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# WATER-TABLE DEPLETION NON-STEADY FLOW TO WELLS EFFECT OF SOIL EVAPORATION

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Abstract

In this paper, the flow pattern induced by multi-wells installed in the vicinity of a line source (recharge source) is presented. It is further assumed that the aquifer is confined and of infinite extent. The permeability of the soil immediately below the bed of the line source is taken different (much less) from the permeability of the main aquifer so as to establish a relationship between the two media. The effect of soil evaporation on the depletion of watertable caused by pumpage has also been taken into account. To begin with, the aquifer may or may not be in an equilibrium state i.e. the present state of the aquifer is being governed by the present recharge rates into the aquifer, the present withdrawal out of aquifer and the evaporation losses from the soil surface under the present water-table conditions. As the water-table depletes, by the installation of project wells, the evaporation losses from the soil decrease at an exponential rate. This means a decrease in natural pumpage due to the reduction of evaporation rates. In other words, it causes an invisible leakage into the aquifer. The change in infiltration, as a result of water-table depletion, from different sources is relatively much smaller and decreases with the depletion of water-table.

## Introduction

M.S. Hantush, and C.E. Jacob[1] (1955) have given a solution for a non-steady radial flow to a pumped well pumping at a constant

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rate in an infinite, confined and leaky aquifer. The leakage is, however, linear one i.e. it is assumed to be vertical and proportional to the draw down.

In the present paper we are taking up a case in which the leakage is of exponentional nature and increases with the draw down. Consider the water-table depletion problem of a given vast area. Let the conditions be such that inflow rates into the aquifer, withdrawal rates out of aquifer and evaporation losses from the soil surface under the present watertable conditions be all in a balanced state. Later on we shall generalise this case by including the non-equilibrium states of the aquifer. When the watertable depletes, the effect of natural pumping caused by soil surface evaporation decreases. This means indirectly an increased leakage into the aquifer.

S.M. Lang[2] (1961) has dealt with a case in which wells are installed, in an infinite non-leaky aquifer, in the vicinity of a line source (a recharge source). Here the permeability of soil below the bed of the recharge source is assumed to be the same as that of the aquifer.

In the present paper we have solved a two-layer case i.e. the permeability of a layer just below the bed of the recharge source is assumed to be different (much less) from that of the main aquifer.

M.S. Hantush[3] (1961), as a result of detailed mathematical analysis, has proved that a partially penetrating well behaves as if completely penetrating the aquifer when $(r/b) > 1.5$ where $r$ is the radial distance of the observation well from the pumped well and $b$ the thickness of the aquifer. Similar is the case when the value of pumpage time $t$ is sufficiently large i.e. $t > b^2S/2T$ where $S$ is the storage coefficient and $T$ the coefficient of transmissibility of the aquifer respectively. In our analysis both the values of $(r/b)$ and $t$ will be greater than the above prescribed limits. Consequently, although the wells will be partially penetrating the aquifer, we shall for calculation purposes assume as if fully penetrating the aquifer.

## Statement of the Problem

It is aimed to determine the variation with time of the drawdown induced by a well steadily discharging from an infinite leaky aquifer where leakage is induced as a result of reduction in natural pumpage caused by decreasing soil-evaporation rates as the watertable depletes from its initial value at time $t$ equal to zero. The initial head is assumed to be uniform. Stated mathematically, the boundary

value problem is :

$$\partial^2 s/\partial r^2 + (1/r)\partial s/\partial r - (E_a/T)\{1-\exp(-\lambda s)\} = (S/T)\partial s/\partial t \quad \ldots \quad (1)$$

$$s(r, 0) = 0, \; r \geq 0 \quad \ldots \quad \ldots \quad \ldots \quad (2a)$$

$$s(\infty, t) = 0, \; t \geq 0 \quad \ldots \quad \ldots \quad \ldots \quad (2b)$$

$$\lim_{r \to 0} r\partial s(r,t)/\partial r = -Q/2\pi T, \; t > 0 \quad \ldots \quad \ldots \quad (2c)$$

As the pumped well will be partially penetrating so we further impose the following conditions on draw-down measurements :

$$(r/b) > 1.5 \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

$$t > b^3 S/2T \quad \ldots \quad \ldots \quad \ldots \quad (3b)$$

where

| | | |
|---|---|---|
| $s(r,t)$ | = | draw-down (ft) at any time $t$ and any distance $r$ from the well subject to conditions (3a) & (3b) |
| $r$ | = | radial distance from the axis of the well(ft) |
| $b$ | = | thickness of the aquifer (ft) |
| $T$ | = | Kb, the coefficient of transmissibility of the aquifer ($ft^2$/sec), K being the coefficient of permeability of the aquifer (ft/sec) |
| $S$ | = | Storage coefficient of the aquifer (dimensionless) |

Note: To get the representative values of T&S, actual pumping tests are performed in the project area before hand.

