**Paper No. 310**

# Tests of bs - Theory

BY


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Lacey did not take into consideration the effect of sides in his formulae of the design of channels in alluvium. Again with observed data his formulae gave different values of the silt factor. Blench and King modified Lacey theory and gave the following three fundamental equations for the design of channels and claimed to remove the above discrepancy,

| Blench | King |
|---|---|
| $b = \frac{V^2}{D_B}$ .....(1) | $b = \frac{V^2}{Dm}$ .....(1′) |
| $s = \frac{V^3}{Wm}$ .....(2) | $s = \frac{V^3}{Ws}$ .....(2′) |
| and $\frac{V^2}{gD_BS} = C\left(\frac{VWm}{v}\right)^{\frac{1}{4}}$ ...(3) | $\frac{V^2}{gDmS} = C'\left(\frac{VWs}{v}\right)^{\frac{1}{4}}$ .....(3′) |

In these equations

$V$ = mean velocity of flow

$D_B$ = depth of water in that portion of the cross section that contains no point vertically above a cohesive side.

$Wm$ = area of the cross section of flow divided by $D_B$

$Ws$ = water surface width.

$Dm$ = area of the cross section of flow divided by $Ws$.

$S$ = water surface slope.

$v$ = kinematie viscosity of water

$g$ = gravity.

and C and C′ are constants.

$b$ and $s$ are bed and side factors.

The above three equations can be put in a form suitable for design as follows :—

Blench's relations will be taken.

From equations (1) and (2) we have

$$V^2 = b \times D_B \qquad \ldots\ldots(4)$$

and $$V^3 = s \times Wm \qquad \ldots\ldots(5)$$

$$\therefore \quad V^5 = b \times s \times D_B \times Wm$$

and $$V^6 = b \times s \times D_B \times Wm \times V$$

$$= b \times s \times A \times V$$

$$= b \times s \times Q$$

where A and Q denote respectively the area of the cross section of flow and the discharge.

$$V = 6\sqrt{b \times s \times Q} \qquad \ldots\ldots(6)$$

Putting this value of V in (4) and (5)

we get

$$Wm = \sqrt{\frac{b}{s}} \cdot Q^{\frac{1}{2}} \qquad \ldots\ldots(7)$$

and $$D_B = 3\sqrt[3]{\frac{s}{b^2}} \cdot Q^{\frac{1}{3}} \qquad \ldots\ldots(8)$$

Now inserting the value of V from

(6) and the values of $Wm$ and $D_B$ from

(7) and (8) in (3) we get

$$S = \frac{b^{5/6}\, s^{1/12}\, Q^{-1/6}}{\left(\frac{gC}{v^{\frac{1}{2}}}\right)} \qquad \ldots\ldots(9)$$

Equations (7), (8) and (9) are the design equations.

Messrs Blench and King each thought his theory to be correct but did not know the value of the constant C in equations (3) and (3′).

In 1947, when Mr. Blench was the Director of the Irrigation Research Institute, Lahore the writer was asked to carry out the following analysis for

the enormous amount of daily hydraulic data pertaining to some channels of the Punjab Province collected by the Mathematical Section of the Institute. The following daily data existed for the various channels.

(*i*) Q, discharge

(*ii*) A, area of cross section

(*iii*) $S \times 10^3$ where S is water surface slope

(*iv*) D, average depth on flat bed

(*v*) W, water surface width,

(*vi*) V, mean velocity of water

(*vii*) T, temperature of water, centigrade and some other parameters.

Equation (3) is

$$\frac{V^2}{gD_BS} = C\left(\frac{VWm}{v}\right)$$

Taking logarithms of both the sides we get

$$\text{Log}\left(\frac{V^2}{gD_BS}\right) = \log C + \tfrac{1}{4}\log\left(\frac{VWm}{v}\right)$$

It shows that if Blench's analysis is correct then on plotting $\log\left(\frac{V^2}{gD_BS}\right)$ against $\log\left(\frac{VWm}{v}\right)$ we should get a straight line with slope as 1/4 and the value of the constant will be determined from the intercept on the vertical axis.

