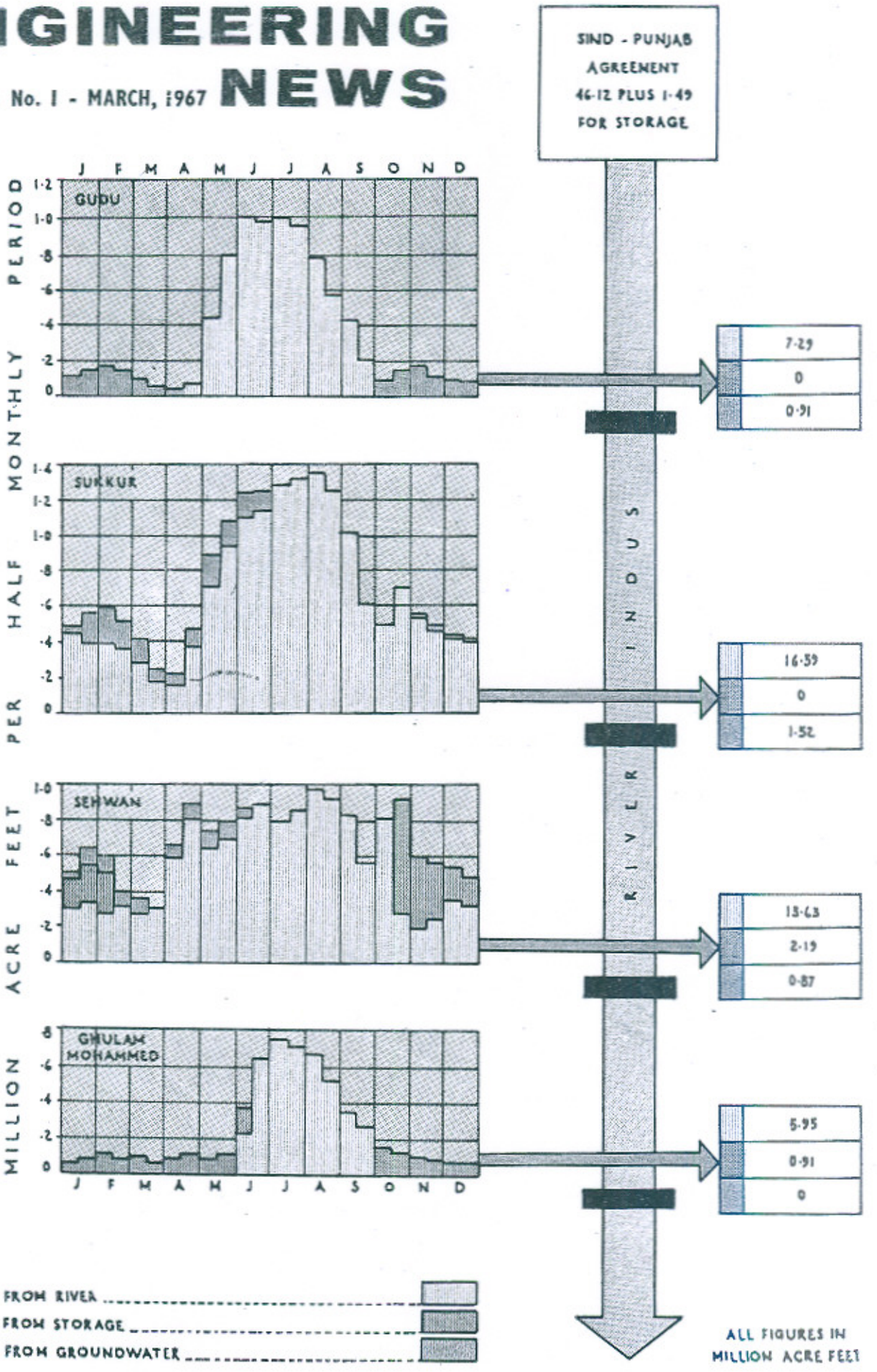
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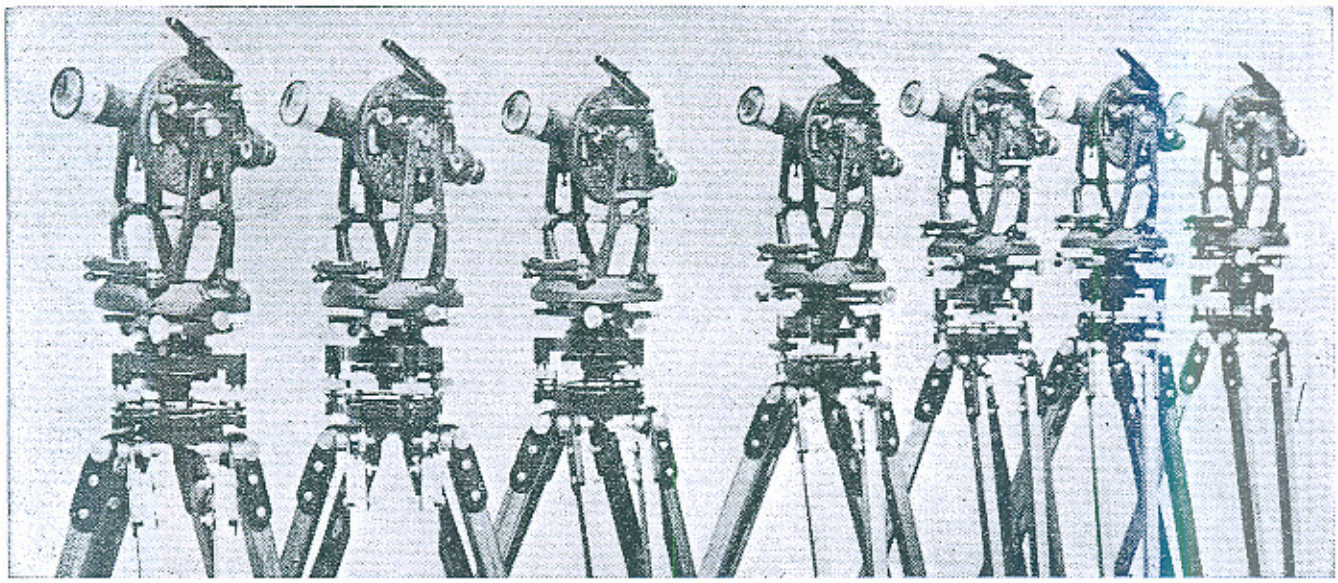
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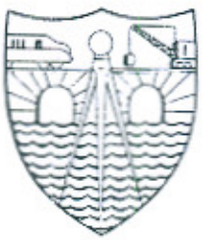
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"ENGINEERING NEWS" DELAYED