$Q$ = constant discharge pumped by the well (cusec)

$E_a$ = Soil surface evaporation with average water-table position of the project area just before pumpage (cusec/sq.ft)

$\lambda$ = constant, being the slope of the line drawn between percent evaporation from soil with depth to watertable on a semi-log paper ($ft^{-1}$)

Solution of the Problem

Due to uneven topography in the given area in which depletion of watertable is desired, the position of watertable may be different in different localities. Consequently we divide the project area into a number of sub-zones according to water-table ranges, for example 0 - 5 ft, 5 - 10 ft, greater than 10 ft etc. Consider the sub-zone with 0 - 5 ft range. Taking the average water-table at 2.5 ft, the evaporation losses when water-table is at 2.5 ft are given by substituting $x = 2.5$ in :-

$$E_x = E_o \exp(-\lambda x) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where

$E_o$ = maximum soil surface evaporation with water-table at zero depth

The type of curve connecting soil surface evaporation and depth to watertable is given in figure 1. When the data of this curve is plotted on a semi-log paper and a best line through the points is drawn, we get a straight line shown dotted in figure 2. Since soil surface evaporation when the water-table is at the surface is taken as 100 percent so the dotted line is shifted parallel to itself (keeping the slope constant) such that the line cuts the Y-axis at 100 percent evaporation when the watertable is at zero depth (full line of fig.2). Consequently the slope $\lambda$ of this line is known.

Substituting the above value of $\lambda$ in eq. (4) and knowing the value of $E_o$ for the locality by experimentation on free water

surface evaporation, the value of $E_{2.5}$ is known. This value of $E_{2.5}$ when multiplied by the total area of the sub-zone having the watertable range 0 to 5 ft., gives the amount of total evaporation from this sub-zone under the initial conditions before the pumpage is started. Likewise we calculate similar contributions by other sub-zones in the Project area. Adding up all these values for all the sub-zones gives the total amount of soil surface evaporation from the entire project area before the start of pumpage. Dividing this total soil surface evaporation by the entire Project area, we get the amount of average soil surface evaporation over a unit area (cusec/sq ft).

A perusal of figure 1 shows that the soil surface evaporation becomes negligibly small when the watertable goes down to 10 ft. As stated above the aquifer, before the start of pumpage, is assumed to be in an equilibrium state. This means the amount of average soil surface evaporation per unit Project area, as calculated above, must be considered as a leakage into the aquifer when watertable reaches a depth of 10 ft. from the surface. Now our draw down $s$ is zero before the start of pumpage, At the same time, the leakage is also zero when the draw down is zero. As the watertable depletes, the leakage into the aquifer due to change of soil surface evaporation increases exponentially till it becomes approximately equal to the value of Soil Surface Evaporation per unit project area under the conditions of watertable which were prevalent before the start of pumpage. Thus the exponential expression mentioned in eq. (1) is fully explained.

The eq. (1) due to the presence of the exponential factor is not readily soluble. The author, however, is trying his best to find out the exact solution if possible. In the meantime an approximate solution is putforth to solve such a watertable depletion problem.

Consider the expression :

$$E_a \{1-\exp(-\lambda s)\}$$

The slope of the line in fig. 2 is 0.434. Thus the value of the above expression for $s = s_m = 10$ ft approaches very nearly to $E_a$. Thus the above expression may be approximated as :

$$\frac{E_a\, s}{s_m}$$

This will hold good mainly near the end points and not for the intermediate points of the path of the draw-down. But our ultimate aim is

Total resistance R offered to the flow by the medium when connected in series is :

$$R \propto \frac{h_1}{K_1} + \frac{h_2}{K_2} = \frac{h_1 K_2 + h_2 K_1}{K_1 K_2}$$

