Paper No 319

Specific discharge gauges of Chenab river sites

By

ABDUL LATIF (1)

The regime of a river means its prevailing conditions of flow *e.g.*, discharge, cross-section, slope and levels etc. The regime of a river is seldom stable. It is changing and the change in regime is indicated by the level of water in it for the same discharge. The causes of the change are many and they differ from reach to reach. A very thorough description of these is given in Punjab Irrigation Branch Paper No. 16 class B. The main causes of this change are given as

(*i*) The occurrence of a cycle of high or low tides at river mouth on change of level at outfall in case of tributaries.

(*ii*) Change in extent of delta at river mouth.

(*iii*) Changes in meandering resulting in changes of river length.

(*iv*) Construction of weirs, barrages and dams on the river.

(*v*) Changes in quality and quantity of sediment change, a higher silt charge needs a steeper slope.

(*vi*) Changes in discharge

(*vii*) Set of the river towards the gauge site.

The effect of (*i*) to (*iii*) is to change the specific discharge gauges without a change in slope and the changes occur from downstream upwards and of (*iv*) to (*vi*) is to change the slope from upstream to downstream. Effect of (*vii*) is temporary and fluctuating.

The method of studying the variation in the regime of a river is by the analysis of the specific discharge gauges. In Indus and its tributaries the supplies are high in monsoon months and there are low supplies in winter, In the earlier or rising stage there is more erosion of sides and bed as the river rises and in the falling stage there is more silting. Daily values of gauge and discharge of a river at a discharge site are plotted for the rising and falling stages of the river separately and smooth curves are drawn through these points. From these curves values of the gauge are read corresponding to specific discharges both for the rising and the falling stage for each year. This gives us two values for one specific discharge for one year denoted by R and F for rising and falling stage respectively. As the

1. Abdul Latif, Assistant Research Officer (Mathamatics), Irrigation Research Institute, Lahore.

river is an entity in time therefore the R and F values are taken one after the other in the time series *i.e.*, the falling stage point of one year is followed by the rising stage point of the following year.

Such a record is maintained by the Discharge Division of our Irrigation Department for various discharge sites of different rivers. Let us denote the specific discharge gauge by y and the time by x. There will be two values of x for each year for rising and falling stages and the corresponding two values of y. When we plot the specific discharge gauges (y) against time (x) and join the plotted points we get a zig-zag curve. This represents the effect of all types of variations. These variations are classified as

(*i*) Secular-trend *i.e.*, long time growth or decline occurring within the data. The period covered should include not less than ten years. Strictly speaking the distinction between a short term or a long term is arbitrary as ten years would be very small when we are considering the recurrence of ice ages or the growth and decay of civilisations.

(*ii*) Seasonal variation, *i e.*, the more or less regular movement within the twelve month period.

(*iii*) Cyclic fluctuation *i.e.*, rhythmical sweep from high to low and back to high.

(*iv*) Accidental variation *i e*, occasional abrupt and extreme deviations due to some unusual disturbances.

We are not interested so much in the other variations as we are in the long range variations *i.e.*, the secular trend.

There are many methods of working out the long range variation *e g*.

(*i*) Free hand method A line is drawn through the plotted points in such a way as to describe what appears to the eye to be the long range variation.

(*ii*) Semi average method The data is split up into two equal periods and the average is taken for each. These average values are plotted at the middle of the two periods and joined by a straight line. (If the number of points is odd then take two parts approximately equal)

(*iii*) Moving average method. A finite number of terms of the y-Series are averaged and this average is taken against the middle of the corresponding x-terms. The first term of the y-series is left and next terms of the y-series equal to the previous number are averaged and this average taken against the middle of the corresponding x-terms and so on.

These methods though simple to use have got some defect or the other. By far the most powerful method is the Method of Least Squares.

Method of Least Squares.

Suppose we have a set of given equations (called the equations of condition)

$$a_1x^1 + b_1y^1 + c_1z^1 + \ldots\ldots + f_1t^1 = n_1$$
$$a_2x^1 + b_2y^1 + c_2z^1 + \ldots\ldots + f_2t^1 = n_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_s x^1 + b_s y^1 + c_s z^1 + \ldots\ldots + f_s t^1 = n_s$$

where s is greater than the number of unknown quantities $x^1, y^1, \ldots\ldots t^1$ and we have to find out the values of $x^1, y^1, \ldots\ldots t^1$, in such a way that the equations are satisfied as nearly as possible, That is the errors.

$$E_1 = a_1x^1 + b_1y^1 + c_1z^1 + \ldots\ldots + f_1t^1 - n_1,$$
$$E_2 = a_2x^1 + b_2y^1 + c_2z^1 + \ldots\ldots + f_2t^1 - n_2,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$E_s = a_s x^1 + b_s y^1 + c_s z^1 + \ldots\ldots + f_s t^1 - n_s,$$

are to be as small as possible.

For this Legendre gave the principle that of all possible sets of values of $x^1, y^1, z^1, \ldots\ldots$ the most satisfactory is that which makes the sum of squares of the errors a minimum

i. e. $E_1^2 + E_2^2 + \ldots\ldots + E_s a^2$ minimum.

$$E_1^2 + E_2^2 + \ldots\ldots + E_s^2 = (\Sigma a^2)\, x^{1^2} + (\Sigma b^2)\, y^{1^2} + \ldots\ldots\ldots$$
$$+ 2\,(\Sigma ab)\, x^1 y^1 + 2\,(\Sigma ac)\, x^1 z^1 + \ldots\ldots$$
$$- 2\,(\Sigma an)\, n^1 - 2\,(\Sigma bn)\, y^1 \ldots\ldots + \Sigma n^2$$

$x^1, y^1, z^1, \ldots\ldots$ are to be so chosen that the above becomes a minimum therefore its partial differential coefficients with respect to $x^1, y^1, z^1, \ldots\ldots$ must be zero.