Similarly if King's theory is correct then we should get a straight line on plotting $\log\left(\frac{V^2}{gD_mS}\right)$ against $\log\left(\frac{VWs}{v}\right)$ with a slope of 1/4 and the constant will be determined from the intercept on the vertical axis. Calculations were made to work out from the above mentioned data values for each day of $Wm$ and $Dm$ and then $\frac{V^2}{gD_BS}$, $\frac{VWm}{v}$, $\frac{V^2}{gD_mS}$ and $\frac{VWs}{v}$ and then the logarithms of the last four functions. The value of $v$ was taken as $10^{-5}$.

These calculations were done for the channels given in Table 1 for a number of months.

The values of $\log\left(\frac{V^2}{gD_BS}\right)$ were plotted against those of log

$\left(\frac{VWm}{v}\right)$ in figure (1) and the values of log $\left(\frac{V^2}{gDmS}\right)$ were plotted against log $\left(\frac{VWs}{v}\right)$ in figure (2). There was a great scatter in the plotted points and no unique secular trend was visible.

Mr. Blench had gone to Canada by that time. Mr. King examined both the graphs but did not give any-thing definite.

If one took the plot to be a linear trend $y = a_0 + a_1 x$, say, where $x =$ log $\left(\frac{VW}{v}\right)$ and $y = \log \left(\frac{V^2}{gDS}\right)$ then the most plausible values of $a_0$ and $a_1$ could be determined from the Least Squares principle by the following equations

$$\Sigma y - a_0\, n - a_1 \Sigma x = 0$$

and $$\Sigma xy - a_0 \Sigma x - a_1 \Sigma x^2 = 0$$

From these the proper power of $\left(\frac{VW}{v}\right)$ and the value of the constant C could be determined. Polynomials and other relations could also be fitted and tested analytically for goodness of fit but the extreme scatter did not justify the labour involved in carrying out all these analyses.

Mr. Blench has now finalised his theory and published it in his book 'Hydraulics of Sediment Bearing Canals and Rivers' published by Evans Industries Limited, Vancouver B.C., Canada. It also appears in 'Civil Engineering Reference Book' by Probst and Comrie.

He has now given the value of the constant and his design equations become

$$W = \sqrt{\frac{b}{s}} \cdot Q^{\frac{1}{2}}$$

$$D = \sqrt[3]{\frac{s}{b^2}} \cdot Q^{\frac{1}{3}}$$

$$S = \frac{b^{5/6}\, s^{1/12}\, Q^{-1/6}}{2080 \left(\frac{10^{-5}}{v}\right)^{\frac{1}{4}}} \qquad \ldots.(10)$$

Obviously logarithms of all the above three equations the variables on will yield straight lines. He has therefore plotted the theoretical values of double log paper and thus worked out the design diagrams in figures (3), (4) and (5), attached, for specific values of $\frac{b}{s}$, $\frac{s}{b^2}$ and $b^{5/6}\, s^{1/12}$.

Comparing equations (9) and (10) we get $C\,g = 2080 \times 10^{-5/4}$

$$\text{or } C = \frac{2080 \times 10^{-5/4}}{32.2}$$

$$\text{or } \log C = 0.5602$$

$$\text{or } C = 3.6325$$

Therefore the basic equation (3) now becomes

$$\frac{V^2}{gD_BS} = 3.6325 \left(\frac{VWm}{v}\right)^{\frac{1}{4}}$$

$$\text{or } \log \left(\frac{V^2}{gD_BS}\right) = \log (3.6325) + 1/4 \log \left(\frac{VWm}{v}\right)$$

$$= 0.5602 + 1/4 \log \left(\frac{VWm}{v}\right) \qquad \text{.....(11)}$$

From this straight line values of

$\log \frac{(V^2)}{(gD_BS)}$ for the values 5.7 and 7.1 of $\log \frac{(VW_m)}{(v)}$ work out as 1.9352 and 2.3352. These two points were plotted on the graph in figure (1) and joined as shown in the figure. This is the fit of the Blench's line to the above mentioned Punjab data. There is too much scatter to justify any analytical test of the goodness of fit. As such no great reliance can be placed on Blench's theory for the design of channels. The author has also tried to test Blench's theory against the daily data pertaining to the channels of the L. C. C. system given in Table 2 as follows :—