This issue of the Engineering News is going to the press rather late. This always happens in a country where there is not only dearth of good articles but also very often there are financial set-backs.

The Engineering News is issued by a staff of honorary workers whose official duties are innumerable and very often these overshoot to the extent that the collection of the material is delayed. This time it especially happened as the Chief Editor had several official assignments, which delayed the publication.

Another serious handicap is the finance. The share of the Engineering News out of the subscription of each member is Rs. 10. On the roll of the Congress there are more than 1,100 members. If every member be conscious to pay the subscription in time, there could be no financial worry to the journal but due to the inattention of the members, the due subscriptions are not paid in time to the Congress, so that the share of the Engineering News is delayed. It will perhaps be quite a surprise for the members to know that the total unpaid subscription by March 1967 amounted to about Rs. 38,000. The break up from various trades was Rs. 12,000 from Irrigation, Rs. 10,000 from B. & R., Rs. 4,000 from Electricity Branch, Rs. 1,300 from Railways, Rs. 2,220 from M.E.S. and Rs. 8,400 from miscellaneous services. Naturally the financial position of the Congress is becoming too tight particularly after purchasing the land for the centre of the Congress at a cost of Rs. 70,000

which has been paid to the Improvement Trust, Lahore. We have tried our best to get some help from the Engineering and Industrial Organizations but they calculate in terms of gains out of the advertisements. The instance of Indus of WAPDA is worth noting. In spite of a large body of consultants and contractors getting a large share out of the funds of WAPDA, the Indus which used to run on profit, now gets hardly a few advertisements even for its survival. The Engineering Congress is in quite a different position when compared to the advantages attained by the manufacturers and suppliers to WAPDA. Very few manufacturers and industrialists find a direct gain from advertisements in the Engineering News although this serves one of the largest community of Engineers who have to utilize the manufactured goods of the Industry.

We are trying to surmount these difficulties. A much quicker solution, however, lies with the members who if they discharge their responsibilities and pay the subscription in time, much of the worries of the Engineering News will vanish. We thus appeal to the members to help in clearing the Congress dues for the periods they have received the literature and other publications of the Congress. They have been receiving the Engineering News and at the same time we are sure they do like the publication to continue.

We hope for a quick and healthy response to this appeal.

Tubewell Pumping Machinery and their Trouble Shooting

By **Dr. NAZIR AHMAD**

Synopsis

We have started a series of articles on various engineering aspects of tubewells with regard to their performance, yield, troubles in operation and such other irformations, which might be useful to the engineers in general and to those installing tubewells for pumping water for irrigation purposes. In Vol. 11 No. 1 and 2 of the Engineering News, an article on measuring discharges of a tubewell was issued. It dealt with all the simple yet scientific methods of recording the discharge of a well. In this article we are discussing some aspects of the tubewell machinery and their troubles which appear while pumping ground water. The various points to locate the troubles are put forth. The strainer, its performance and troubles will be discussed in another article to be issued subsequently.

