PAPER No. 470

Theory of Minimum Rate of Energy Dissipation and its Application

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SUMMARY

The theory of minimum rate of energy dissipation states that when a system is in its dynamic equilibrium condition, its rate of energy dissipation is at its minimum value which is compatible with the constraints applied to the system. When a system is not at its dynamic equilibrium condition, its rate of energy dissipation is not at its minimum value. However, the system will adjust itself in such a manner that its rate of energy dissipation can be reduced to a minimum value and regain equilibrium. For open channel flows under certain special conditions, the minimization of rate of energy dissipation is equivalent to the minimization of stream power or unit stream power. The theory of minimum rate of energy dissipation or its simplified versions of minimum stream power or minimum unit stream power can be applied to explain the variation of velocity distribution, bed form and roughness, channel width, channel geometry, channel pattern, and channel profile. The theory can also be used for stable channel designs. The relationship between minimization of rate of energy dissipation and maximization of sediment transport rate or friction factor is explained. Laboratory and field data are used to support the validity of the theory.

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Chih Ted Yang[1]

INTRODUCTION

The development of hydraulics and river mechanics is mainly based on Newton's laws of motion. Equations of motion, or their simplified versions, are solved with proper initial and boundary conditions from vectorial approach. This approach is well suited for solving fluid mechanics problems where the magnitude and direction of the forces are given and the boundary is fixed. For natural rivers of irregular shape and changing direction, it is very difficult to define the direction of forces acting on a fluid particle along its course of flow. Our common practice in river engineering is to assume a one-dimensional flow condition knowing that natural river phenomena are three dimensional. Because a river can adjust its channel geometry and longitudinal profile, the boundary conditions become variables which need to be determined before the equations of motion can be applied. The existence of different bed forms further complicates the problem by changing the resistance to flow under different flow and sediment conditions. It is extremely difficult, if not impossible, to solve these problems theoretically from Newton's laws of motion alone. Consequently,

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hydraulic engineers are forced to make some simplified assumptions and use some empirical or semiempirical equations based on their experience in solving river engineering problems. Strictly speaking, our practice in river mechanics and engineering is not a precise science but an art.

Parallel to the vectorial approach is the scalar or the variational approach. The variational approach replaces the force by the work of the force which is often replaced by potential energy. Instead of solving equations of motion, the calculus of variation or other minimization techniques are used as the mathematical tool for the variational approach. Thus, a river hydraulic problem can become a minimization or optimization problem. Because the minimum value depends on the constraints which define the optimum path, the scalar approach is that of minimization with constraints instead of solving differential equations with boundary conditions.

Vectorial and scalar approaches are both important to the development of the science of mechanics. Due to the influence of Newton, our developments in river mechanics are mainly limited to vectorial approach. The importance of scalar or minimization approach has gained increasing recognition in recent years due to our inability to solve river hydraulic problems analytically from the vectorial approach alone. This paper briefly describes the theory of minimum rate of energy dissipation and some of its simplified versions based on the scalar approach. Laboratory and field data are used to illustrate how this theory can be applied to explain or to solve complicated river hydraulic problems.

THEORIES AND HYPOTHESES

Based on U.S. Geological Survey river gauging station data, the hydraulic and channel geometry characteristics can be approximated as power functions of water discharge by the following equations:

$$V = kQ^m \tag{1}$$

$$D = cQ^f \tag{2}$$

$$B = aQ^b \tag{3}$$

$$S = iQ^j \tag{4}$$

in which V = average flow velocity; Q = water discharge; D = average flow depth; B = channel width; S = channel slope; and k, m, c, f, a, b, i, j = constants. Leopold and Maddock (1953) called these empirical relationships the hydraulic geometry relationships. They also found that the ranges of variation of hydraulic exponents m, f, b, and j are relatively small compared with those of coefficients k, c, a, and i.

Langbein (1964) hypothesized that the changes in the variables shown in equations (1) through (4) are such that the total effect, action, work, or adjustment is a minimum. At a cross section of a river, Langbein's hypothesis would suggest that all variables strive to resist any imposed changes, with the net result of having equal amount of adjustments among the dependent variables, subject to the external restrictions. The "variance" used by Langbein represents the square of hydraulic exponent. Thus, minimum variance means the sum of the squares of the hydraulic

exponents is a minimum which is consistent with local restrictions. One of the basic problems of applying Langbein's hypothesis is the selection of correct combination of variables to be used in the minimization process. Different combinations of variables could lead to different answers.

Leopold and Langbein (1962) introduced the concept of entropy to the study of river morphology. The entropy concept was further explained by Langbein (1964). Kennedy, et al (1964) pointed out some mathematical inadequacy in Leopold and Langbein's derivation. No further advancement on entropy analogy was made until 1971.

Yang (1971a) reintroduced the entropy concept to the study of natural river systems. He considered the only useful energy per unit mass or weight of water in a river system is its potential energy. He further assumed that the potential energy and elevation of a river system are equivalent to thermal energy and absolute temperature, respectively, of a heat system. Based on this analogy and the direct application of entropy concept in thermodynamics, it can be shown that

$$\frac{dY}{dt} = \text{a minimum} \tag{5}$$

in which Y = potential energy per unit weight of water in a river system, and t = time. Thus, Yang concluded that during the evolution toward its equilibrium condition, a natural river chooses its course of flow in such a manner that the rate of potential energy dissipation per

unit weight of water along its course is a minimum. The minimum value depends on the constraints applied to the river. Unlike the minimum variance hypothesis by Langbein, equation (5) gives a clearly defined objective function and equation for minimization. Equation (5) can be rewritten as

$$\frac{dY}{dt} = \frac{dx}{dt}\frac{dY}{dx} = VS = \text{a minimum} \tag{6}$$

in which x = distance along the course of flow; V = average flow velocity; and S = energy slope. Yang (1973) defined the VS product as the unit stream power.

Yang and Song (1985) developed a comprehensive theory of minimum energy and energy dissipation rate. The system considered by them is a closed and dissipative system, and the total energy is the sum of potential energy E_p and kinetic energy E_k. Because energy is nonnegative, it is possible to write the total energy E as

$$E = E_p + E_k \geq 0 \tag{7}$$

For a closed and dissipative system, energy can only decrease with respect to time. It follows that

$$\frac{dE}{dt} \leq 0 \tag{8}$$

In the end, when all motions cease

$$\frac{dE}{dt} = 0 \tag{9}$$

Thus, the state of static equilibrium is a state of minimum energy. The minimum value depends on the constraints applied to the system.

A closed dynamic system will dissipate energy during its evolutional process to the final static equilibrium. Let ϕ represent the rate of energy dissipation so that

$$\phi = - \frac{dE}{dt} < 0 \tag{10}$$

Because static equilibrium is a state of minimum energy, a dynamic state that is not too far from the static state should have

$$\frac{d\phi}{dt} = - \frac{d^2E}{dt^2} \leq 0 \tag{11}$$

That is, the rate of energy dissipation of all closed dynamic systems must eventually tend to decrease or stay the same after sufficient time. Thus, the state of dynamic equilibrium is a state of minimum rate of energy dissipation. The minimum value depends on the constraints applied to the system. When a system is not at its dynamic equilibrium condition, its rate of energy dissipation is not at its minimum value. However, the system will adjust itself in such a manner that its rate of energy dissipation can be reduced to a minimum value which is compatible with the constraints and regain equilibrium.