Equation (10) is

$$S = \frac{b^{5/6}\ s^{1/12}\ Q^{-1/6}}{2080\left(\frac{10^{-5}}{v}\right)^{\frac{1}{4}}}$$

$$\therefore \quad \log S = \frac{5}{6} \log b + \frac{1}{12} \log s - \frac{1}{6} \log Q$$

$$- \log (2080) - \tfrac{1}{4} \log \left(\frac{10^{-5}}{v}\right) \qquad \text{.....(12)}$$

Calculations were made to work out from the existing data the values $W_m$, $b$, and $s$ by the help of equations (1) and (2). Then the values of the logarithms of the various terms were determined and the necessary arithmetic worked out. The values of the kinematic viscosity were read for the given temperatures from the kinematic viscosity—temperature graph. The anti-logarithm of equation (12) gave the theoretical values of the Slope by Blench's theory. This value of S was multiplied by 1000 and the results were compar-

ed with the actually observed values of the slope. Percentage differences of the observed and the calculated values were worked out for each day and from these daily values monthly averages were worked out. The results are given in Table (2). It shows that the calculated slope is in most cases, smaller than the observed by about 40%. This analysis was done for both the cases when the effect of temperature is taken into account and when $v$ is taken equal to $10^{-5}$. It was also noted from the analyses that instead of assuming $v = 10^{-5}$ for all values of T if we take actual values of T and crorresponding values of $v$ the variation between the above mentioned percentage differences in the two cases is about five for low temperatures and it goes on decreasing as temperature approaches the value for which $v = 10^{-5}$.

**Summary and conclusions :**

Blench and King modified Lacey theory of the design of channels in alluvium and gave desing equations. They introduced a side factor in addition to the bed factor and claimed to remove the discrepancy which results from Lacey relations by working out from the same data the silt factor by different formulae.

The modified theory has been tested against some Punjab data and it has been shown that it is not safe to rely on this theory

**TABLE No. 1.**

LOWER JHELUM CANAL

| Serial No. | Section | Name of Branch or Distributory. | R. D. |
|---|---|---|---|
| 1 | Rurala .. | Northern Branch .. | 227000 |
| 2 | Manguana .. | Northern Branch Upper .. | 272600 |
| 3 | Rurala .. | Northern Branch .. | 204200 |
| 4 | Manguana .. | Manguana Distributory .. | 3000 |
| 5 | Manguana .. | N. B. Lower .. | 281500 |
| 6 | Gullapur .. | Kirana .. | 21675 |
| 7 | Rurala .. | Rurala .. | 1700 |
| 8 | Rurala .. | Pindi .. | 1550 |
| 9 | Satghara .. | Mithalak .. | 4000 |
| 10 | Rurala .. | Northern Branch .. | 210100 |
| 11 | Satghara .. | N. B. Lower .. | 73500 |
| 12 | Manguana .. | Sillanwali .. | 7000 |
| 13 | Gullapur .. | Southern Branch .. | 102500 |
| 14 | Ludewala .. | Bhek .. | 1500 |
| 15 | Ludewala .. | Sulki .. | 2300 |

**TABLE No. 2**

LOWER JHELUM CANAL (YEAR 1942)

