problems of fluid dynamics.

Mathematical solution of the problems by the application of conjugate functions, graphical solution by drawing flow nets, flow lines through capillary space, Electrical Analogy and Hydraulic Models [1-15] all are based upon this equation in one form or another.

All these methods have generally been applied to homogeneous medium, a condition seldom met with in nature. So far the use of hydraulic models which are made to small scale, for the study of flow line and pressure is the only means of achieving reasonably satisfactory results.

In this note several cases of flow lines through hetrogeneous stratum as determined experimentally are given, which represent conditions in an earth dam, embankment, cone of depression to a well and flow lines under a weir.

Stratification in Nature:—

All alluvium deposits are stratified, grading from largest gravel layer at the bottom to fine clay at the stop [16]. Often in such deposits local faults are noticeable which are pockets of silt or clay. This is particularly the case in the aquifer of the Punjab Doabs.

In such stratified media permeability varies in different directions although in the same horizontal plane it is more or less the same. The permeability variation in the horizontal and vertical directions is of the order of 5 to 10 times [17].

Stratification in soil media is universal and exists even in artifically constructed embankments, although in a less uniform way than alluvial deposits under natural agencies.

In a saturated medium with flow conditions stabilized, the path of stream line flow is the final accumulative effect of all stratifications. When the flow is at right angle to the stratified layers, vertical or horizontal, the flow lines are uneffected by change in the nature of medium. The change in the path of the flow lines occurs when the incident flow lines make an angle with the stratified medium.

Experimental and Discussions:—

(*a*) *Hydraulic Gradient in Hetrogeneous Media.*

The laboratory investigations in a small tank 4 ft. x 3 ft., having a width of nine inches with a front of glass plate were undertaken. A section of a bank was made in sand and stratification of variable width of coarse material was tried in different positions. The path of flow lines as traced out without core wall is shown in Fig. 1 (a).

A thin vertical core wall, 6/32 times the width of the bank at the top and extending to the bed level was made starting from the mid point of the bank.

Flow lines when traced were found to have lowered down abruptly at the entry into the coarse medium. This deviation is shown in Fig. 2 (a) by a B. P. The photograph has been deleted for economy.

Two more cases were tried by keeping the coarse core wall at a slope of 15° and 30° to the vertical. In the latter case the core wall was too wide and covered a large area of the bank.

A study of the flow lines. shown in Fig. 2 (b) and 2 (c) revealed that whereas a core wall with a 15° slope was nearly as effective in creating a deviation in the path of flow as a vertical one, the wider core wall was ineffective, did not produce any deviation and flow lines were practically horizontal. The effect of the position of vertical pervious core wall was next studied. A single core wall was tried at position 1/8 and 1/3rd the width of the bank at top. Flow lines for the first case are shown in Fig. (b) and for the second in Fig. 2 (e).

The last two cases were studied with two corewalls. The lowering of the path of flow lines is shown in Fig. 2 (f) and 2 (g).

In Table 1 the position of flow lines at four points has been measured below the top level of the bank. These four positions were—

(1) at entry into corewell,
(2) at exit from core wall,
(3) just below the end of bank and
(4) at emergence on the downstream slope.

Leaving the entry point all other positions of stream lines are important and are considerably lowered down with the result that the stability of the bank is increased.

TABLE No. 1.

	Position of S. L. below the bank surface at entry into core wall (cm).	Position of S.L. below the bank surface at emergence out of core wall (cm).	Position of S.L. just below the end of bank (cm).	Position of S.L. at emergence on the D/s slope of bank (cm).
Case of Fig. 1 (*a*) without core wall	7.0	8.0	14.0	24.0
Case of Fig. 2 (*e*) core wall at 1/3 B. width	10.0	18.0	24.0	29.0
Case of Fig. 1 (*a*)	8.0	9.0	14.0	24.0
Case of Fig. 2 (*a*) core wall at 1/2 width	13.6	18.0	20.0	27.0
Without core wall	6.0	7.0	14.0	24.0
With 2 core wall Fig. 2 (*f*)	8.0	14.0	21.0	26.0
Without core wall	5.0	6.0	14.0	24.0
With 2 core wall Fig. 2 (g)	8.0	21.0	27.0	28.0

It seems that:—

(*i*) The maximum lowering of the hydraulic gradient takes place when the core wall starts at the point when the incident line makes the largest angle with the normal to the interface. This position is with the core wall at one third the width of the bank or even closer to the beginning of bank. Shifting the position further down-stream the incident angle of flow lines becomes smaller and deviation also decreases.