The total rate of energy dissipation Φ required to transport a given amount of water discharge Q and a sediment discharge Q_S through an open channel with a reach length of L is

$$\Phi = \Phi_W + \Phi_S = (Q\gamma + Q_S\gamma_S)\, LS \tag{12}$$

In which, Φ_W, Φ_S = rate of energy dissipation due to the transportation of water and sediment, respectively; γ, γ_S = specific weight of water and sediment, respectively; and S = slope.

When the rate of energy dissipation due to sediment transport is small and can be neglected, equation (12) can be reduced to

$$\Phi = (QS)\gamma L \tag{13}$$

In this condition, the minimization of Φ is equivalent to the minimization of stream power QS. Thus, the theory of minimum stream power is a special and simplified version of the theory of minimum rate of energy dissipation. When the velocity is fairly uniform across a channel with a given cross-sectional area A, equation (13) can be rewritten as

$$\Phi = \gamma LA(VS) \tag{14}$$

In this condition, the minimization of Φ is equivalent to the minimization of unit stream power VS. Therefore, the theory of minimum unit stream is another special and simplified version of the theory of minimum rate of energy dissipation.

The application of the theory of minimum energy and energy dissipation rate is not limited to open channel systems. It can be applied to a system of either solid, liquid, gas, or a combination of them as long as the system is closed and dissipative. An open system can be converted to a closed system under equilibrium conditions. Detailed deviations and applications of the general minimization theory and its simplified versions are given by Yang and Song (1985).

APPLICATIONS

A river is a dynamic system. It can adjust its velocity distribution, bed form and roughness, channel geometry, pattern, and longitudinal profile either individually or collectively to maintain a balance between the rate of energy dissipation and the rate of work being done in transporting water and sediment. For a graded river or a regime channel, water discharge and sediment load are considered as independent variables, and the emphasis is on long-term adjustments. For short-term adjustments, water discharge and slope are considered as independent variables and sediment transport rate may become a variable to be adjusted. Laboratory and field data will be used to demonstrate that all these adjustments are consistent with the theory of minimum rate of energy dissipation or its simplified versions. This theory can also be applied to stable or regime channel designs.

1. Velocity Distribution

For rigid boundary open channel flows or alluvial channels with plane bed or very small bed forms, the resistance to flow or roughness can be considered as a constant. With fixed channel geometry, slope, and roughness, the only way to minimize its rate of energy dissipation is through the adjustment of velocity distribution. It was shown by Yang and Song (1979) that under the condition of gradually varied subcritical flow where the effect of sediment movement can be neglected, the rate of energy dissipation is a minimum when the velocity profile satisfies the equation of motion. Song and Yang (1979) also showed that vertical velocity distribution can be determined from the theory of minimum rate of energy dissipation under a given set of constraints.

Song and Yang (1979) assumed that the velocity distribution of turbulent open channel flow can be divided into two parts. The outer region can be approximated by a parabolic distribution while the inner region can be approximated by a logarithmic distribution. In order to fully define the velocity distribution, seven coefficients need to be determined. These coefficients can be solved by the application of the theory of minimum rate of energy dissipation with proper boundary conditions. An example of comparison between the theoretical solution thus obtained and the measured velocity distribution is shown in figure 1. The agreement is very good. The average sand wave height is also shown in figure 1. Because the

sediment concentration of 120 ppm by weight is not high and the ripple size is small, the effect of sediment transport and bed form on the rate of energy dissipation can be neglected in this case. Readers who are interested in the theoretical derivations should refer to the original publication.

2. Bed Form and Roughness

A major difference between the hydraulics of rigid and alluvial boundary is that the former can treat roughness as a constant while the latter cannot. Yang (1976) used the laboratory data collected by Guy, et al (1966), as shown in figure 2, to illustrate the variation of roughness due to the existence of different bed forms. Our ability to determine roughness of an alluvial channel is mainly limited to the grain roughness which is directly related to the sediment particle size. Although different approaches have been suggested for the determination of form roughness, most of them are empirical in nature, and none of them have gained universal acceptance. Figure 2 clearly demonstrates the high degree of variation of form roughness due to the existence of different bed forms and its impact on total roughness. Strictly speaking, Manning's equation or other resistance equation cannot be applied unless the roughness value under different flow and sediment conditions are given or can be reliably predicted.

Yang (1976) applied the theory of minimum unit stream power to determine the velocity, slope, and depth of alluvial channels with given

width, water discharge, and sediment concentration without the use of Manning's equation or other resistance equations. The equations used by Yang (1976) are:

The continuity equation of water discharge

$$Q = WDV \tag{15}$$

in which Q = water discharge; W = channel width; and D = average water depth.

Yang's (1973) sediment transport equation

$$\log C = 5.435 - 0.286 \log \frac{\omega d}{\nu} - 0.457 \log \frac{U_*}{\omega}$$

$$+ \left(1.799 - 0.409 \log \frac{\omega d}{\nu} - 0.314 \log \frac{U_*}{\omega}\right) \log \left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \tag{16}$$

in which C = total sediment concentration, in parts per million by weight; ω = terminal fall velocity of sediment particles; d = median sieve diameter of sediment particles; ν = kinematic visiosity; $U_* = \sqrt{gDS}$ = shear velocity; g = gravitational acceleration; VS = unit stream power; and $V_{cr}S$ = critical unit stream power required at incipient motion.

The equation of minimum unit stream power

$$VS = V_m S_m \tag{17}$$

in which $V_m S_m$ = minimum unit stream power.

With the preceding three equations, the three unknowns, V, S, and D can be solved. Good agreements between computed and measured laboratory and river data were obtained (Yang, 1976). Figure 3 shows an example between the measured and computed values from minimum unit stream power which have the subscript m. Once V, D, and S are determined, the roughness coefficient can be computed from Manning's equation. Figure 4 shows an example of comparison between the measured Manning's roughness coefficient n and the computed value n_m based on minimum unit stream power. In view of the large range of variation of roughness coefficient under different flow and sediment conditions as shown in figure 2, the results in figure 4 are satisfactory.

The results shown in figures 3 and 4 indicate that an alluvial channel can adjust its roughness in order to minimize its rate of energy dissipation or unit stream power. Recently, Davies and Sutherland (1983) proposed a hypothesis of maximum friction factor. They noticed the increase of friction factor from plane bed to ripple and dune and concluded that an alluvial channel adjusts itself in order to maximize its friction factor. As shown in figure 2, the friction factor initially increases when the bed form is changing

from plane bed to ripple. However, the friction factor actually decreases when the bed form is changing from ripple to dune and to transition. Thus, the hypothesis of maximum friction factor only describes one means of adjustment which is available to an alluvial channel under some special conditions for the purpose of minimizing its rate of energy dissipation. It was shown by Yang and Song (1985) that the friction factor can either be maximized or minimized in the process of minimizing its rate of energy dissipation.