A TUBEWELL has two major components. One is the strainer and the blind pipe. Water flows into the strainer from the water bearing formation. The second component is the pumping machinery which draws the accumulated water from the strainer.

The pumping system has further two components. One is the source of power which is made available either through an electric motor, a diesel engine, a crude oil engine or even through the utilization of a tractor, or by any other mechanical device. The power works the pumping system which sucks and lifts the water.

Power Consumption for Lifting a given Discharge

The unit of measurement of power is foot lbs. per second. One cubic ft. of water weighs 62.5 lbs. If this much water is lifted through a height of one foot, every second, it will need a power of 62.5 ft. lbs. per sec. A common unit of power is called a horse-power. It is equal to the lifting of 550 ft. lbs. per sec. or 33000 ft. lbs. per minute. If a machine is hundred percent efficient, a horse-power will lift one cubic foot of water per sec. through a height of 8.8 feet.

$$\text{Lift in ft.} = \frac{\text{Horse-Power}}{\text{Weight of one cubic ft. of water.}}$$

$$= \frac{550}{62.5} = 8.8 \text{ ft.}$$

or

$$1 \text{ H. P.} = \frac{62.5 \times 8.8}{550}$$

Power is the work done per unit of time, and while pumping water it is expressed as $\text{WHP} = \frac{65.5\ QH}{550} = \frac{QH}{8.8}$. Where Q is the discharge in cubic feet per second and H is the total lift in feet.

If Q is expressed in gallons per minute then the above equation changes to

$$\text{W.H.P.} = \frac{8.33\ QH}{33{,}000} = \frac{QH}{3{,}960}.$$

On the basis of these equations H.P., needed to pump a given discharge, lifted through a given distance is worked out. It may be noted that these relations are on the basis of the assumption that the machine employed is 100 percent efficient. This is seldom the case. The output is always less than the input. The ratio of the two is called the efficiency and is expressed as

$$EP = \frac{\text{Output}}{\text{Input}} = \frac{\text{W.H.P.}}{\text{B.H.P.}}$$

where B.H.P. is the British Horse Power.

Both the power unit and the pumping unit have their own efficiencies. For an electric motor it is fairly high, being of the order of 80 to 90 percent. The maximum efficiency of a centrifugal pump is about 73 percent. The overall efficiency of the two components thus falls to 50 or 60 percent. In Table No. 1 below, assuming 50% efficiency of a pumping plant, the Horse-Power needed to lift different qualities of water through certain height is worked out.

TABLE No. 1.

Horse-Power required to lift different quantities of water to an elevation of 10 to 80 ft.

Efficiency of pumping plant 50% of theoretical.

Discharge in			Horse-Power required to lift water from (in ft.).					
Gall. per minute.		Cubic ft. per second.	10	30	50	60	70	80
100	..	0.22	0.5	1.5	2.5	3.0	3.9	4.0
250	..	0.56	1.3	3.8	6.3	7.6	8.8	10.1
450	..	1.00	2.3	6.8	11.4	13.6	15.9	18.2
700	..	1.56	3.5	10.6	17.7	21.2	24.8	28.3
900	..	2.01	4.6	13.6	22.7	27.3	31.8	36.4
1500	..	3.34	7.6	22.7	37.9	45.4	53.0	60.6

Type of Pumping Set

There are several types of pumps which are used to lift water. The common ones are the centrifugal, the turbine, the piston or reciprocating type and the air-lift pump.

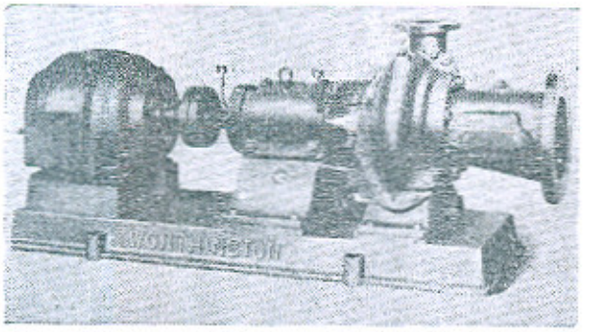


Fig. (1*a*) Horizontal Centrifugal Pumping Set.