Hence the equations giving the values of $x^1, y^1, z^1, \ldots\ldots$ are

$$(\Sigma a^2)\, x^1 + (\Sigma\, ab)\, y^1 + (\Sigma ac)\, z^1 + \ldots = \Sigma\, a\, n$$
$$(\Sigma\, ab)\, x^1 + (\Sigma\, b^2)\, y^1 + (\Sigma\, bc)\, z^1 + \ldots = \Sigma\, b\, n$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

These are called Normal Equations and as their number is equal to the number of $x^1, y^1, z^1 \ldots\ldots$ these can be solved by ordinary algebra.

There is a big difficulty in the practical use of this method. Suppose we have fitted a second degree curve to a given data and we want to fit a third degree curve, For this we shall have to form the normal equations afresh and solve them again.

R. A. Fisher has by the help of Orthogonal polynomials developed a special technique by which we have simply to add another term to our polynomial when we want to fit the next higher curve. A brief outline of the method is given below for fitting a fifth degree polynomial.

In this method however the values of the independent variable x should be equally spaced,

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	x	Specific Gauge y	1st sum.	2nd sum.	3rd sum.	4th sum.	5th sum.	y^2
R	1	y_1	y_1	y_1	y_1	y_1	y_1	y_1^2
F	2	y^2	y_1+y_2	$2y_1+y_2$	:	:	:	y_2^2
R	3	y_3	$y_1+y_2+y_3$	$3y_1+2y_2+y_3$	:	:	:	y_3^2
F	4	y_4	$y_1+y_2+y_3+y_4$	$4y_1+3y_2+2y_3+y_4$	:	:	:	y_4^2
R	5	y_5	:	:	:	:	:	:
F	6	y_6	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
:	n	y_n	$y_1+y_2+\ldots+y_n$	:	:	:	:	y_n^2
Total		S_1	S_2	S_3	S_4	S_5	S_6	Σy^2

The values of the specific gauges are written against the R and F stages of the years as shown above. The values of the gauges y are succesfully added and the columns are then totalled to give the sums S_1, S_2, ..., S_6. The value of any item in each column of successive summations is equal to the sum of one just above it in the same column and the one to its left in the previous column.

Let the polynomical to be fitted to the x, y values be

$$y=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5.$$

We transform it into

$$y=A+B\mathfrak{Z}_1+C\mathfrak{Z}_2+D\mathfrak{Z}_3+E\mathfrak{Z}_4+F\mathfrak{Z}_5 \text{ where}$$

$$\mathfrak{Z}_1=x-\bar{x}$$

$$\mathfrak{Z}_2=\mathfrak{Z}_1^2-\frac{n^2-1}{12}$$

$$\mathfrak{Z}_3=\mathfrak{Z}_1^3-\frac{3n^2-7}{20}\mathfrak{Z}_1$$

$$\mathfrak{Z}_4=\mathfrak{Z}_1^4-\frac{3n^2-13}{14}\mathfrak{Z}_1^2+\frac{3(n^2-1)(n^3-9)}{560}$$

$$\mathfrak{Z}_5=\mathfrak{Z}_1^5-\frac{5(n^2-7)}{18}\mathfrak{Z}_1^3+\frac{15n^4-230n^2+407}{1008}\mathfrak{Z}_1$$

Where $\bar{x}$ is mean of x values and n is the number of observations.

From the sums S_1, S_2, ... a new series of quantities a, b, c,... ... are obtained as follows

$$a=\frac{S_1}{n}$$

$$b=\frac{1\times2}{n(n+1)}S_2$$

$$c=\frac{1\times2\times3}{n(n+1)\times(n+2)}S_3$$

$$d=\frac{1\times2\times3\times4}{n(n+1)(n+2)(n+3)}S_4$$

$$e=\frac{1\times2\times3\times4\times5}{n(n+1)(n+2)(n+3)(n+4)}S_5$$

$$f=\frac{1\times2\times3\times4\times5\times6}{n(n+1)(n+2)(n+3)(n+4)(n+5)}S_6$$

From these another series of quantities

$a^I, b^I, c^I, \ldots$ are obtained as follows

$$a^I = a$$

$$b^I = a - b$$

$$c^I = a - 3b + 2c$$

$$d^I = a - 6b + 10c - 5d$$

$$e^I = a - 10b + 30c - 35d + 14e$$

$$f^I = a - 15b + 70c - 140d + 126e - 42f$$

The values of A, B, C, ... are then obtained as follows

$$A = a^I$$

$$B = \frac{6}{n-1} b^I$$

$$C = \frac{30}{(n-1)(n-2)} c^I$$

$$D = \frac{140}{(n-1)(n-2)(n-3)} d^I$$

$$E = \frac{630}{(n-1)(n-2)(n-3)(n-4)} e^I$$

$$F = \frac{2772}{(n-1)(n-2)(n-3)(n-4)(n-5)} f^I$$

The test of goodness of fit of a polynomial of a particular degree is made by the usual t—test.