| Serial No. | Section | Name of Branch or distributory | R. D. | Data analysed | Monthly averages of percentage differences of observed and calculated slopes | |
|---|---|---|---|---|---|---|
| | | | | | When effect of temperature is taken into account. | When $\nu$ is taken as $10^{-5}$ |
| 1 | Sagar | M. L. L. | 149000 | 11 Months | (Jan. —36.8), (Feb. — 42.7) (March —45.9), (April —42.3) (June —48.39), (July. —46.18) (Aug. —47.59). (Sep. —49.43) (Oct. —49.1), (Nov. —49.2) (Dec. —44.0) | (Jan. —42.18), (Feb. —46.64) (March. —47.8), (Apl. —47.2) (June. —48.5), (July. —46.98) (Aug. —47.22), Sep. —49.2) (Oct. —49.48), (Nov. —50.85) (Dec. —47.81) |
| 2 | Nanuana | Jhang Branch | 7260 | 7 months | (Jan. —33.7), (Feb. — 35.9) (March. —40.7), (Aug. —39.91) (Sep. —40.1), (Nov. —38.4) (Dec. —36.9) | (Jan. —39.1), (Feb. —40.3) (March. —42.5), (Aug. —40.5) (Sep. —40.1), (Nov. —41.2) (Dec. —41.1) |
| 3 | Nanuana | Rakh Branch | 7260 | 2 months | (Jan. —6.83), (Feb. —4.26) | (Jan. —14.4), (Feb. —10.53) |
| 4 | Uqbana | Rakh Branch II | 235620 | 1 month | (August +17.61) | (August +19.9) |
| 5 | Sagar | U. G. Branch | 2500 | 2 months | (Jan. —31.4), (Feb. —35.59) | (Jan. —37.65), (Feb. —39.4) |
| 6 | Sagar | U. G. Branch | 6500 | 9 months | (March —38.02), (Apl. —34.91) (June —48.13), (July — 43.6) (Aug. —46.01), (Sept. —46.93) (Oct. —49.26), (Nov. —45.15) (Dec. —41.24) | (March. —39.54), (Apl. —42.98) (June. —48.63), (July. —43.52) (Aug. —45.95), (Sep. —47.14) (Oct. —50.36), (Nov. —47.1) (Dec. —44.99) |
| 7 | Sagar | Hafizabad | 3050 | 2 months | (Jan. +4.2), (Feb. +8.9) | (Jan. —4.80), (Feb. —1.80) |