3. Channel Width

Analytical solution of open channel hydraulics, such as the use of Manning's equation, is based on the assumption that the channel width is given or predetermined. Due to the lack of theoretical solutions, our approach to the determination of channel width is mainly empirical. The use of regime equations is a prime example of empirical approach. The width thus obtained can be used only to those conditions which are similar to those from where the regime equations were derived from. Two shortcomings of using the regime equations are the lack of theoretical basis and the possibility of obtaining different answers from using different regime equations.

Chang (1979) applied the Engelund and Hansen's (1967) resistance relationship, DuBoys' (1879) bedload transport equation, and the theory of minimum stream power to determine the regime channel width. Figure 5 shows good agreements between measured width and computed width based on the theory of minimum stream power.

4. Channel Geometry and Regime Channel Design

The empirical relationships shown in equations (1) to (4) are called hydraulic geometry relationships. Most of the studies in the past were made for the determination of hydraulic exponents m, f, b, and j from river gauging station records. These studies showed an average value of f = 0.41 for the rivers in the United States.

Yang, et al (1981) applied the theory of minimum rate of energy dissipation to the determination of hydraulic exponents. The equations used included equation (12), Manning's equation for discharge, and Yang's (1973) unit stream power equation in its basic exponential form. By using the Lagrangian technique for minimization, the values of hydraulic exponents obtained for uniform materials are

$$b = f = 9/22 \tag{18}$$

$$m = 2/11 \tag{19}$$

$$j = -2/11 \tag{20}$$

The theoretical value of f = 9/22 is almost identical to the average value of 0.41 observed from stable rivers in the United States (Leopold and Maddock, 1953). The fact that both b and f should converge to the value of 9/22 was also confirmed by laboratory and field studies conducted by Barr, et al (1980). This confirmation is shown in figure 6.

The regime equations have been used extensively in India and Pakistan for stable channel designs. Leopold and Maddock (1953) showed that regime equations can be converted to the basic form of hydraulic geometry relationships as shown in equations (1) to (4). Among all the regime equations, Lacey's (1929) equations are the most commonly used in Pakistan. Lacey's equations for stable velocity and slope are

$$V = 1.1547 \sqrt{f_s R} \tag{21}$$

$$S = 0.0005423 \frac{f_s^{5/3}}{Q^{1/6}} \tag{22}$$

in which R = hydraulics radius; and f_s = Lacey's silt factor. From equations (18) and (19), the power relationship between velocity and depth is m/f = 4/9. This is almost identical to the power relationship of 1/2 between velocity and hydraulic radius in equation (21). The difference of 1/2 - 4/9 = 1/18 is due to the difference between the use of hydraulic radius and average depth. The theoretical value of j = -2/11 as shown in equation (20) is also almost identical to Lacey's empirical value of -1/6 given in equation (22). These results strongly suggest that the theory of minimum rate of energy dissipation can be applied to the study of channel geometry and regime channel designs. The theory also gives us the theoretical basis to improve empirical regime equations and eventually transfer

the regime type of equations to be applied outside India and Pakistan. The possibility of using the theory of minimum stream power for stable channel design was demonstrated by Chang (1980).

5. Channel Pattern

A natural river can be fairly straight, meandering, or braided. Under most conditions, meandering rivers are more common and more stable. The rotation of the earth, maximization or minimization of energy loss, local disturbances, bank erosion and sedimentation, etc., have been suggested in the literature as the basic reasons for river meandering. Yang (1971b) made a critical review of these theories and hypotheses and concluded that none of them can provide a satisfactory explanation of the basic reason for a river to meander. He further concluded that the basic reason for a river to meander is to reduce its rate of energy dissipation.

Chang (1980) applied the theory of minimum stream power to verify natural river patterns. Chang considered a river to be meandering when the sinuosity is equal to or greater than 1.3. Sinuosity is the ratio of valley slope to channel slope. Based on the theory of minimum stream power, he found that there is only one solution for straight channel which is equal to the valley slope. However, for meandering rivers, two minimum slopes can be obtained. One minimum value is larger while the other is smaller than the valley slope. Because it is physically impossible for the channel slope to be steeper than the valley s ope, the one which is smaller than the

valley slope must be the solution for meandering river. The condition for channel braiding was also investigated by Chang. His results are shown in figure 7. Thus, the change of channel patterns is also consistent with the theory of minimum rate of energy dissipation or the simplified theory of minimum stream power.

6. Channel Profile

Most natural rivers have concaved overall longitudinal riverbed profiles with local variation of slope in the form of pools and riffles. Yang (1971c) studied the basic reasons for the formation of the pool-riffle sequence at low flows. He considered the case of water flowing from point A to C as shown in figure 8. It can take a straight profile ADC or a mild pool profile AB followed by a steeper riffle profile BC. The location of B can be anywhere within AMCN. For all possible profiles, the total horizontal distance is X and the vertical drop is Y. Because Y is a constant, the only way to minimize the rate of energy dissipation for a given water discharge through the study reach is to increase its time of travel. Figure 9 shows the variation of computed time of travel through different hypothetical paths. It is apparent that the straight line path ADC has the shortest time of travel. For a given constant drop of Y, it means that the rate of energy dissipation along ADC is the maximum among all possible paths. According to the theory of minimum rate of energy dissipation, a river will avoid the straight profile and form the pool-riffle sequence. Because the minimum value depends on the

the pool-riffle sequence. Because the minimum value depends on the constraints, the rate of energy dissipation cannot be zero as long as the discharge is not zero. As the discharge decreases, the rate of energy dissipation should also decrease. This is why the pool-riffle phenomenon is more dominant at low flows. As discharge increases, the rate of energy dissipation increases accordingly, and the profile becomes more uniform.

It is interesting to note that the locations of pools and riffles are closely associated with bends and crossings of a meandering river. This is an indication that a river can minimize its rate of energy dissipation in the lateral direction by meandering and in the vertical direction by the formation of pools and riffles (Yang, 1971 c).

The overall longitudinal riverbed profile of a river can also be explained by the theory of minimum rate of energy dissipation. Let us consider the simple case of minimizing stream power. The theory requires that the stream power QS be minimized in the longitudinal direction, i.e.,

$$\frac{d}{dx}(QS) = S\frac{dQ}{dx} + Q\frac{dS}{dx} = 0 \qquad (23)$$

Because Q increases in the downstream direction for most natural rivers, dQ/dx is positive. In order to satisfy equation (23), dS/dx must be negative and the slope decreases in the downstream direction. Thus, the overall concaved longitudinal riverbed profile is also consistent with the theory of minimum rate of energy dissipation.

For irrigation canals, the discharge actually decreases in the downstream direction, i.e., dQ/dx is negative. In order to satisfy equation (23), channel slope must increase in the downstream direction. This is consistent with Lacey's slope regime equation. In equation (22), a decrease of discharge in the downstream direction will cause an increase of slope. Thus, stable regime canal profiles in Pakistan and India are also in compliance with the theory of minimum rate of energy dissipation or minimum stream power.