The above method was made use of in working out theoretically the trends of the specific discharge gauges of the following sites on the Chenab river for various specific discharges.

Serial No.	Site.	Distance in canal miles along the river from		
		Head of Indus delta	From its confluence with Indus at Mithankot	Station to Station
1.	Panjnad	679	58	58
2.	Haveli	860	239	45
3,	Trimmu	868	247	8
4,	Rivaz Bridge	892	271	10
5.	Khanki	1048	427	4
6.	Alexandra Bridge	1059	438	11
7.	Marala	1088	467	4

The values of the specific gauges are shown in figures 1 to 7.

The theoretical equation of the trend is

$$y = A + B\xi_1 + C\xi_2 + D\xi_3 + E\xi_4 + F\xi_5$$

and this is known if the values of A, B, ... , F are known. The values of A, B, C, ... , F have been worked out and are given in table 1 for the different degrees of polynomials fitted. The secular trends are depicted graphically in figures 1 to 7. The polynomial values were worked out with the help of Fishers' Tables for given values.

From the above analysis the total and the annual linear rates of rise or fall of the specific gauges were also worked out for the different cases and these are given in table 2. A brief note about each site is given below.

TABLE

Values of A, B, . . , F for equations of

No.	SITE	Specific Discharge	A	B
1	Panjnand Below ...	5,000	1·5971	—·05792
2	,,	10,000	1·6850	—·08226
3	,,	25,000	1·4192	—·0822
4	,,	50,000	1·2320	—·09334
5	,,	100,000	·3060	—·0948
6	,,	150,000	—·023934	—·0890185
7	Haveli ..	5,000	2·074	·01897
8	,,	10,000	2·5081	·021638
9	,,	25,000	1·1698	·0248
10	,,	50,000	·6447	·0321637
11	,,	100,000	1·1610	·0245
12	Trimmu Below ..	5,000	—·0283	—·1011598
13	,,	10,000	1·5326	—·095363
14	,,	25,000	·4750	—·084885
15	,,	50,000	1·249	—·0784
16	,,	100,000	·1693	—·065589
17	Rivaz Bridge ...	5,000	1·787	·0754
18	,,	10,000	2·3386	·0799998
19	,,	25,000	2·9566	·07609
20	,,	50,000	3·236	·0708

trends of specific discharge gauges.

C	D	E	F
·0017777	—·00011909	—·000002216	·0000009549
·001626	—·000002468	—·000005176	·00000016778
·0018005	·00002258	—·000004734	—
·001750	·0000470	—·00000340	—
·00101	·00003	—·000006	—·0000003
·0012584	·00003633	—·0000065698	—
·000984	·000143	·00000353	—
·00084396	·00015367	·00000431504	—
·0008573	·0001654	·0000063178	—
·00152773	·00015235	·00001029	·0000002205
·001177	·0000989	·00001426	—
—·000342	+·00043264	—	—
—·00043472	·00037536	—	—
·000185	·000294	—	—
·000206	·000229	—	—
—·0005329	·00016473	—·000012317	—
—·0001	—·0000389	—·00000466	—·000000225
—·00023241	—·00002702	—·0000039809	—·00000029236
—·00035	—·0000043	—·0000029	—·00000020
—·00013	—·00000546	—·00000377	—

TABLE

Values of A, B, . . , F for equations of

No:	SITE	Specific Discharge	A	B
21	Khanki Below ..	5,000	2·6651	·0371306
22	,,	10,000	1·7849	·02906
23	,,	25,000	·7976	·0199
24	,,	50,000	·8056	·009039
25	,,	100,000	·8741	·00436
26	,,	200,000	·8117	·01842
27	Alexandra Bridge	5,000	·7111	·0129
28	,,	10,000	1·243	·011
29	,,	25,000	·7452	·008893
30	,,	50,000	·925	·0055
31	,,	100,000	·803	·0052
32	Marala Below ...	5,000	6·1907	·05287
33	,,	10,000	2·4791	·03531
34	,,	25,000	1·7048	·003672
35	,,	50,000	·6333	·009466
36	,,	100,000	·5019	·0008485

C	D	E	F
—·0013413	—	—	—
—·0006591	—·000037446	·00000083289	·00000008746
—·00018	—·000024	·00000031	·00000015
—·0000073764	—·00004901	·000002525	·00000013393
—·00025918	—·00006404	·0000008231	·00000013705
—·00021808	—·00010241	—·0000006534	·0000002387
—·0006	+·0001	—	—
+·00053	·000063	—	—
—·0005782	·00002986	—	—
—·0004	·0000341	·00000127	—
—·0004	·000037	·0000017	·00000021
—·002045	·0001881	·00003764	—
—·002392	·0001405	·00003937	—
—·002449	·0001119	·00003859	—
—·002599	·000001751	·00002105	—
—·001770	—·0001296	—·000002955	—