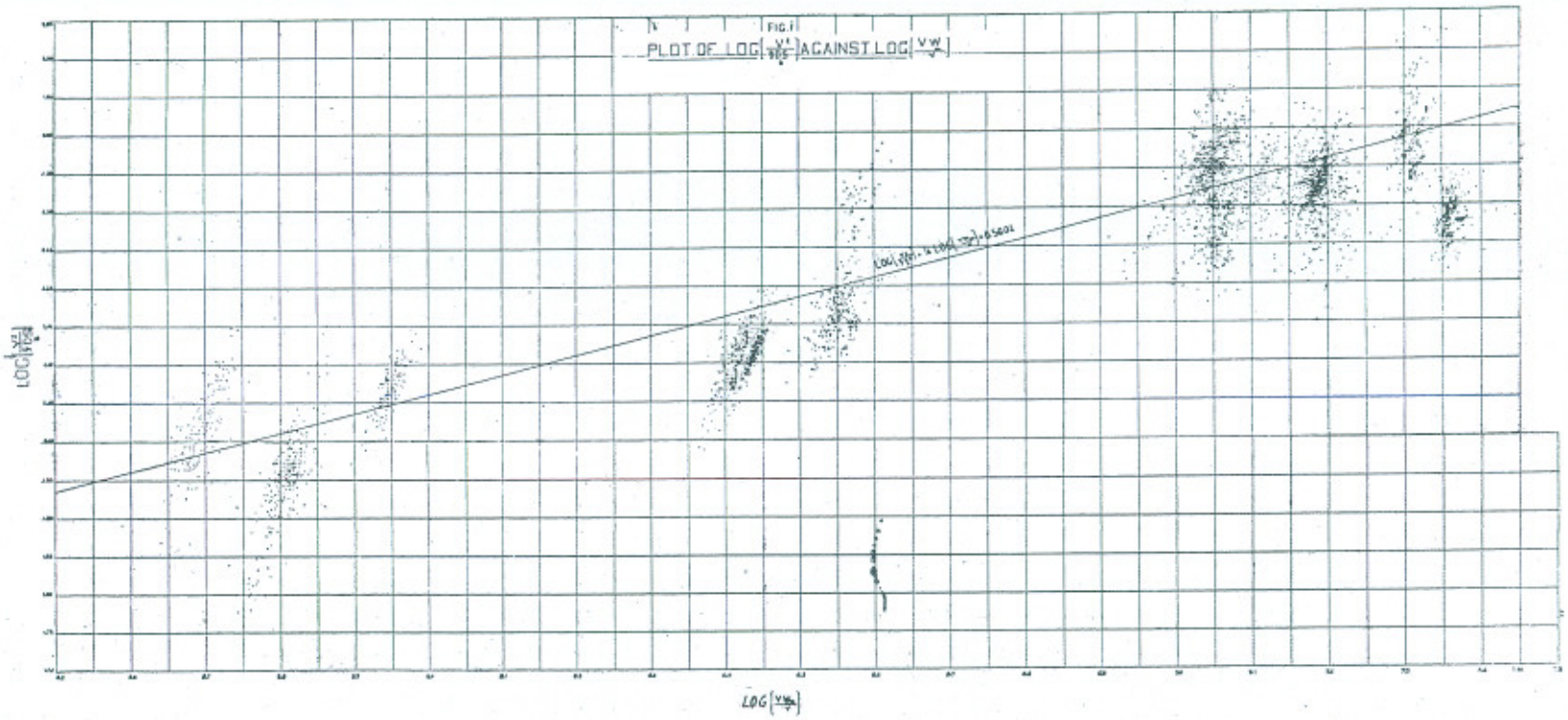
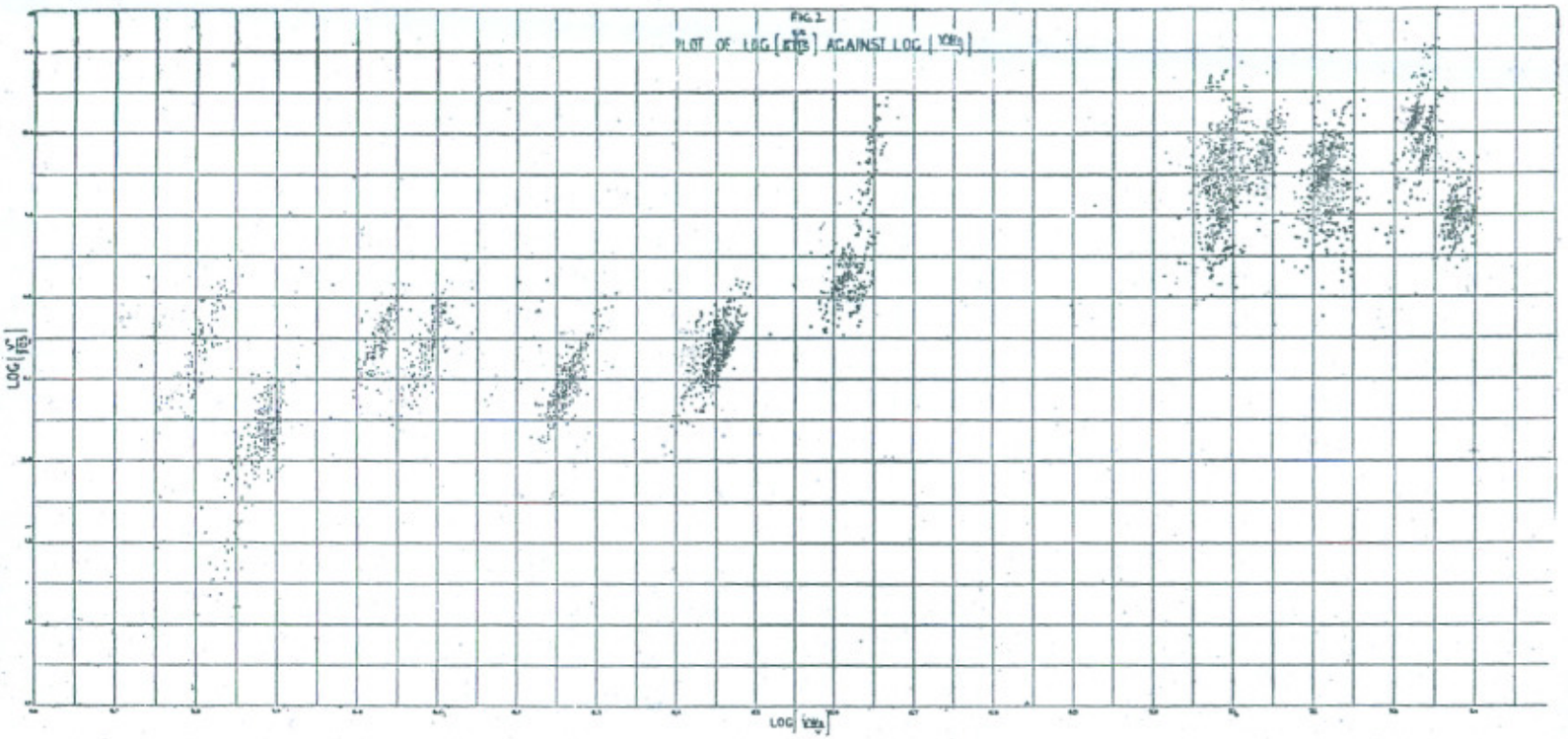