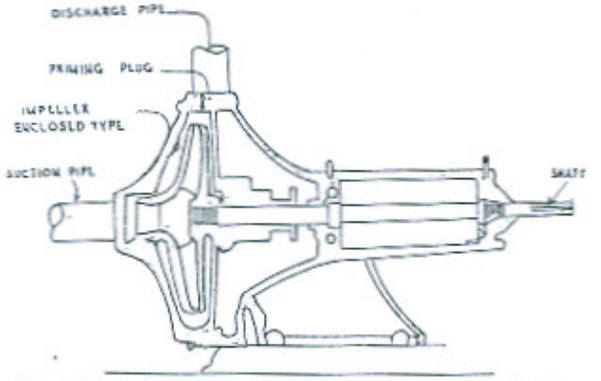


Fig. (1*b*) Horizontal Centrifugal Pump for Surface or Pit Installation.

In the earlier stages piston types or reciprocating pump was commonly used. A hand-pump often used to lift domestic water, is an example of the piston type pump. With a mechanical arrangement to move the piston up and down or in any other direction, fitted to this type of pump, it lifts water. Such pumps are generally called reciprocating. These are now no longer used commonly, although these have some advantages when c mpared with a centrifugal type of pump.

Centrifugal pumps are of recent development. See Fig. 1*a* and 1*b*. These create suction as a result of centrifugal force created by a rotating column of water. These pumps have high efficiencies and can operate under varied conditions of flow. In fact much of the progress of pumping ground water is a result of the development of centrifugal type of pumps. Both the reciprocating pump, and the centrifugal pump work on the principle of suction and have thus the same limitations of pumping water from great depths. These cannot pump if the water level exists at a depth of more than one atmospheric pressure which at sea level is considered to be about 34 ft. The atmospheric pressure varies with the site elevation and so is effected by the suction depth. An example of variation of atmospheric pressure with change of elevations above sea level and the practical suction lift for reciprocating and centrifugal pump is shown in Table No. 2.

TABLE No. 2

Suction Lifts.

Altitude above the sea-level (in ft.)	Barometric pressure at 32°F — Lbs. per sq. inch.	Equivalent water head in ft.	Practical Suction lift in ft. — Reciprocal	Centrifugal
Sea Level ..	14.70	33.93	22.0	15.0
500 ..	14.40	33.26	21.50	14.5
1000 ..	14.20	32.63	21.0	14.0
1500 ..	13.90	32.01	20.5	13.5
2000 ..	13.60	31.42	20.0	13.0
4000 ..	12.60	29.11	18.0	11.0

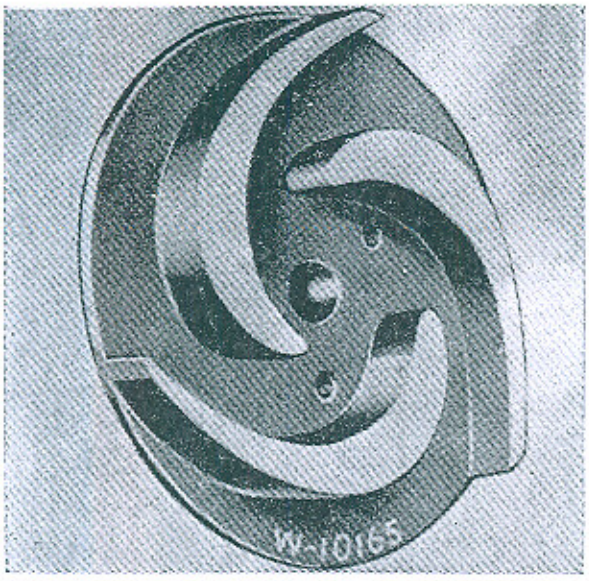


Fig. 2. Open Impeller.



Fig. 3. Semi-open Impeller.

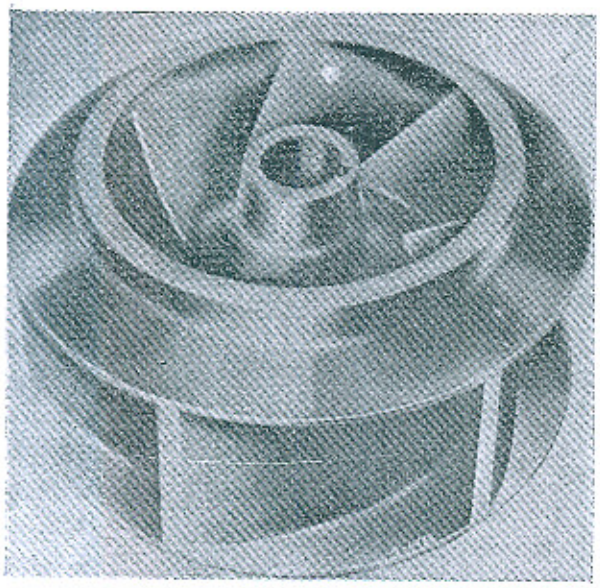


Fig. 4 Enclosed Impeller.