(*ii*) The point of emergence on the downstream slope is also lowered with the imposition of a core wall. Combining this device with a filter at the toe will be of great advantage.

(*iii*) A second core wall is less efficient.

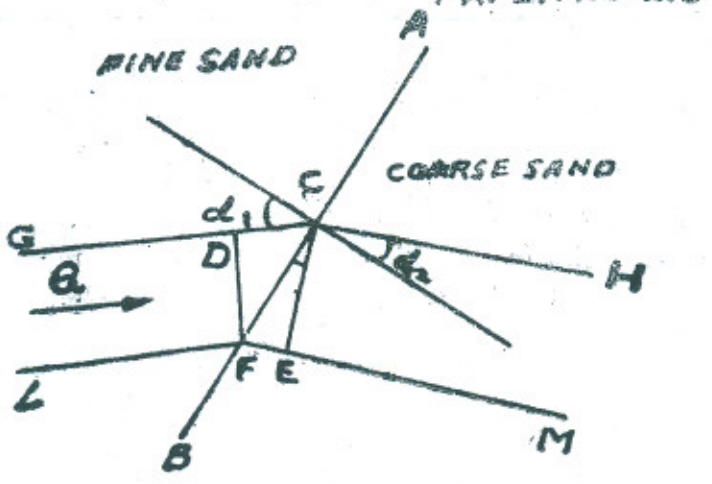
(*iv*) Core wall being a low resistance medium, attracts stream lines from other regions close to it.

Deviation of Flow Lines as Related to Transmission Constant.

Suppose AB is the interface of two media having different permeabilities K_1 and K_2. Suppose a stream tube GC and LF (Fig. 3) is incident on the interface at point C & F.

FIG. 3

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Suppose CH and FM are the deviated stream lines. Draw FD and CE normal to stream lines. These represents the equipotential lines.

As discharge Q is a constant through the stream tube we have

$$Q = K \frac{\Delta h}{\Delta L} \times \text{width of stream tube}$$

$$= \frac{K_1 \Delta h}{DC} \times FD$$

Similarly

$$Q = K_2 \frac{\Delta h}{FE} . EC$$

$$\because \frac{DC}{FD} = \tan x_1 \text{ and } \frac{FE}{EC} = \tan x_2$$

$$\therefore \quad Q = K_1 \frac{\Delta h}{\tan x_1}$$

$$= K_2 \frac{\Delta h}{\tan x_2}$$

$$\text{or } \frac{K_1}{K_2} = \frac{\tan x_1}{\tan x_2}$$

An experimental verification of this relation was attempted by measurement of angles of incident and deviation and the permeabilities of the two media from the six cases mentioned in Fig. 2 (*a*), (*b*), (*c*), (*e*), (*f*) and (*g*).

In all these cases the topmost stream tube was only considered and the angles of incidence and deviation with the normal to the surface were measured at two points where the line entered into the coarse medium and where it emerged out of this medium. Angles at entry into coarse medium were denoted by i_1 and i_2 and emergence by r_1 and r_2.

The measurement of angles, however, presented one difficulty. As all the flow lines were observed by injecting potassium permanganate, the stream tube spread out too much particularly in the coarse medium and measurement of central line was always carried out depending upon the best judgment of the experimenter.

The observed angles for all these cases are given in table No. II where in values of natural tangents and the ratio of tan i_1 to tan i_2 and tan r_1 to tan r_2 are also shown.

TABLE II

Statement of deviation of flow lines through hetrogeneous medium.

Serial No.	Characteristics of case.	Fig. No.	Angle from fine to coarse		Angle from coarse to fine		Tan i_1	Tan i_2	Ratio of		Tan r_1	Tan r_2	Ratio of	
			Incidence i_1	Deviation i_2	Incidence r_1	Deviation r_2			tan i_1	tan i_2			tan r_1	tan r_2
1	3	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Single Central Core Wall	2a	12°	40°	45°	35°	·213	·839	1	4	1·00	0·7	1·3	1
2	Single Core side 15° to vertical.	2b	40°	60°	25°	20°	0 84	1·7	1	2	0·47	0·86	1·3	1
3	Single Core side 30° to vertical.	2c	15°	45°	30°	13°	0·28	1·0	1	3	0·6	0·23	2·6	1
1a	Core Wall at middle from beginning of bank.	—	13°	37°	25°	10°	0·23	0·75	1	3	0·47	0·18	2·6	1
5	Core Wall at 1/3 from beginning of bank.	2e	20°	42°	48°	20°	0·36	0·9	1	3	1 11	0·36	3·1	1
6	Two Core Walls first at 1/8 from beginning of bank, second at 1/8 from bank.	2f	15°	45°	40°	15°	0·28	1·0	1	3	0·48	0·28	3	1
7	Two Core Walls 1st close to beginning of bank and second beginning at mid point of bank.	2g	25°	70°	32°	5°	0·47	2·75	1	5	0·63	0·19	3·3	1