FIG. 1
PLOT OF LOG (V²/905) AGAINST LOG (VW/√)
LOG (V²/905)
LOG (VW/√)

FIG. 2

PLOT OF LOG [ ] AGAINST LOG [ ]



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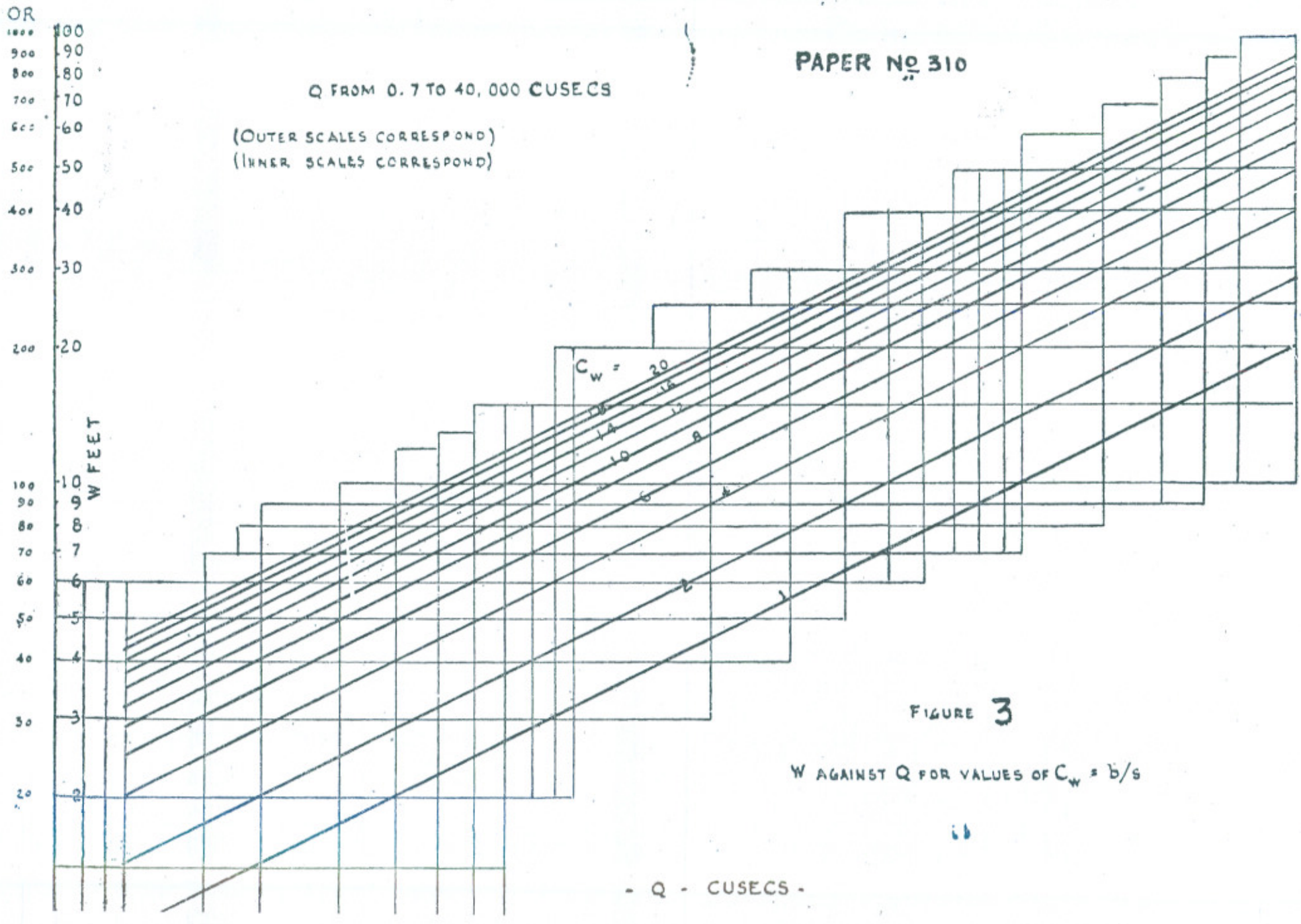