7. Sediment Discharge

The theory of minimum rate of energy dissipation, when applied to an alluvial system, can be interpreted as a river will adjust itself in such a manner that the given amount of water discharge and sediment load can be transported in a most efficient way, i.e., with a minimum rate of energy dissipation. This also means that for a given rate of energy dissipation and water discharge, a river should increase its efficiency by maximizing the rate of sediment transport. White, et al (1982) hypothesized that a river adjusts itself in order to maximize its sediment transport rate. They also found that the solution based on maximization of sediment transport rate or concentration is identical to that obtained from minimization of stream power for a given constant discharge as shown in figure 10. It is apparent that maximization of sediment transport rate or concentration is a tool available to a river or alluvial channel to minimize its rate of energy dissipation when and where sediment discharge can be treated as a dependent variable.

8. Dynamic Adjustment

An originally stable river system can become unstable due to engineering activities. According to the theory, once a river becomes unstable, its rate of energy dissipation will not be at its minimum value. However, the river will adjust itself in such a manner that its rate of energy dissipation can be reduced and regain equilibrium. The South Fork Forked Deer River in Tennessee, United States, was a stable equilibrium river. Between 1964 and 1966, the main-stem Forked Deer River from the confluence of the north and south forks to its confluence with the Obion River was enlarged and straightened. These activities made the river unstable and caused the river to adjust itself. Data collected by Robbins and Simon (1983), as shown in figure 11, indicate that unit stream power increases immediately after these man-induced activities and then decreases gradually as the river regains its equilibrium. Thus, the history of the variation of a river's rate of energy dissipation can provide us with an indication of its stability.

SUMMARY AND CONCLUSIONS

Mechanics can be classified into the vectorial and the scalar approaches. The basic difficulties and deficiencies of using vectorial approach alone in solving complicated fluvial hydraulic problems are explained in this paper. The scalar approach based on the theory of minimum rate of energy dissipation is used as a unified theory to explain some interesting phenomena commonly observed in alluvial channels and rivers. This study reaches the following conclusions:

1. The theory of minimum stream power and minimum unit stream power are special and simplified versions of the general theory of minimum rate of energy dissipation.

2. The variation of velocity distribution, bed form and roughness, channel width, channel geometry, channel pattern, and channel profile are all consistent with the theory of minimum rate of energy dissipation or its simplified versions. The validity of the theory is supported by laboratory and field observations.

3. The basic regime relationships can be derived directly from the application of the theory of minimum rate of energy dissipation. This theory can provide a rational basis for further development and improvement of regime equations for stable channel designs.

4. When a system is changed from equilibrium to nonequilibrium condition, its rate of energy dissipation will increase initially and then reduce gradually to regain equilibrium.

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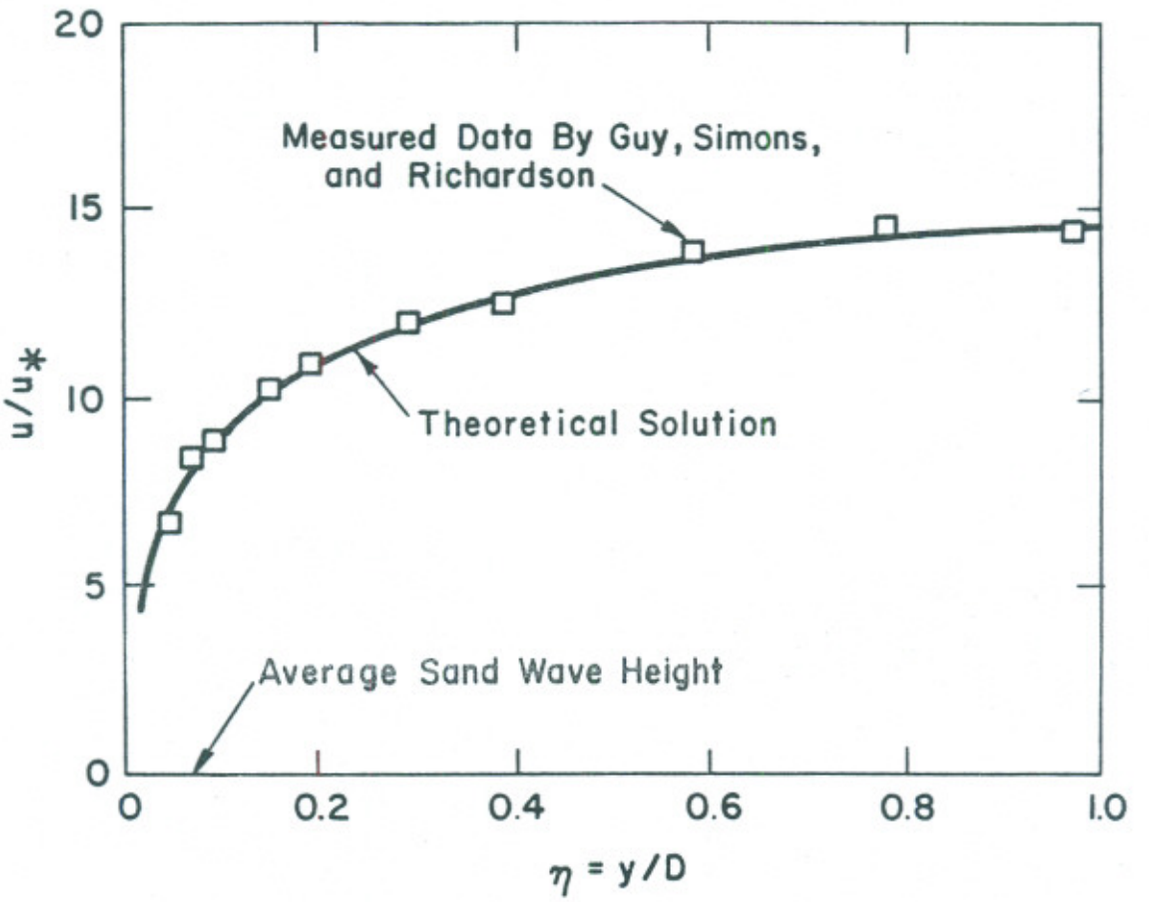


Figure 1. - Comparison between measured and theoretical velocity profiles with low sediment concentration (from Song and Yang, 1979).