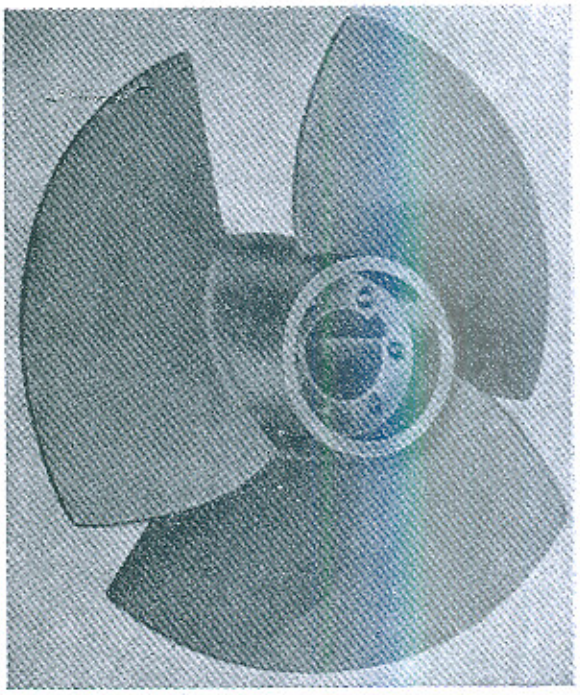


Fig. 5. Propeller-Type Impeller.

The centrifugal pump works efficiently with a suction of 13-15 ft. where as a reciprocating pump is efficient with suction up to 22 ft. If the suction lift increases, the efficiency of both the types of pumps decreases. It was soon found out that in order to attain the maximum efficiency, the location of the pump was to be as close to the water level as possible. This idea was the forerunner of the modern turbine or submersible pump. A brief information with regard to the essential components of the pumps and their working, together with the appearance of some trouble is explained in this note.

Centrifugal Pump

An essential component of this type of pump is an impeller. It is fixed to a shaft and is housed in a casing, and when it rotates at a high speed, it develops suction as a result of the centrifugal action. Some common types of impellers for these pumps are shown in Figs. 2, 3, 4 and 5. The simplest impeller consists of a few vanes fixed to a plate. The central hub is used for mounting them on a shaft (Fig. 2). Another type of impeller is the semi-open as shown in Fig. 3. It has a shroud or an impeller back wall. The third type is shown in Fig. 4. It is an enclosed impeller and is used very commonly. When a centrifugal pump is to be used for pumping sewage and there is a possibility of trash coming along with water, it is preferable to use propeller type impeller as shown in Fig. 5. As already stated, this type of pump works on the principle of suction which is created by a high speed of centrifugally rotating water. If there is slight leakage in the enclosing box or at the bushes, the pump fails to suck water. A leak-tight arrangement of the stuffening box and the casing of the impeller is most essential. The impeller has to be rotated at a high speed.

The power consumption and the discharge pumped out has a direct relation with the speed, width of the impeller and its diameter. Simple relations exist between the various parameters for a centrifugal pump. If we express the various parameters as :—

Speed = N
Power = P
Discharge = Q
Width of impeller = W
Diameter of impeller = D and
Total water head = H,

then the following relations exist between them.

$$\frac{Q}{Q_o}=\frac{N}{N_o}=\frac{D}{D_o}=\frac{W}{W_o} \quad \ldots.(1)$$

$$\frac{H}{H_o}=\left(\frac{N}{N_o}\right)^2=\left(\frac{D}{D_o}\right)^2=\left(\frac{W}{W_o}\right)^2 \quad \ldots.(2)$$

$$\frac{P}{P_o}=\left(\frac{N}{N_o}\right)^3=\left(\frac{D}{D_o}\right)^3=\left(\frac{W}{W_o}\right)^3 \quad \ldots.(3)$$

$$\frac{Q}{Q_o}=\frac{N}{N_o}=\sqrt{\frac{H}{H_o}}=\sqrt[3]{\frac{P}{P_o}} \quad \ldots.(4)$$

The subscription zero refers to the original conditions of the pumping set for which the characteristics are known.