FIGURE 3

W AGAINST Q FOR VALUES OF $C_w = b/s$

FIGURE 4

D AGAINST Q FOR VALUES OF $C_d = s/b^2$

(FOR UPPER SET OF LINES USE UPPER Q SCALE)

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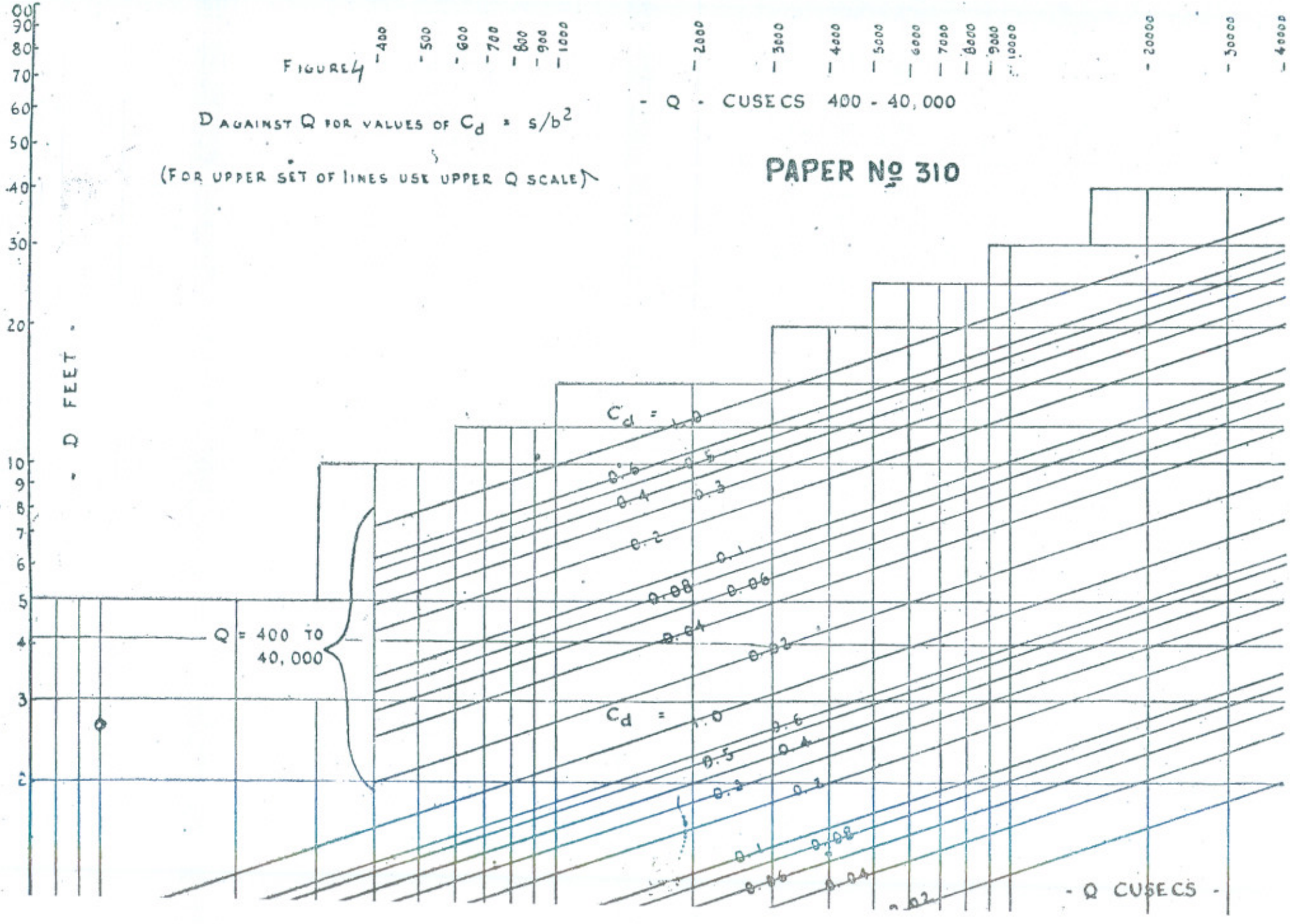