TABLE

Sinear rates of rise or fall in

Serial No.	SITE	Specific Discharge in cusecs
1	Panjnand Below ..	5,000
2	,,	10,000
3	,,	25,000
4	,,	50,000
5	,,	100,000
6	,,	150,000
7	Haveli Below ..	5,000
8	,,	10,000
9	,,	25,000
10	,,	50,000
11	,,	100,000
12	Trimmu Below ..	5,000
13	,,	10,000
14	,,	25,000
15	,,	50,000
16	,,	100,000
17	Rivaz Bridge ...	5,000
18	,,	10,000
19	,,	25,000
20	,,	50,000

the specific discharge gauges

Period	Number of years	Rise (+) or fall (−) of gauge in feet	
		Total	Annual
1927—R to 1952—F	25·5	−2·95392	−·11584
1921—F to 1952—F	31	−5·10012	−·16452
1921—F to 1952—F	31	−5·09640	−·16440
1922—R to 1952—F	30·5	−5·69374	−·18668
1922—R to 1952—R	30	−5·68800	−·18960
1922—R to 1952—R	30	−5·34111	−·17802
1921—F to 1952—F	31	+1·17614	+·03794
1921—F to 1952—F	31	+1·34156	+·04328
1922—R to 1952—F	30·5	+1·51280	+·04960
1922—R to 1952—F	30·5	+1·96200	+·064328
1922—R to 1952—R	30	+1·47000	+·04900
1930—R to 1952—F	22·5	−4·55219	−·20232
1930—R to 1952—F	22·5	−4·29133	−·19072
1930—R to 1952—F	22·5	−3·81983	−·16979
1930—R to 1952—R	22	−3·44960	−·15680
1930—R to 1952—R	22	−2·88592	−·13118
1921—F to 1952—F	31	+4·6748	+·15080
1921—F to 1952—F	31	+4·95999	+·16000
1922—R to 1952—F	30 5	+4·64149	+·15218
1922—R to 1952—R	30	+4·24800	+·14160

Sinear rates of rise or fall in

Serial No.	SITE	Specific Discharge in cusecs
21	Khanki Below	5,000
22	,,	10,000
23	,,	25,000
24	,,	50,000
25	,,	100,000
26	,,	200,000
27	Alexandra Bridge ..	5,000
28	,,	10,000
29	,,	25,000
30	,,	50,000
31	,,	100,000
32	Marala ...	5,000
33	,,	10,000
34	,,	25,000
35	,,	50,000
36	,,	100,000

the specific discharge gauges

Period	Number of years	Rise (+) of fall (−) of gauge in feet	
		Total	Annual
1921—F to 1952—F	31	+2·30210	+·07426
1921—F to 1952—F	31	+1·80172	+·05812
1921—F to 1952—F	21	+1·23380	+·03980
1922—R to 1952—F	30·5	+ ·55138	+·018078
1922—R to 1952—R	30	+ ·26160	+·00872
1923—R to 1951—F	28 5	+1·04994	+·03684
1921—F to 1952—F	31	+·7998	+·02580
1921—F to 1952—F	31	+·6820	+·02200
1921—F to 1952—F	31	+·5514	+·01778
1922—R to 1952—R	30	+·3300	+·01100
1922—R to 1950—F	28	+·2912	+·01040
1921—F to 1942—F	21	+2·2205	+·10574
1921—F to 1942—F	21	+1·4830	+·07062
1922—R to 1942—F	20·5	+1·5055	+·00734
1922—R to 1942—F	20·5	—·3881	—·01894
1921—F to 1942—F	21	+ 0356	+·001697