(1) The discharge pumped out is directly related with the speed, *i.e.*,

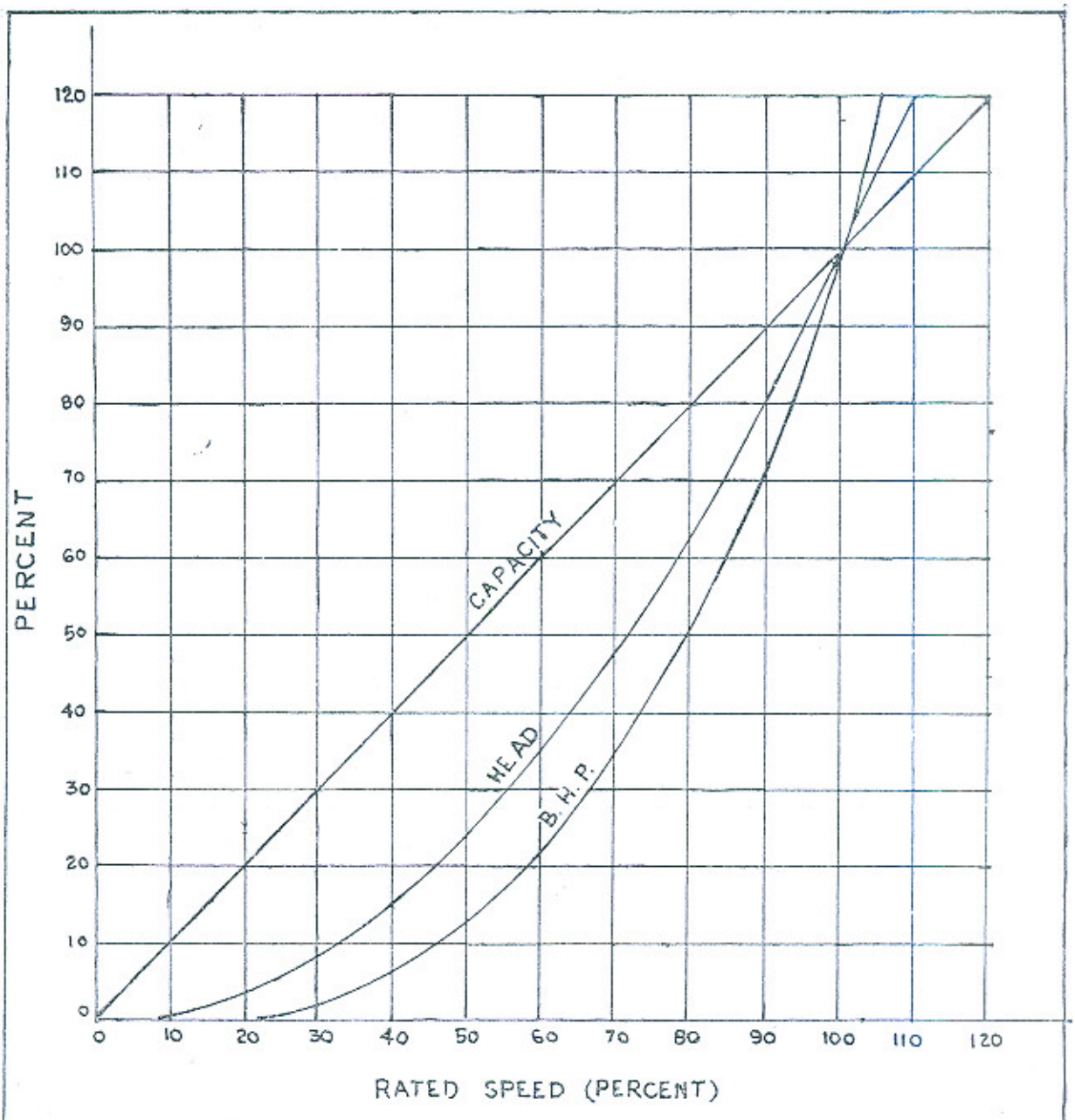
$$Q=Q_o\times\frac{N}{N_o}$$

(2) The total head or the lift is proportional to the square of the speed :

$$H=H_o\times\frac{N^2}{N_o^2}$$

(3) The power is proportional to the cube of the speed :

$$P=P_o\times\frac{N^3}{N_o^3}$$



EFFECT OF SPEED CHANGE ON CENTRIFUGAL PUMP PERFORMANCE.

FIG.- 6

We can express these statements in the form of a diagram. In Fig. 6 the percentage of rated speed is plotted against percentage of capacity, head and horse-power.

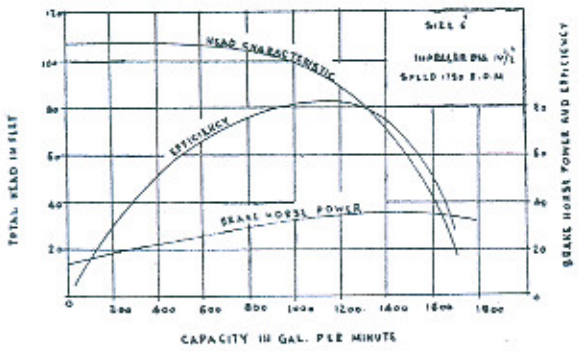
Evidently the speed of a centrifugal pump is an important parameter effecting the total head, power and the discharge. For each pump the most efficient speed of rotation is worked out. These very equations conclude that :—

(1) Power will change as the cube of impeller diameter.
(2) The head will change as the square of the impeller diameter, and
(3) The discharge will change in direct proportion to the impeller diameter.

These relations help in designing a centrifugal pump.