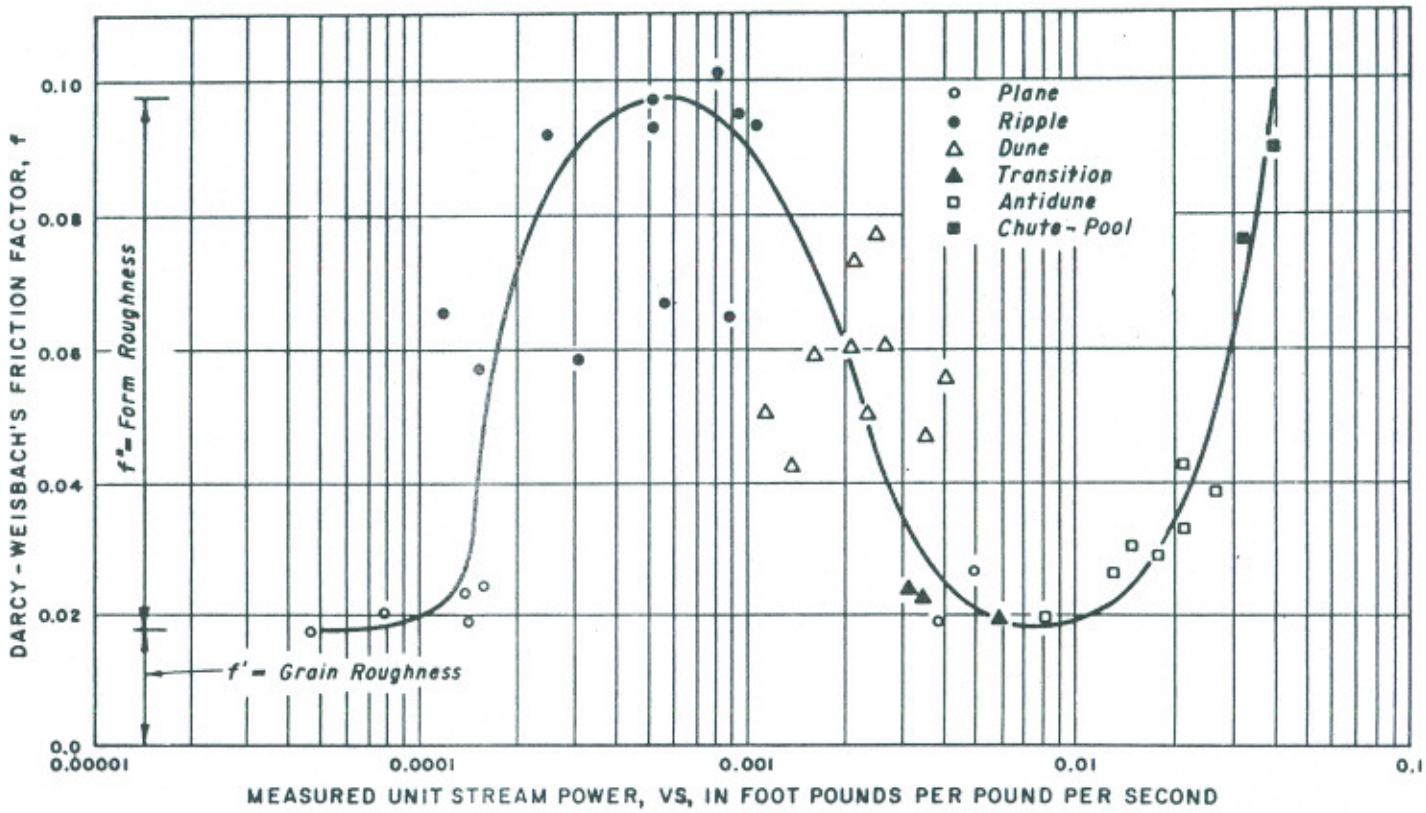


Figure 2. - Variation of friction factor with respect to unit stream power and bed forms (from Yang, 1976).

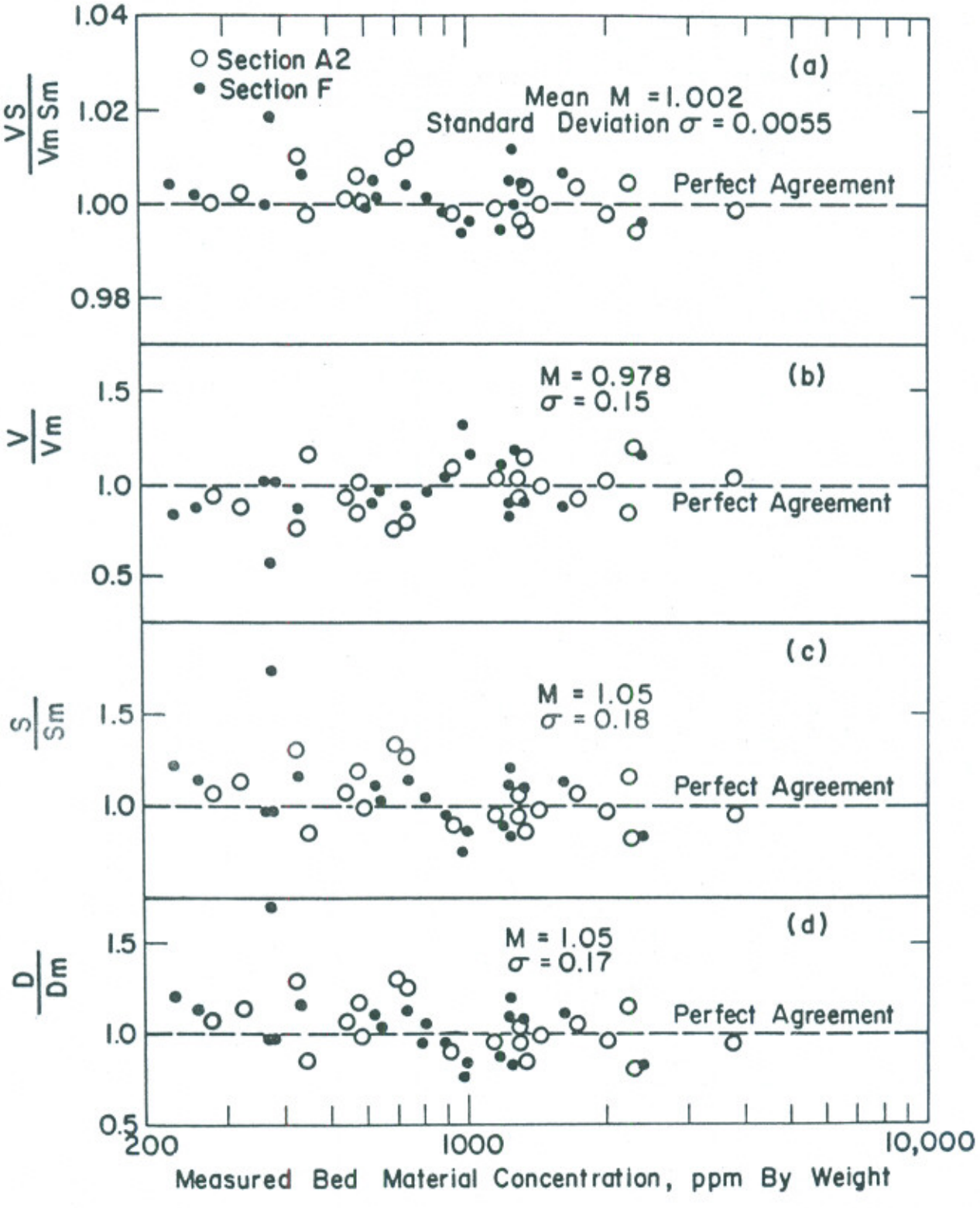


Figure 3. - Comparison between measured and computed results based on the theory of minimum unit stream power for the Rio Grande River near Bernalillo, New Mexico (from Yang and Song, 1979).