TABLE II

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Serial No.	Characteristics of case.	Fig. No.	Angle from fine to coarse		Angle from coarse to fine		Tan i_1	Tan i_2	Ratio of		Tan r_1	Tan r_2	Ratio of	
			Incidence i_1	Deviation i_2	Incidence r_1	Deviation r_2			tan i_1	tan i_2			tan r_1	tan r_2
1	3	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Single Central Core Wall	2a	12°	40°	45°	35°	·213	·839	1	4	1·00	0·7	1·3	1
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It was found that this ratio was approximately 1 to 3.

The material used in these tests consisted of white sand having all grades with a mean diameter of 0.28 mm. The transmission constant for the two sands was 0,00051 ft/sec. and 0.00146 ft/sec. per unit gradient at 20°C.

The ratio of the two transmission constants was approximately 1 : 3. Thus the ratio of transmission constant of two media was found to be a direct proportion to the tangents of the angle of incidence and deviation.

(*c*) Flow Lines through bank with impervious core walls.

Another case of hetrogeneity in an embankment is the imposition of an impervious core well. A core wall is sometimes provided which is neither tied at bottom to an impervious layer nor is extended to the top of the bank. The flow lines under these conditions were studied for four different cases :—

(1) A bank with impervious core wall ; top in level with full supply and bottom in level with canal bed.

(2) Core wall as such but bank resting on pervious bed.

(3) Core wall as such but bank resting on pervious bed starting at the downstream end of the wall.

(4) Core wall extended to the top of bank and bottom as in case 3.

The observed flow lines are presented in Fig. 4 (*a*), (*b*), (*c*), (*d*), and actual photograph for item 2 only is shown in Fig. 5.

The imposition of an impervious stratification modifies the flow lines which cannot get through a high pressure zone of previous strata and are deflected upward where the syphon action of capillary forces helps the flow lines to pass over the top of the core wall, thus seriously deteriorating the stability of the structure.

It is thus obvious that in such a design the core wall if provided must extend right upto the top of the bank, no porous stratum should exist on the up stream side of the core wall as this is an area of low resistance and therefore dissipate very little of the head. This line of high water pressure tends to deflect all the flow lines to pass over the top of the core wall.

Present Punjab Practice for the Bank Design.

According to the Manual of Irrigation Practice Chapter 8. 5, the safe hydraulic gradient for distributries is fixed as below :—

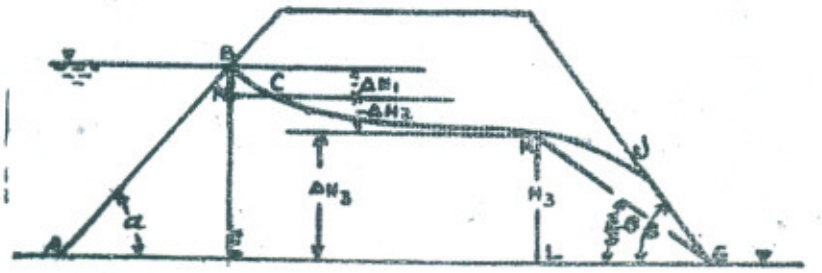
Discharge in cu. ft. per second	Depth in channel in ft.	Gradient.
48	4.0	1:4
199	5.0	1:5
350	6.0	1:6

This does not take into account the variations in soil type. For main canals and branches the gradient has been fixed as 1 : 4 for good soil, 1 : 5 for average soil and 1 : 6 for light soil. As a safety measure a 'Pushta' 3.0 ft. wife for distributries and 6.0 ft. wide for canals and branches with top level, 1.0 ft. lower than the full supply level and with top level, 1.0 ft. lower than than the full supply level and with downstream slope similar to the bank, has been recommended.

In structure of these specifications the flow lines, top-most of which is considered important, do not follow a definite slope but has a characteristic shape, usually that of a parabola with slight deviation at the point of focus and where the flow line enters the sand medium. A characteristic type of flow line observed in the laboratory as well as in the field is shown in Fig. 6.