Now discharge $Q \propto \frac{1}{\text{Resistance}}$

If there had been one uniform medium with $K_1 = K_2$, then :

$$Q_1 = Q_2 \propto \frac{1}{\text{Resistance}}$$

$$\propto \frac{1}{\frac{h_1 + h_2}{K_2}} \propto \frac{K_2}{h_1 + h_2}$$

But in a two layers case :

$$Q_2 \propto \frac{1}{(h_1 K_2 + h_2 K_1)/K_1 K_2}$$

$$\propto \frac{K_1 K_2}{h_1 K_2 + h_2 K_1}$$

or

$$\frac{Q_2}{Q_1} = \frac{K_1 K_2}{h_1 K_2 + h_2 K_1} \cdot \frac{h_1 + h_2}{K_2}$$

$$= \frac{K_1 (h_1 + h_2)}{h_1 K_2 + h_2 K_1}$$

or

$$Q_2 = \frac{K_1 (h_1 + h_2)}{h_1 K_2 + h_2 K_1} \cdot Q_1 = \beta Q_1$$

where

$$\beta = \frac{K_1(h_1+h_2)}{h_1K_2+h_2K_1} \qquad \ldots \qquad \ldots \qquad (10)$$

$$0 \leq \beta \leq 1$$

Thus we have determined the relation between the discharge of the imaginary well (for a two layer case) in comparison to the discharge of the real well for a single layer case. The discharge of the real well is not altered much by the presence of a low permeability layer at the top because it sucks water from the lower layer of higher permeability. On the contrary the line source contribution is greatly affected by the permeability of the medium below its bed. Thus if the permeability of the bed is very low and the thickness of this layer sufficiently great then there will be a negligible contribution of the line source to the aquifer.

Now if there are m wells installed near the recharge source, then eq. (9) can be modified as :

$$s = (Q/4\pi T)\{\sum_{j=1}^{m} W(u_{jr},r_{jr}/B) - \beta\sum_{j=1}^{m} W(u_{jI},r_{jI}/B)\} \qquad \ldots \qquad (11)$$

where

$W(u_{jr},r_{jr}/B)$ = value of the well function for the jth real well

$W(u_{jI},r_{jI}/B)$ = value of the well function for the jth image well

$u_{jr},u_{jI}$ = the value of $u$ for the jth real well and jth image well respectively

$r_{jr},r_{jI}$ = the value of $r$ for the jth real well from the observation well and the jth image well from the same observation well respectively

$\beta$ = as given by eq. (10)

Now we consider the case in which m wells are installed near the recharge source and (n-m) wells away from the recharge source in a given project area such that the total wells remain n. Combining equations (9) & (11) we have the net draw-down s in a given observation well due to the effect of all the wells as :

$$s = (Q/4\pi T)\{\sum_{k=1}^{n} W(u_{kr}, r_{kr}/B) - \beta \sum_{j=1}^{m} W(u_{jl}, r_{jl}/B)\} \quad \ldots \quad (12)$$

Equation (12) holds for the case in which the aquifer is in a balanced state before the installation of the project tubewells. Now we shall consider the case when the aquifer is in an un-balanced state. The eq.(12) is then modified as :

$$s = \{(nQ \pm Q_1)/n\}\{\sum_{k=1}^{n} W(u_{kr}, r_{kr}/B) - \beta \sum_{j=1}^{m} W(u_{jl}, r_{jl}/B)\} \quad (13)$$

where

$n$ = total number of project tubewells installed.

$Q$ = discharge pumped by each project well

$+Q_1$ = excess withdrawal rate (before pumpage) out of the project area over and above the total recharge into the project area (i.e. the aquifer state in which the watertable is gradually depleting)

Thus the resultant discharge pumped by each project well = $(nQ+Q_1)/n$

Likewise

$-Q_1$ = excess recharge rate over and above the total withdrawal out of the project area (before pumpage) (i.e. the aquifer state in which the watertable is gradually rising)

Here the resultant discharge pumped by each project well = $(nQ-Q_1)/n$

The practical application of eq.(12) has been done in the problem of ground watertable depletion of Faisalabad city because, as reported by the Faisalabad Development Authority, the aquifer is in a balanced state. A report[4] to this effect has been issued by the Irrigation Research Institute Lahore.

Now we consider the change in infiltration as a result of watertable depletion from different sources like vast irrigated area or nearby canals etc.

Let A be the irrigated area having a permeability K, h the depth to watertable and a the depth of irrigation applied. Thus contribution $Q_2$ to the watertable is :

$$Q_2 = KA(a+h)/h$$

$$= KA\left(\frac{a}{h} + 1\right) \quad \ldots \quad \ldots \quad (14)$$

Now as watertable depletes, h increases and the limiting value is :

$$Q_3 = KA + KA \lim_{h\to\infty} \frac{a}{h}$$

$$= KA \quad \ldots \quad \ldots \quad (15)$$

Clearly $Q_3 < Q_2$

Equation (15) shows that as the watertable depletes, the recharge rates from the infiltration sources decrease, till a constant value is attained.