FIG I

SPECIFIC GAUGES OF RIVER CHENAB AT PANJNAD BELOW

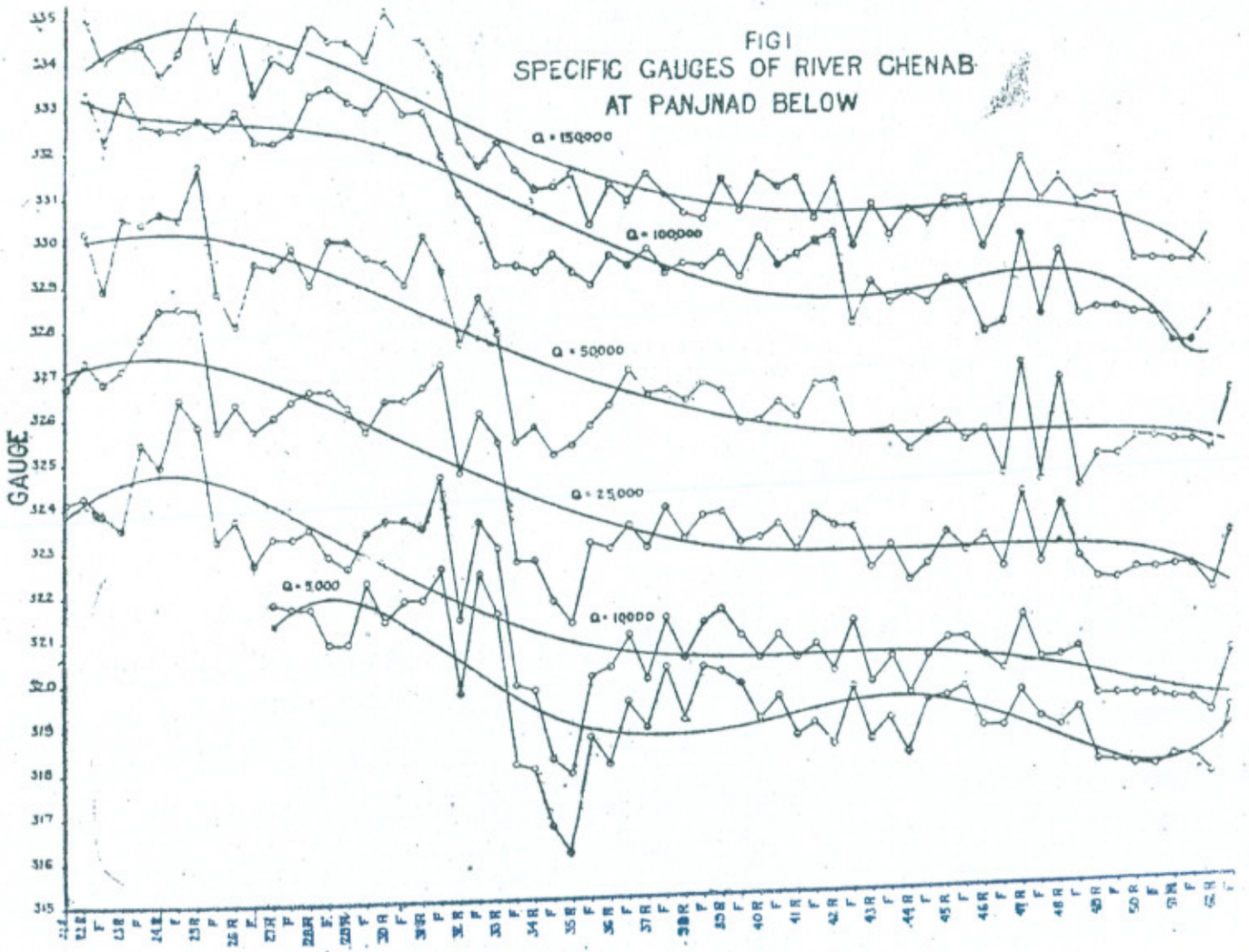


FIG: 2

SPECIFIC GAUGES OF RIVER CHENAB AT HAVELI