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Typical Hydaruiic Gradient IN an embankment



This line has three definite parts, an abrupt deviation at entry in the soil, horizontal portion and an exit with a steep slope usually opening on the down stream slope with position of this point approximately $h/_3$ [18]

above the down-stream water level, where 'h' is the water head on the upstream side.

Provision of a Pushta means widening the bank and this results in increasing the horizontal portion of the flow line, practically uneffecting the point of emergence on the downstream side.

The path of this line is fixed so long the geometry of the bank including the upstream and the downstream water levels are fixed. The nature of materials constituting the bank is immaterial so long it is homogeneous and the flow is laminar. The flow line is uneffected except a small deviation in the top flow due to changes in capillary forces. This does not mean that the stability of the bank is the same in all type of materials. The discharge or the transmission constant of the material is another factor influencing the stability. An impervious bank is more stable than a pervious one of the same geometrical shape.

Below in a tabular form are given the mean values of transmission constant determined after analysis[20] of hundred of samples of the Punjab aquifer and Punjab Typical soil.

The vast variation of the constant with change in soil is quite apparant.

TABLE.

Coarse sand 90% finer than	0.31 mm	is	0.00072	ft/sec/unit gradient at	20°C.
Medium sand ,, ,, ,,	0.254 ,,		0.00041	,,	,,
Fine sand ,, ,, ,,	0.20 ,,		0.00013	,,	,,
Coarse silt between	0.20	to	0.02 mmis 00001	,,	,,
Silt betweeu 0.02 to 0.002mm		is	0.000,001 to 0.0000001		,,
Soil with 20% clay particles smaller than 0.002 mm			is 000000017		,,

It may be noted that provision of a previous core wall as explained earlier can be of help in increasing the stability of the bank, Combination of this device with a pervious stratum on the downstream side still improves the stability. This can also eliminate the imposition of costy pushta as well.

Lines of flow through stratification occuring close to a well or under a weir.

While studying stream lines issuing from a canal and feeding a tubewell it was observed that when a flow takes place from a denser medium to a rarer one deviation must occur. The emergent angle with the normal to the interface is more compared to the incident angle. If the order of densities of the media are reversed the angles of deviation are also reversed. In one case stratification of a coarse medium was introduced.

This deviated the flow line such that the deviated angle was much more than the incident angle.

In the same experiment the stratification was replaced by a finer medium with the result that the angle of deviation decreased compared to the incident angle.

Similar observations have been made by Harding and Wood[21] in a two dimensional experiment while studying the deviation of stream lines by interposing layers of coarse and fine sand in the path of flow lines concentrating on to a sink.

As for change in the path of flow lines while passing through stratified medium under a weir, the work of Schaffernak need special mention who has studied this problem.

Conclusions :—

1. Flow lines through a homogeneous medium depend upon the geometry of the structure including the water levels and are independant of the soil medium.

2. For stability the quantity of seepage is as important as the position of point of emergence of a flow line on the downstream face of a bank and an exit gradient on the end of a floor.

3. Widening a bank either by provision of a 'Pushta' or otherwise is not of much help as the top most flow line is not depressed below the toe of a bank.

4. Out of the several devices adopted to lower the flow line below the toe of a bank, provision of a narrow vertical stratification of a coarse material, of width equal to 1/5 that of the bank at top and position closer than 1/3 the top width from upstream side, is fairly effective,

5. Deviation of the path of a flow line depends upon the angle of incidence to the interface of a stratification and the ratio of the transmission constant of the two media. This holds good both for flow from a finer to a coarser medium or *vice versa*.

6. If an impervious core wall is provided in the middle of the bank it must extent to the top and should have no coarse filter on the upstream side.

A filter on the downstream side is of considerable advantage and often eliminates the necessity of a 'Pushta' on the downstream side.

Acknowledgment.