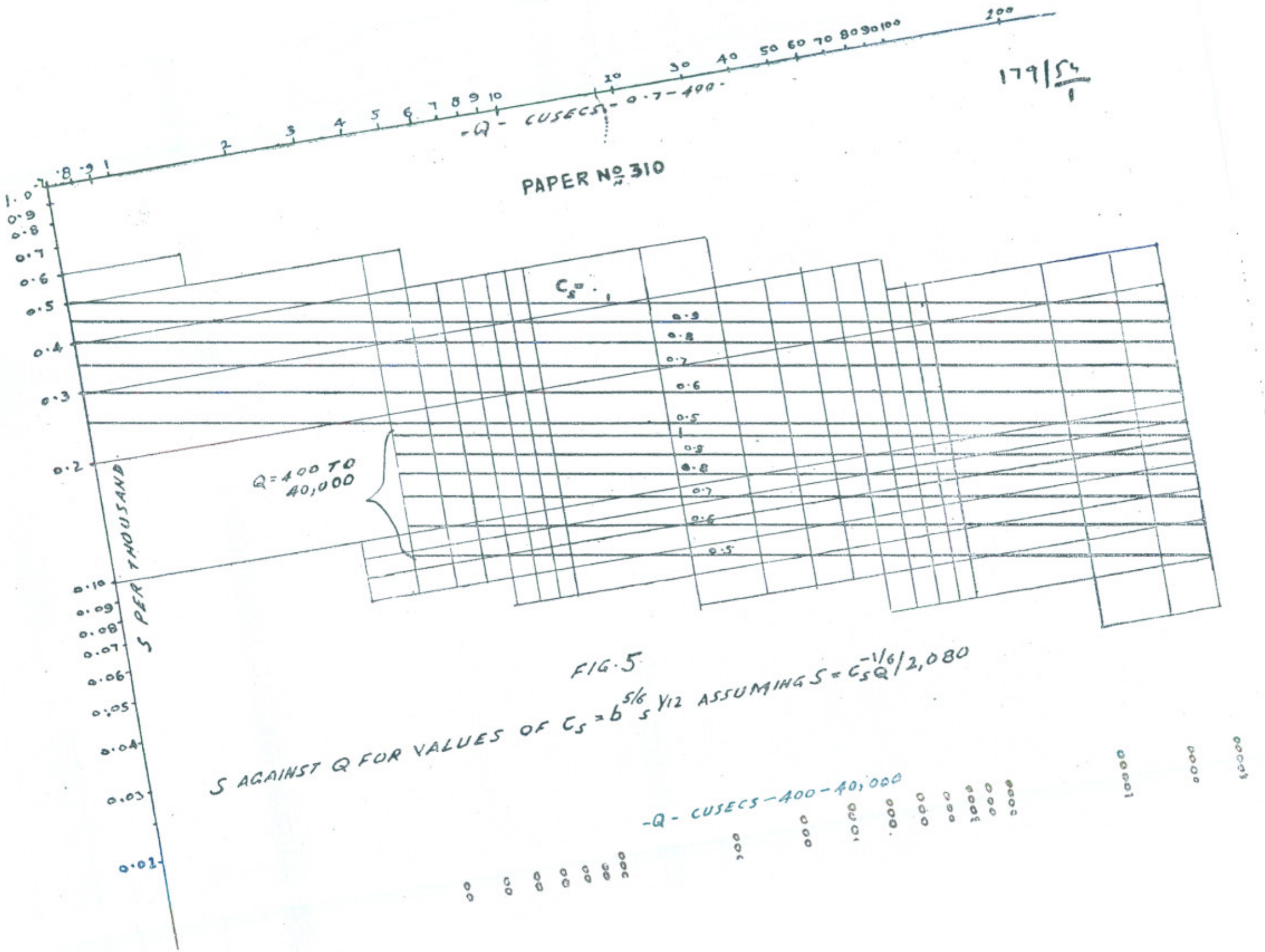


FIG·5

S AGAINST Q FOR VALUES OF $C_s = b_s^{5/6} Y^{1/2}$ ASSUMING $S = C_s Q^{-1/6} / 2,080$