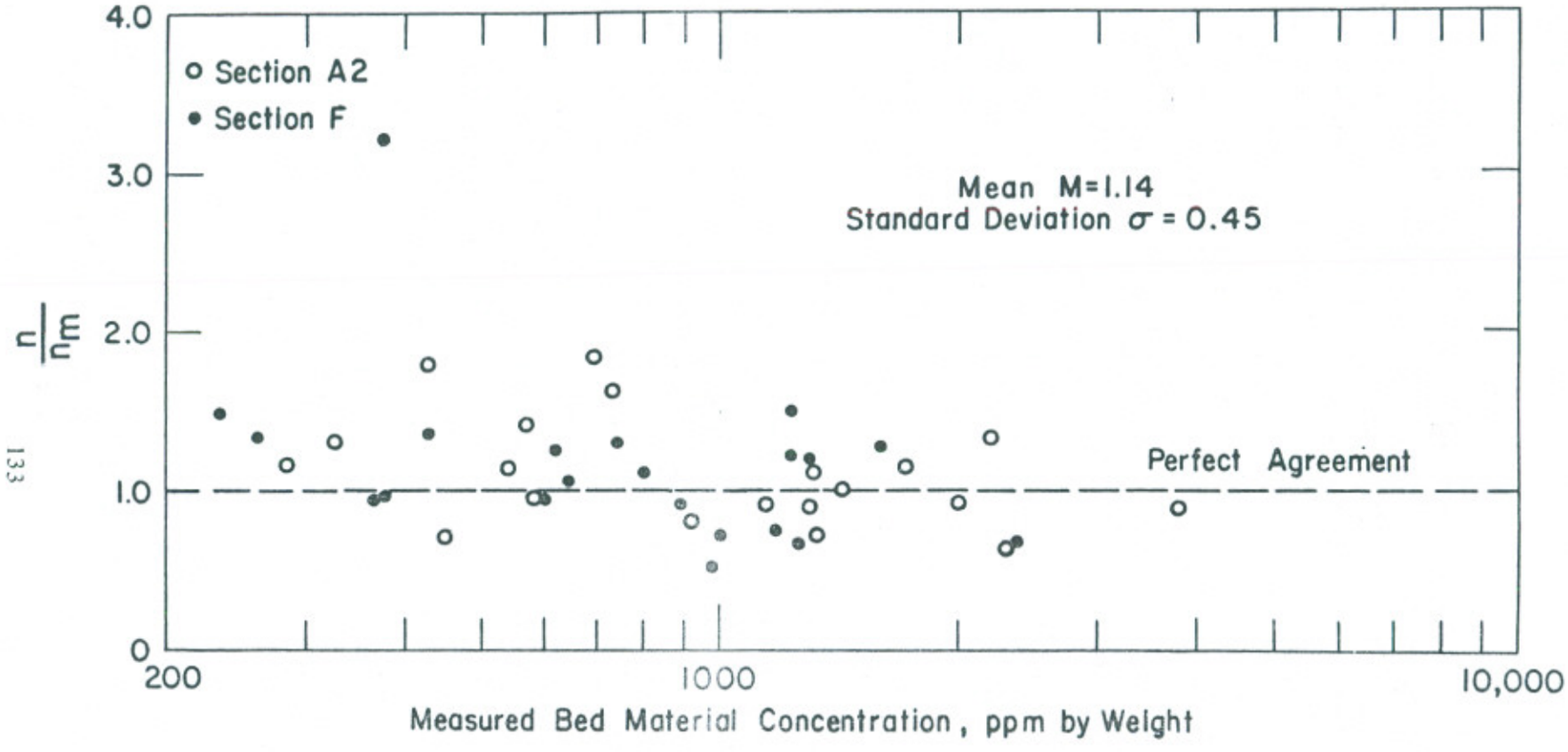


Figure 4. - Comparison between measured and computed Manning's roughness based on the theory of minimum unit stream power for the Rio Grande River near Bernalillo, New Mexico (from Yang and Song, 1979).

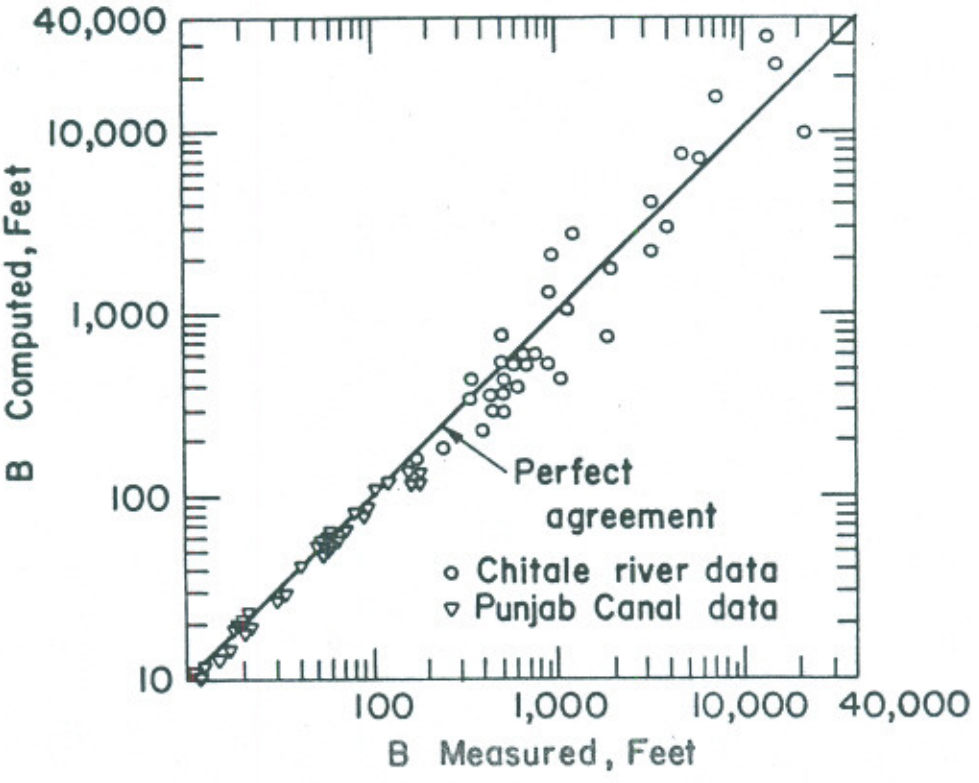


Figure 5. - Comparison between computed channel width and measured channel width (from Chang, 1979).