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Fig. 1(a)

Flow lines through a bank without core wall

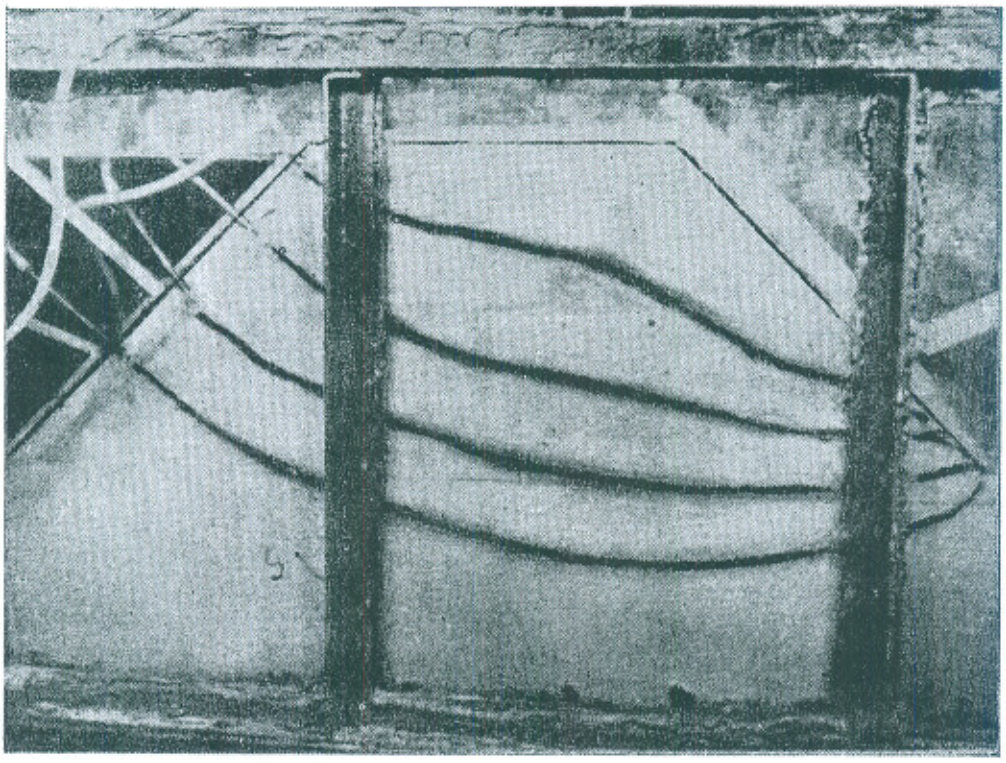


Fig. 1(b)

Flow lines with two previous corewall 1st wall at start of bank 2nd wall at mid point