The measurement of the rate of rise of watertable gives results of opposite nature i.e. the rate of rise was higher when the watertable was deep in comparison to the rates with shallow watertables. There is nothing contradictory because at shallow watertables natural pumping caused by soil surface evaporation becomes very predominant in checking the rapid rise of watertable.

Conclusions

1. A modified mathematical modelling is presented to study ground watertable depletion problem by taking into account the soil surface evaporation.

2. The modelling has also been modified to take into consideration the re-circulation effect if all or some of the wells are installed in the vicinity of a recharge source. The permeability of the layer immediately below the bed of the recharge source has been taken much less from that of the main aquifer.

3. The mathematical modelling has actually been applied for the solution of ground watertable depletion in Faisalabad city and a report[4] has accordingly been issued.

4. The soil-surface evaporation is a very important factor in checking the rise of watertable.

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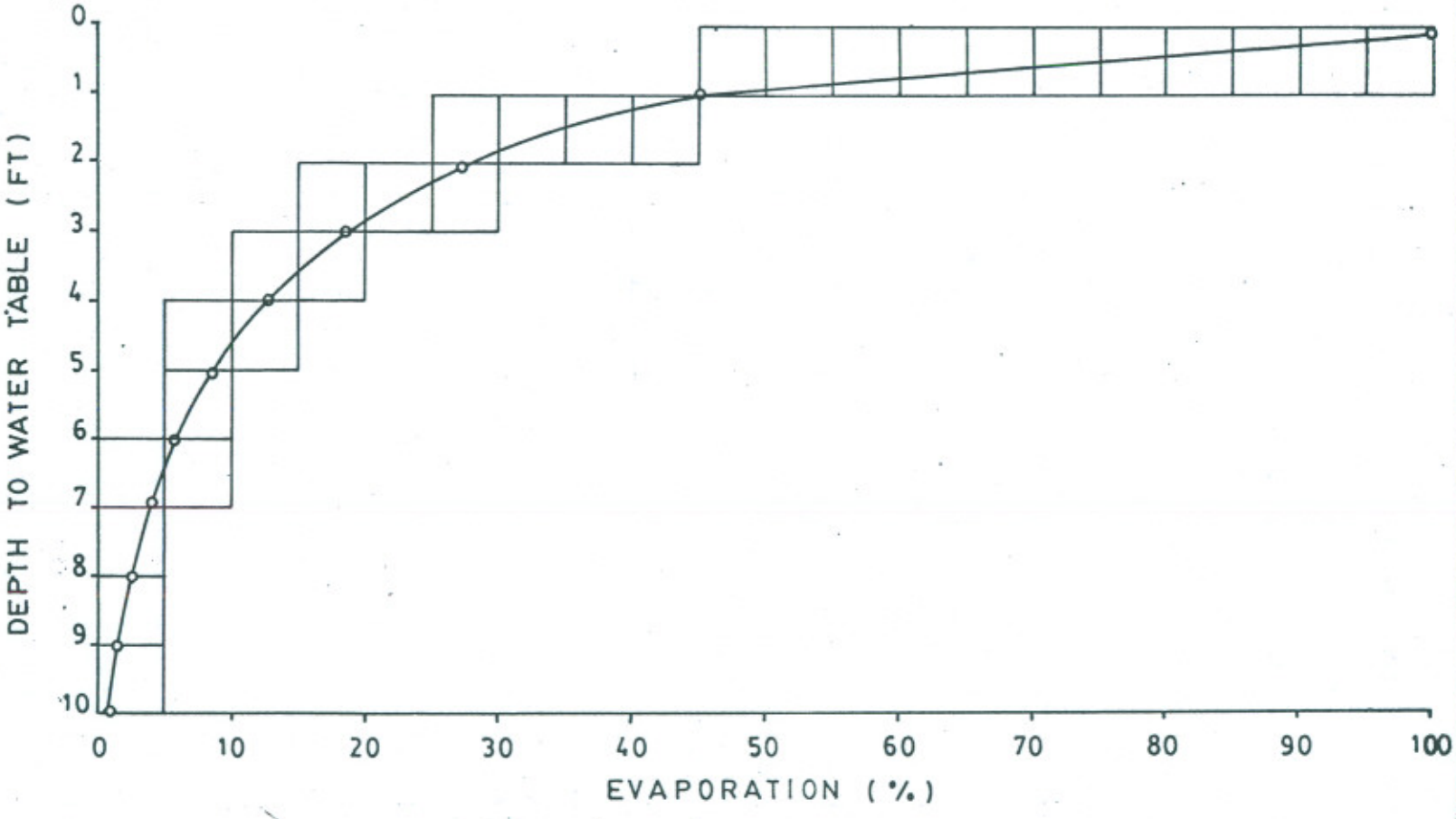
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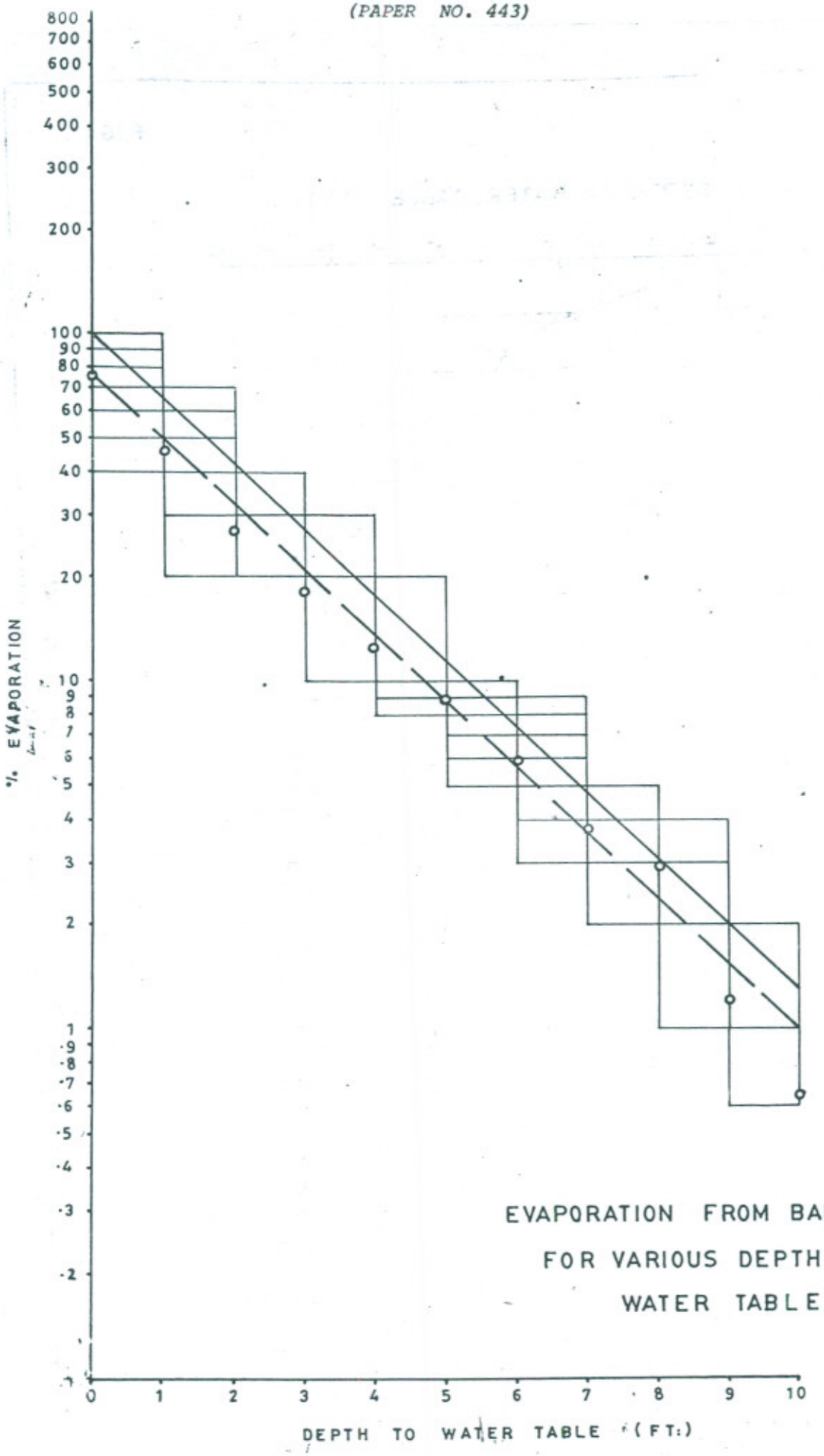
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FIG: 1




EVAPORATION FROM BARE SOIL FOR VARIOUS DEPTHS TO WATER TABLE

FIG :- 2




EVAPORATION FROM BARE SOIL FOR VARIOUS DEPTHS TO WATER TABLE