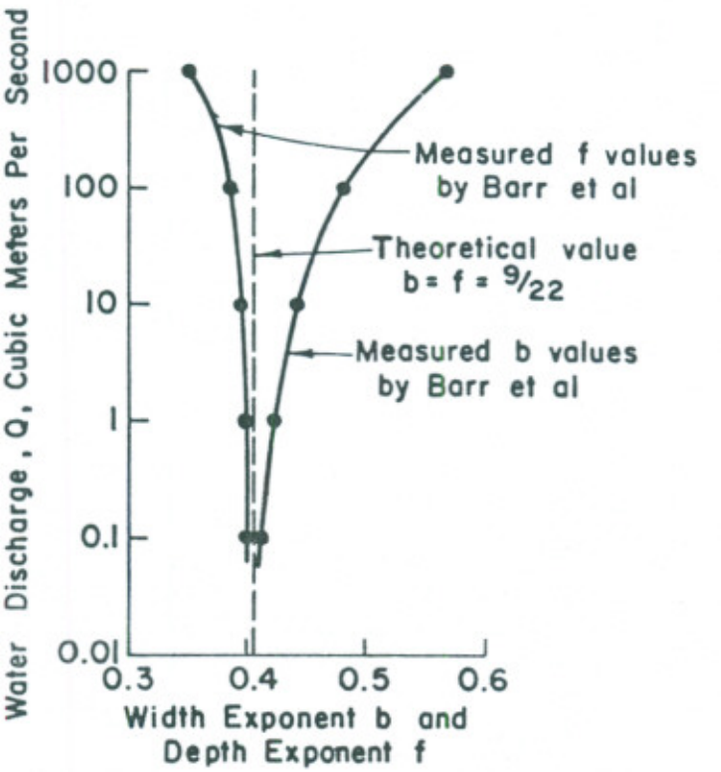


Figure 6. - Comparison between the theoretical values of b and f based on the theory of minimum rate of energy dissipation and the values measured by Barr, et al, 1980 (from Yang, et al, 1981).

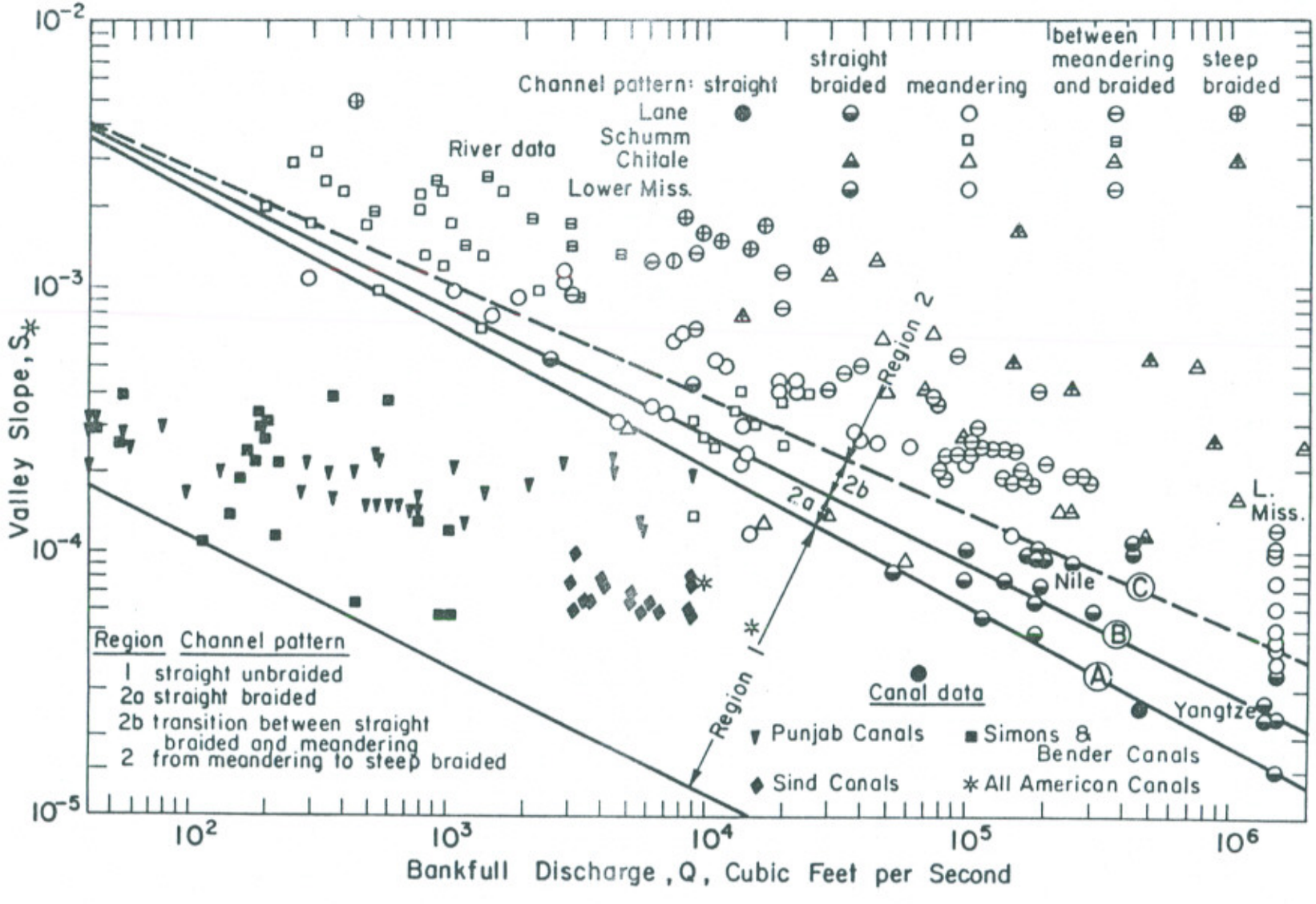


Figure 7. - Channel pattern for sand streams (from Chang, 1980).

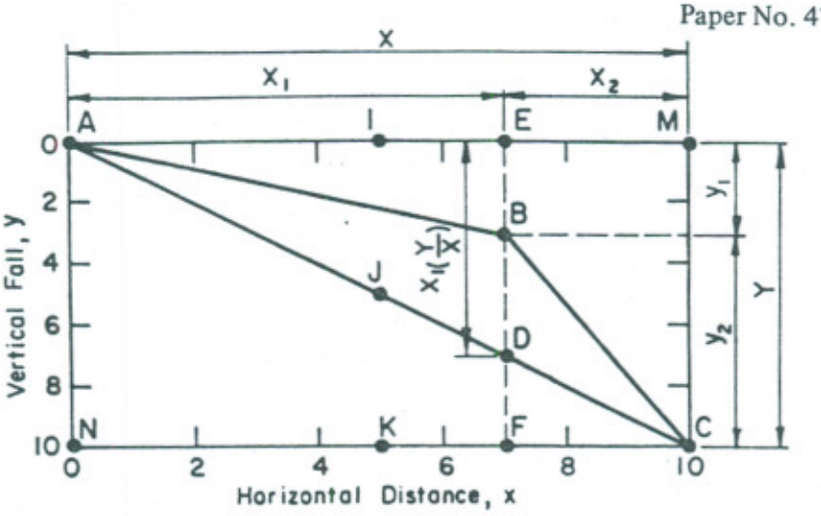


Figure 8. - Alternative paths of a stream in the vertical plane along its thalweg (from Yang, 1971 c).