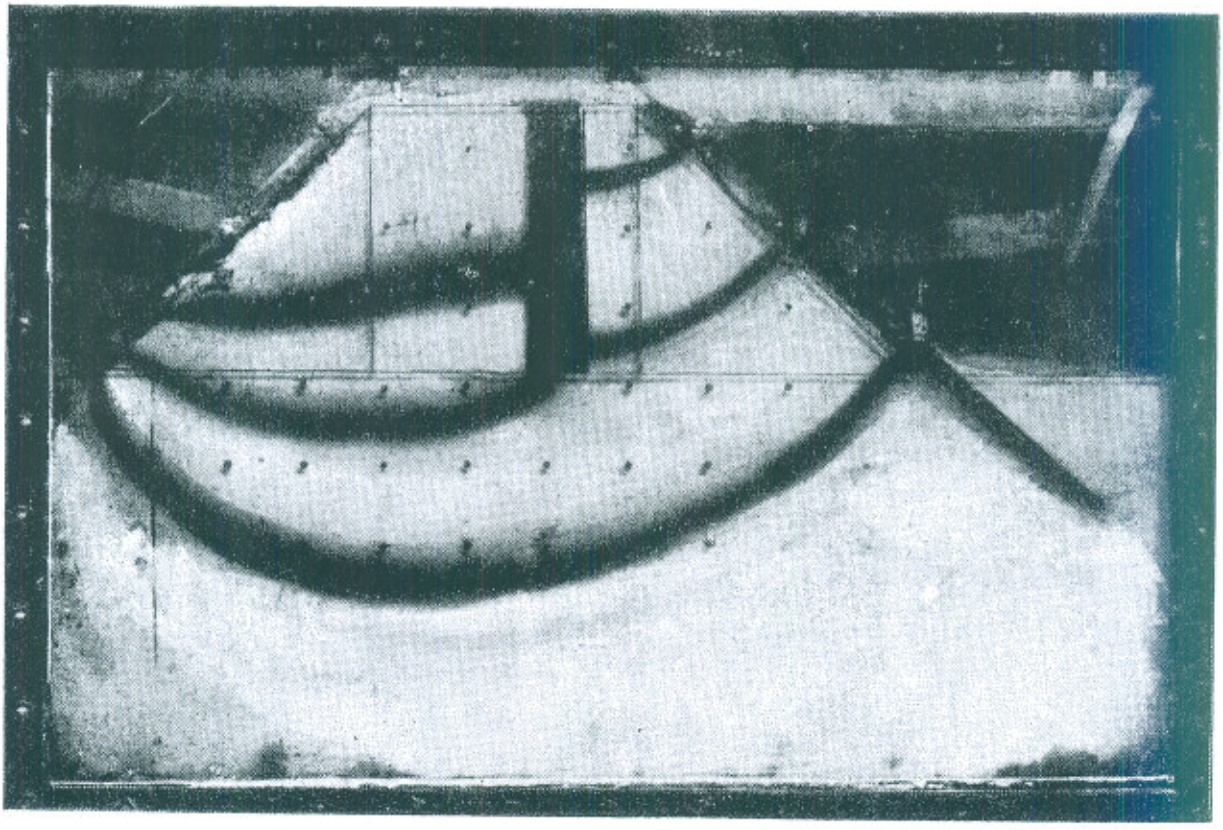
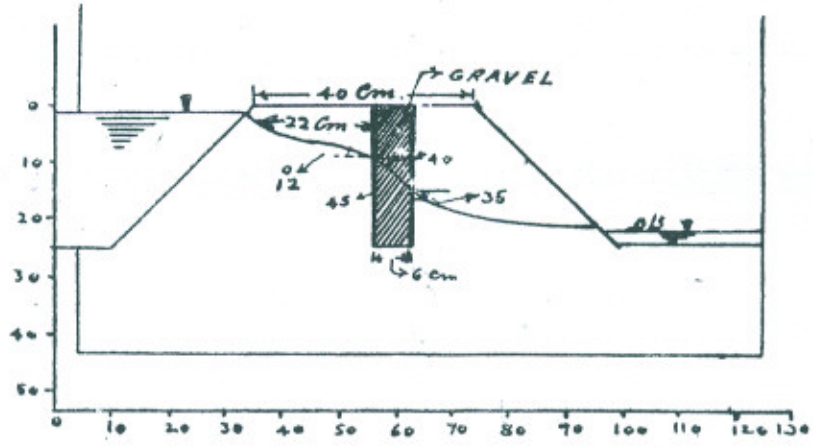


FIG:2 (a)

HYDRAULIC GRADIENT EXPERIMENT WITH SAND CORE WALL

VERTICAL CORE WALL



CORE WALL S. SLOPE 15° TO VERTICAL

FIG:2 (b)

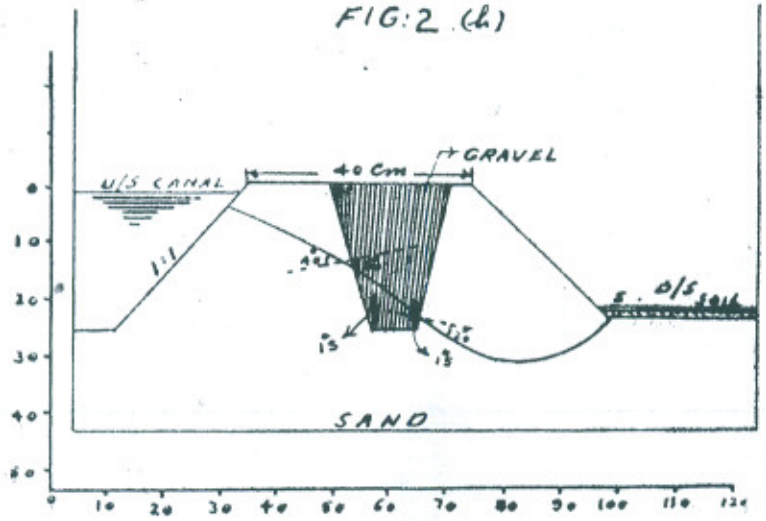


FIG:2 (C)