Characteristics of a Pumping Set

It is customary that the manufacturers determine the operation characteristics of a pumping set. Each manufacturer supplies the characteristics performance of his pumping set for a given speed, efficiency of the pump, delivering a given quantity of water. Such typical characteristics curves are shown in Fig. 7. For a specific discharge and the head, the efficiency is the maximum. From the curves it is apparent the change in power is not as much as the change in yield. Even with zero discharge the pump consumes a certain amount of energy. The head characteristics remains constant for a considerable range of the output.



CHARACTERISTIC PERFORMANCE CURVES FROM TEST RECORDS

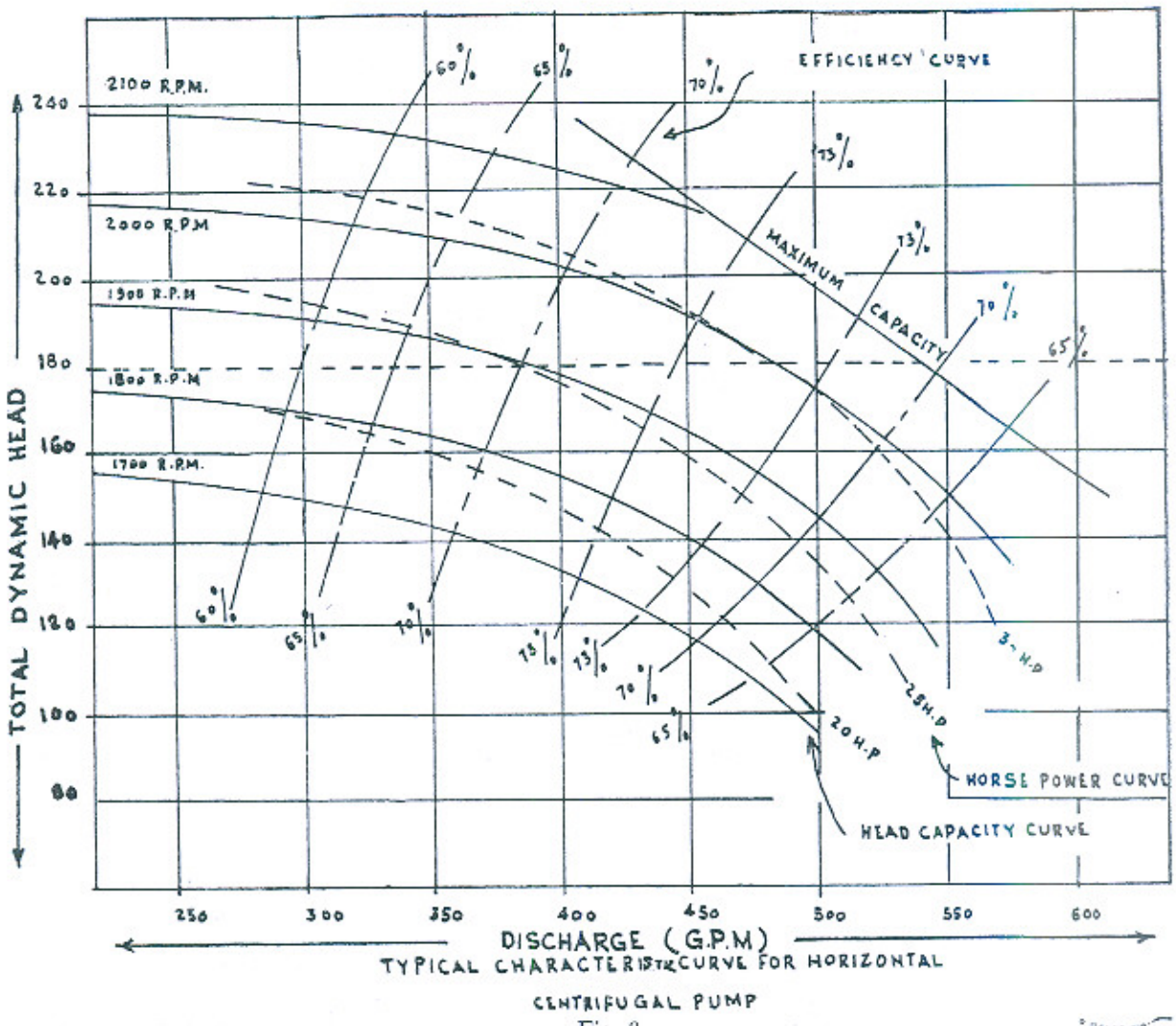
Fig. 7.