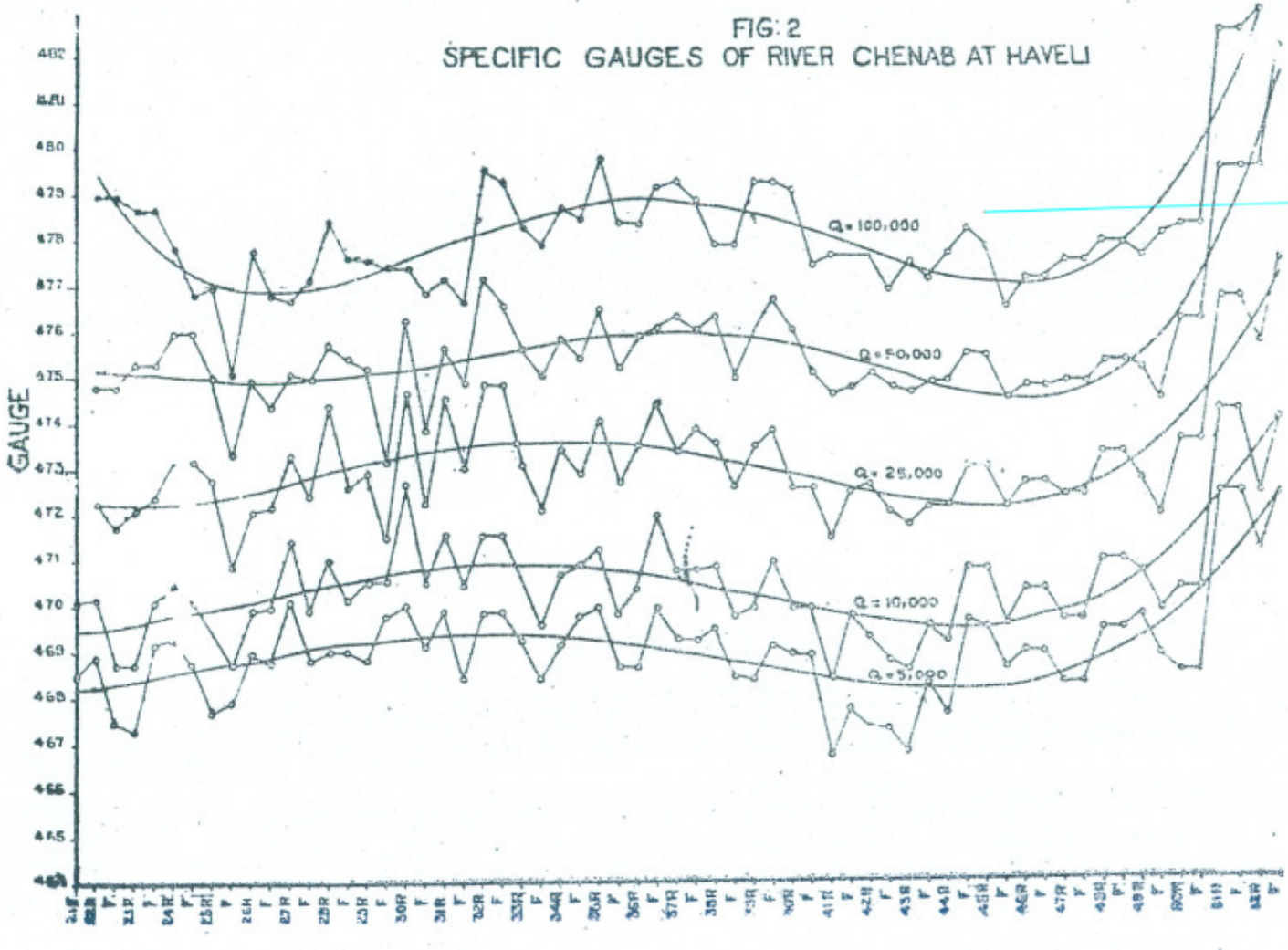


FIG.3
SPECIFIC GAUGES OF RIVER CHENAB AT TRIMMU

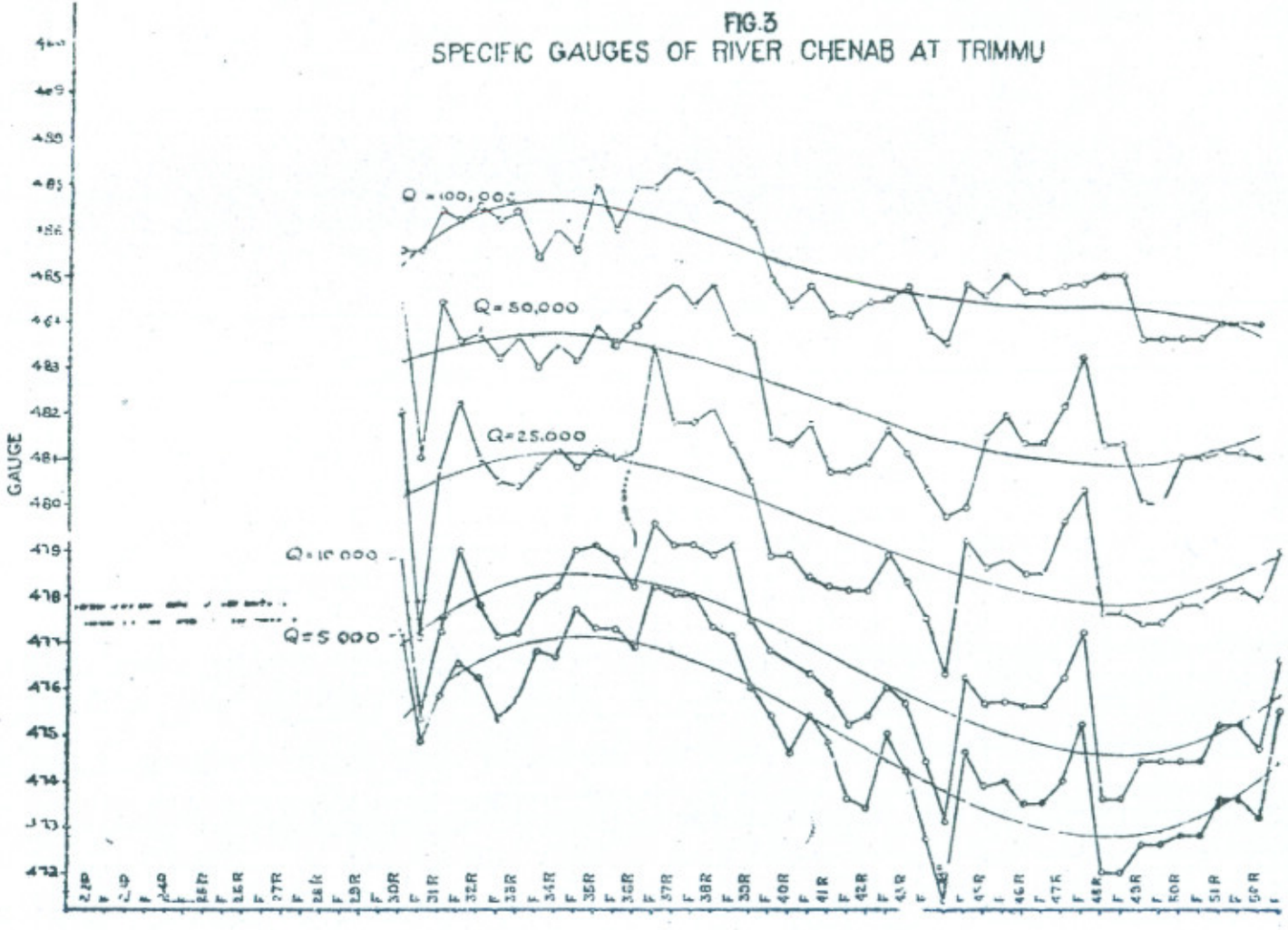


FIG 4
SPECIFIC GAUGE OF RIVER CHENAB AT RIVAZ BRIDGE