CORE WALL SIDE SLOPE 30° TO VERTICAL

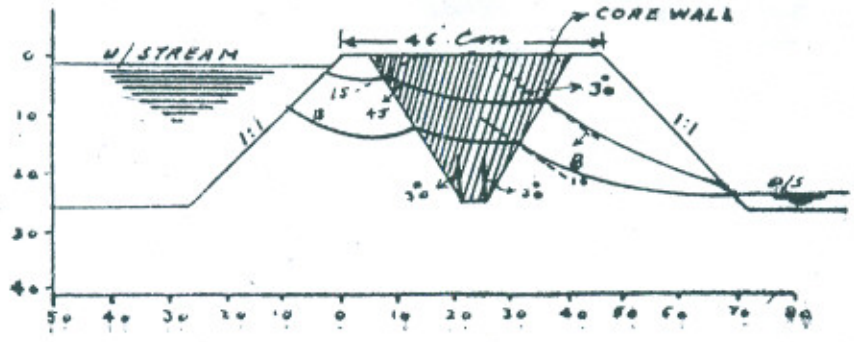
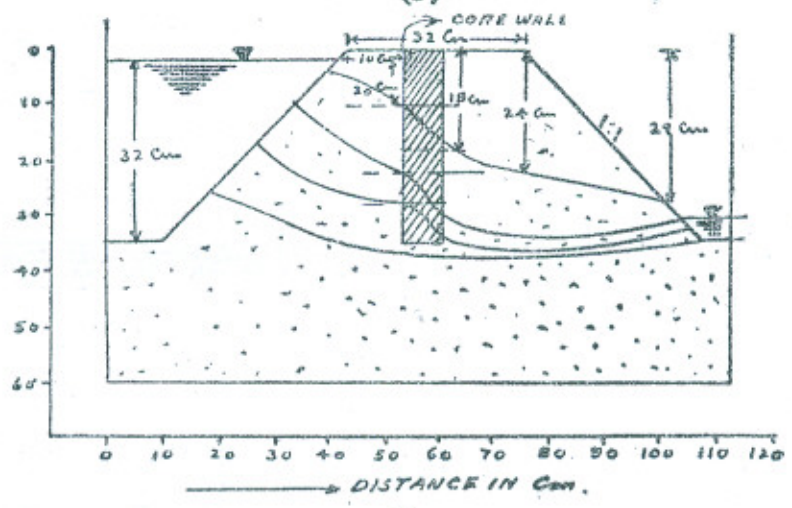


FIG. 2

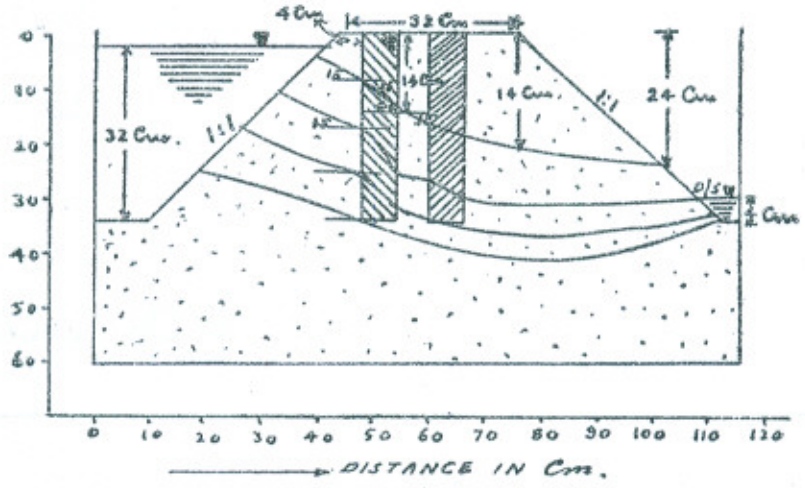
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HYDRAULIC GRADIENT WITH SAND CORE WALL

(e)



(f)



(g)