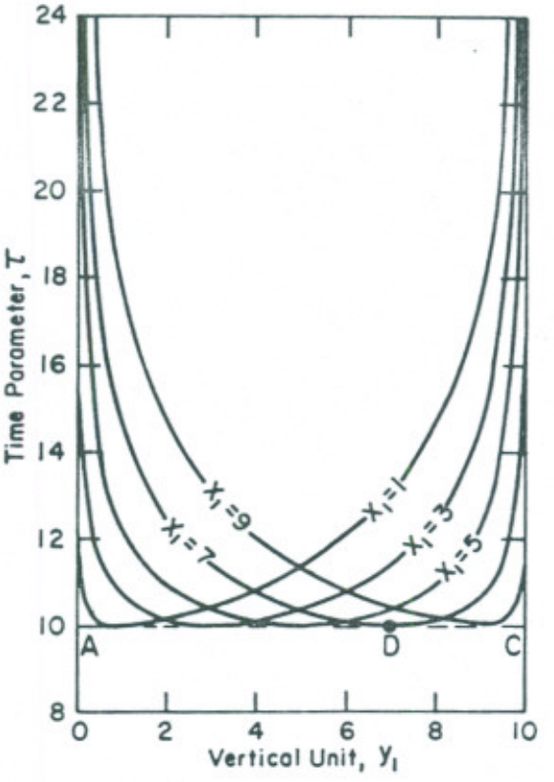


Figure 9. - Traveltime of a stream for different paths in the vertical plane along its thalweg (from Yang, 1971 c).

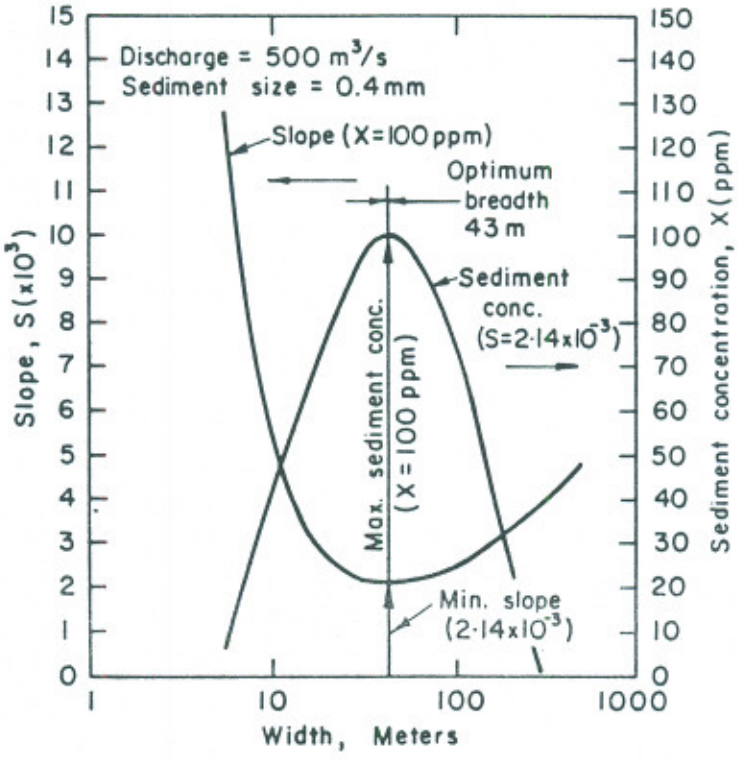


Figure 10. - Slope and sediment concentration versus width (from White, et al, 1982).

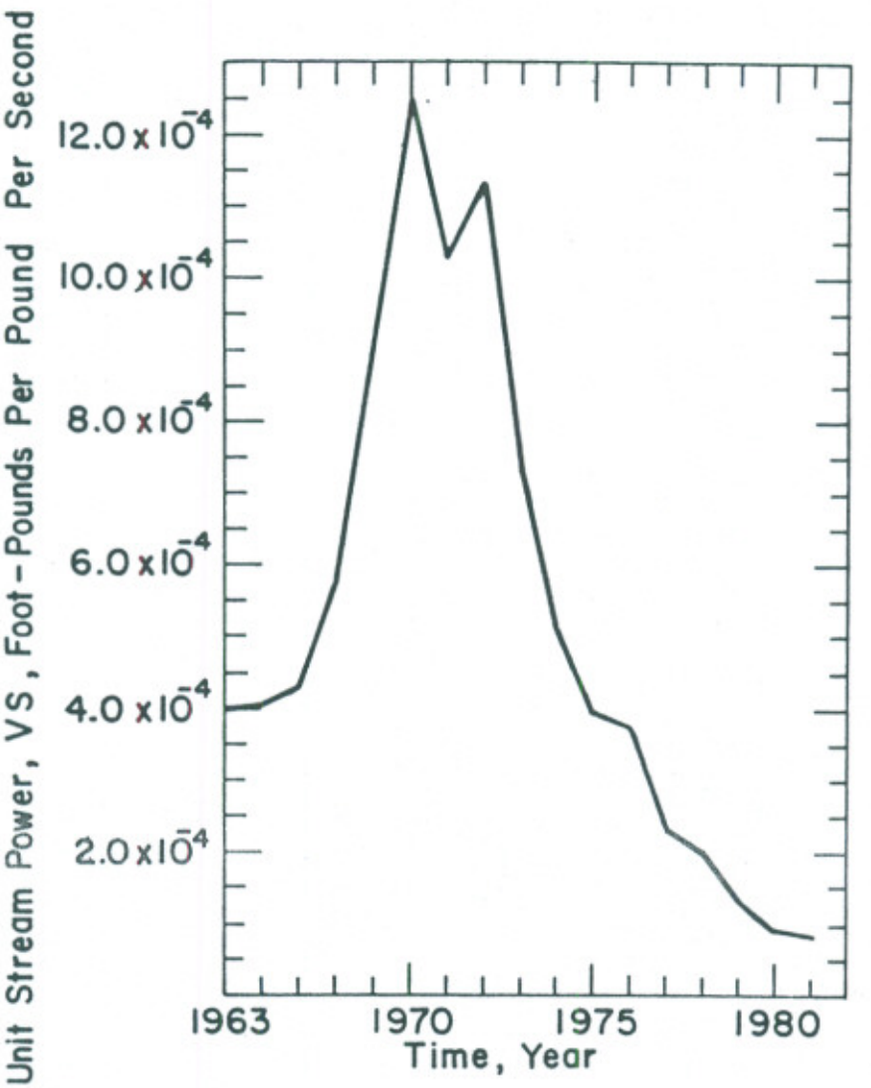


Figure 11. - Variation of unit stream power versus time for gauging station at Halls on the South Fork Forked Deer River, Tennessee, U.S.A. (after Robbins and Simon, 1983).