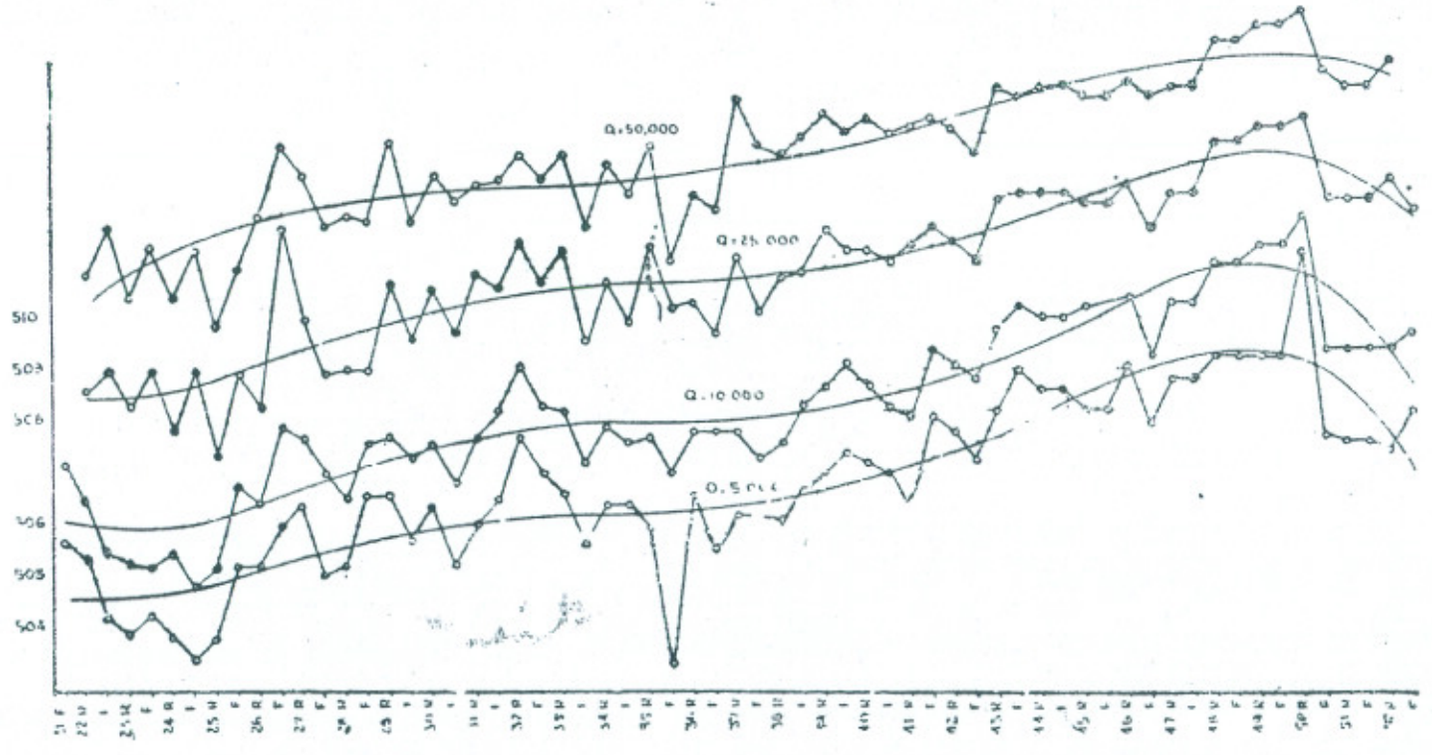


FIG 5

SPECIFIC GAUGES OF RIVER CHENAB

AT

KHANKI

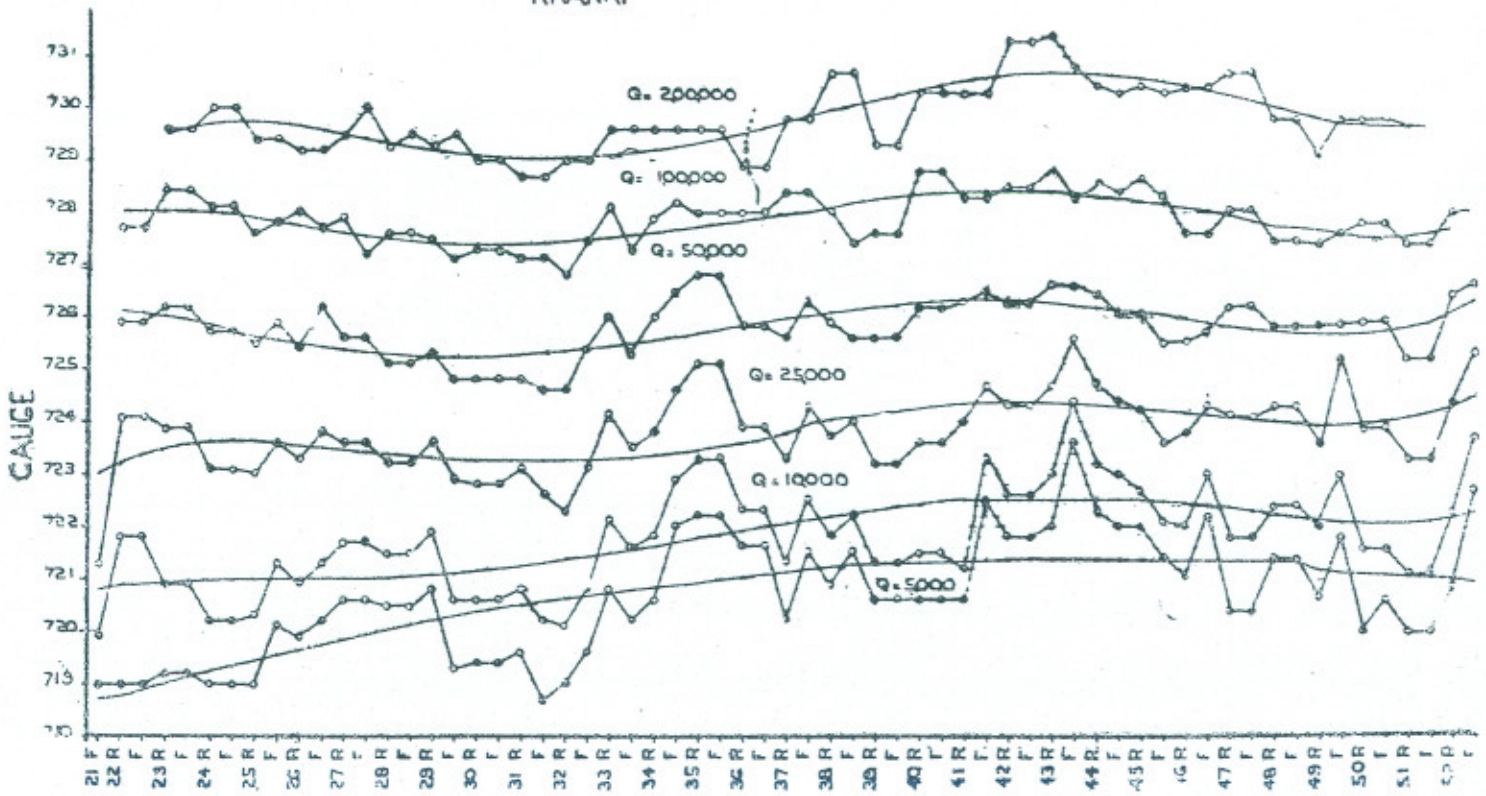