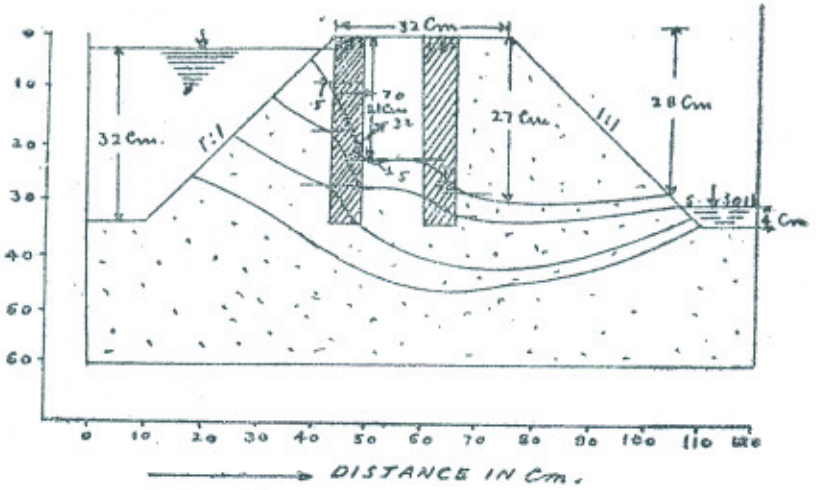


FIG: 4

FLOW LINES THROUGH A BANK WITH IMPERVIOUS CORE WALL

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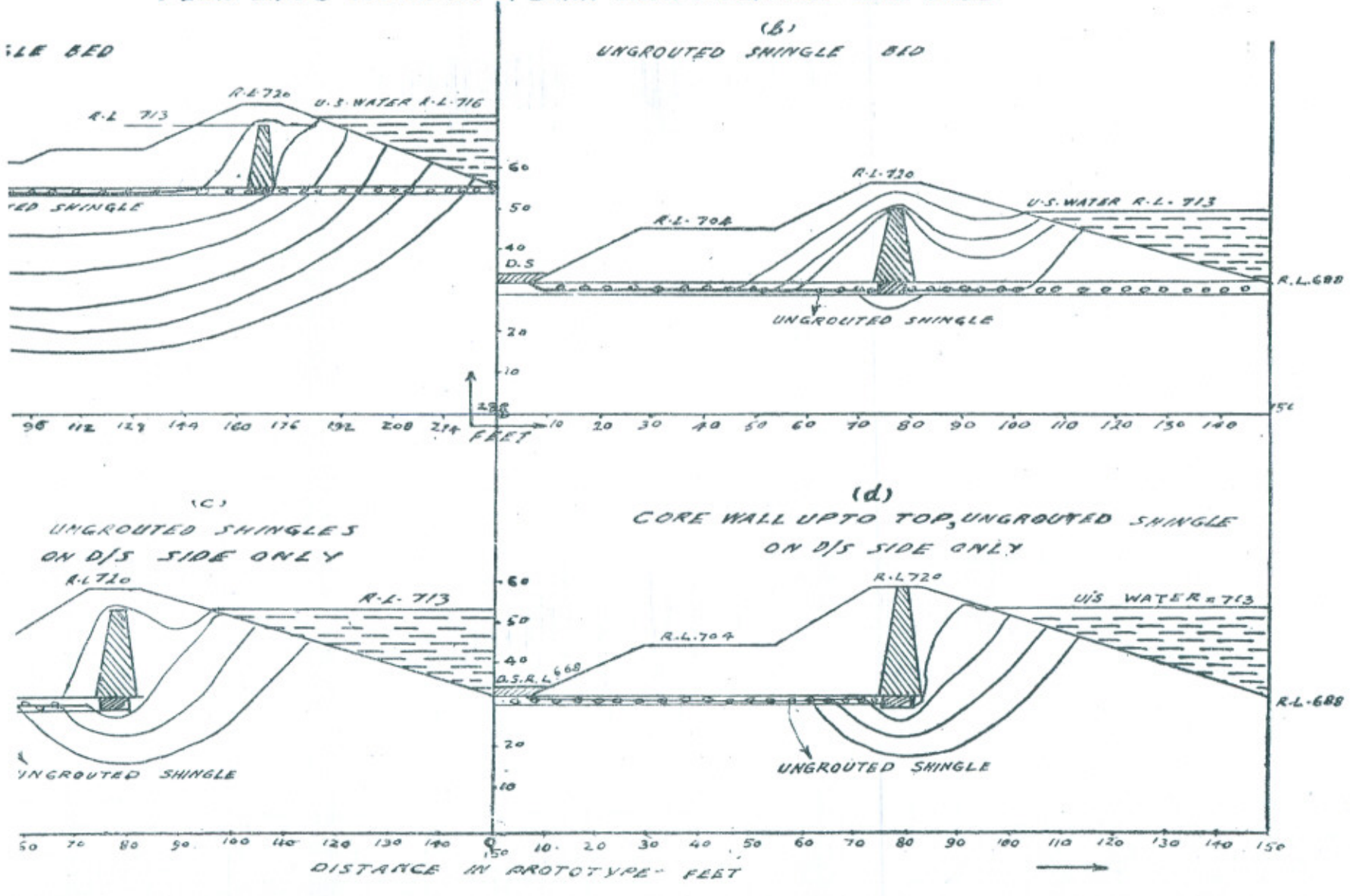


Fig. 5

Flow lines with impervious core wall and previous stratum extending both u/s and D/s of the wall