FIG: 6.
SPECIFIC GAUGES OF RIVER CHENAB AT
ALEXANDRA BRIDGE

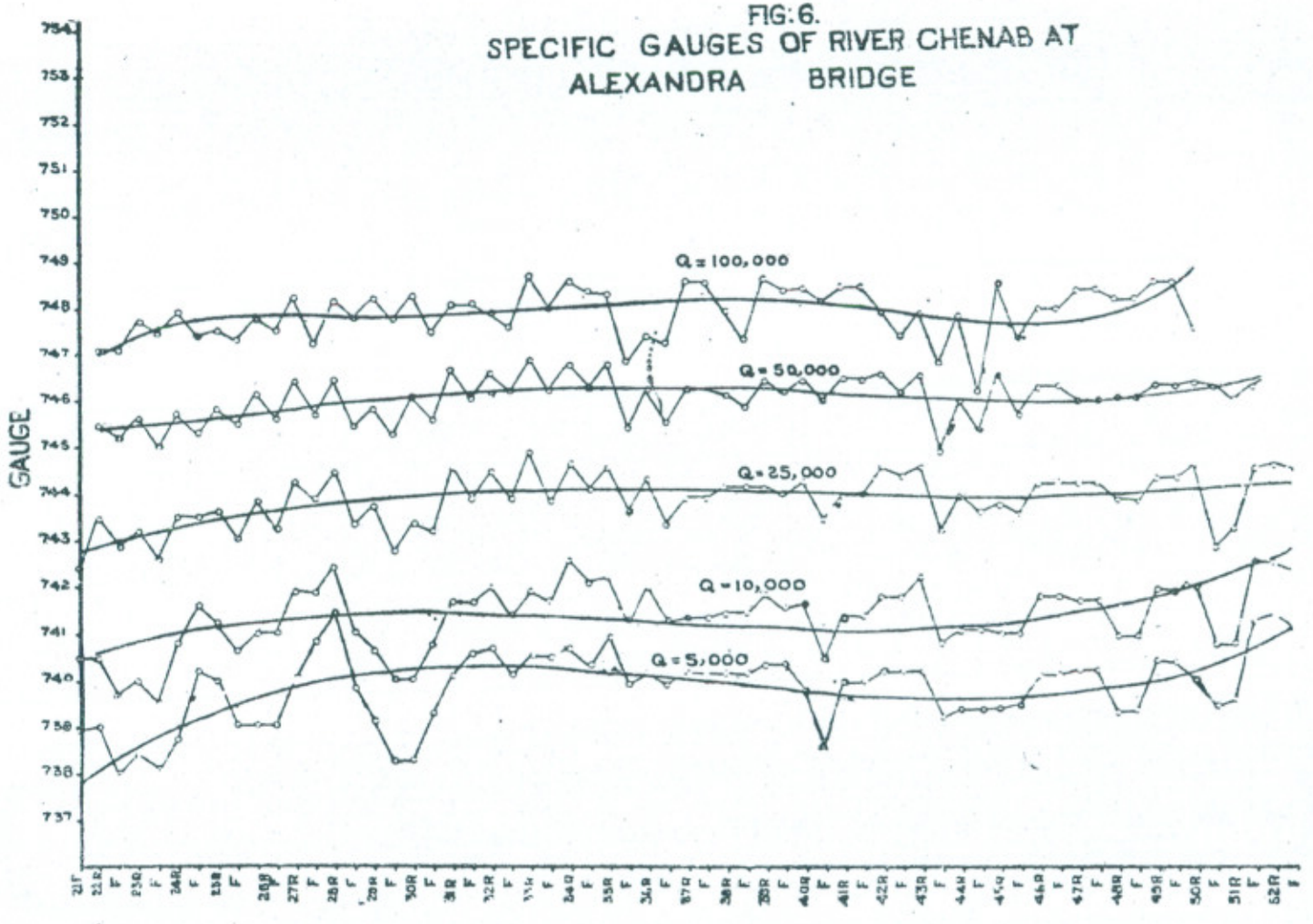


FIG. 7

SPECIFIC GAUGES OF RIVER CHENAB

AT

MARALA