A change in the speed of the centrifugal pump, however, results in a change in the yield as well as the total head. It affects the efficiency of well which is maximum at a particular order of the parameter. Such typical characteristics curves for a horizontal centrifugal pump are shown in Fig. 8. From these curves, the selection of a required pump of a given speed can be made to pump the required discharge to the known head. Suppose it is desired to pump 450 gallons per minute to a head of 140 ft. Then from the curves it seems that a 20 H. P. Motor of 1800 R.P.M. will be just suitable. The efficiency of pumping will be 73 percent. These curves are thus helpful to select the motor of a given H. P. and speed for the desired quantity of water to be pumped to a known head.

TROUBLES WITH CENTRIFUGAL PUMPS

Importance of Priming

As already indicated, these types of pumps work on the principle of suction and any leak, anywhere, results in the inflow of air. The pump stops lifting water. Before a centrifugal pump can start functioning, all air has to be replaced with water from the suction pipe, reflex valve and the impeller. This is called priming of the pump. Water is added to fill the whole pumping box. There should be no leakage on the suction side. If there is slight leakage, the pump will fail to yield. Majority of troubles with centrifugal pump can be traced to suction



TYPICAL CHARACTERISTIC CURVE FOR HORIZONTAL CENTRIFUGAL PUMP

Fig. 8.

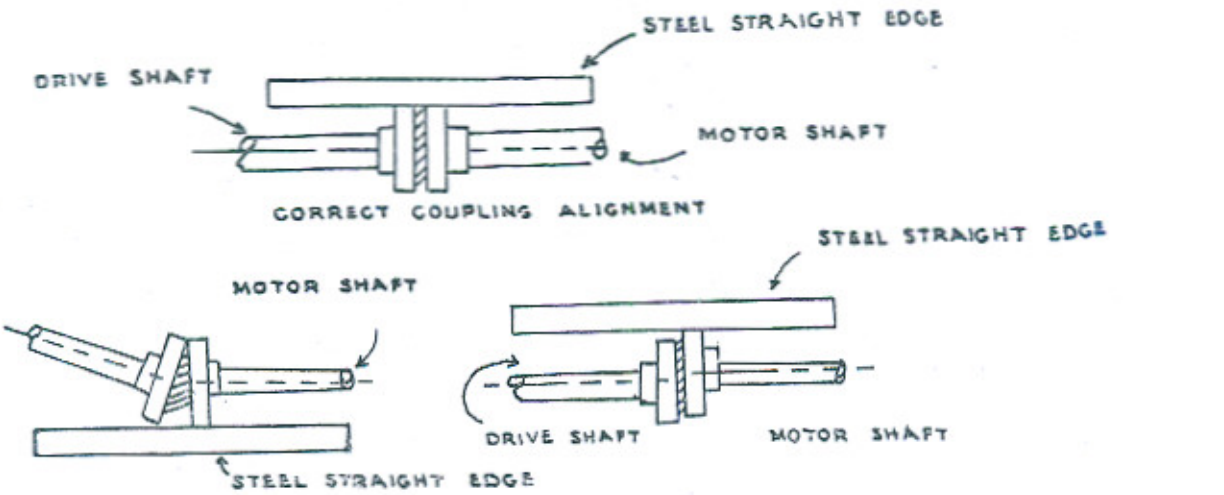
line, its joints, elbows, foot valve and other accessories. Leakage can occur in the threaded connection of the suction line, through the gasket of the flanges or even through the shaft seals where packing may be worne out or improperly greased. All leakage at any place on the suction side must be eliminated before a centrifugal pump can start yielding water.

Defects of Alignment

The pumping set is connected to the motor through a shaft. Both are connected in such a way that there is no loosening of the coupling. If the alignment is incorrect as shown in Fig. 9, it can result in increase consumption of power, vibrations and noise. A bent shaft or a worne-out bearing will soon result in the above defects. The coupling should be perfect.

Improper Speed

The yield of a pump is directly connected with the rated speed of the pump. If the speed is insufficient, the pump will yield insufficient water and will develop insufficient pressure. The fall in speed or the use of too much power by the pump can be due to too tight stuffing box, bent shaft or misalignment of the pump and driving shaft.



COUPLING ALIGNMENT

Fig. 9.

Keeping exact speed-as indicated on the pumping set, is essential.

Wrong Direction of Motion of Motor

Wrong connection of the motor can result is rotation in the reverse direction. There will be no yield of water under that condition. Direction of rotation of the motor should be the same as indicated for the pump.

Leaks at Stuffing Box

When the packing is worne out, improperly lubricated or incorrectly inserted, leakage takes place through it.

Presence of Foreign Materials in the Impeller

Although in case of tubewell there is little possibility for foreign materials to enter in the impeller but this can happen when sewage water is pumped. Any foreign material clogging the impeller, wears them out, resulting in considerable reduction of the suction lift.

Cares for the Reflex Valve and the Check Valve

Each pump has to be provided with a foot valve or a reflex valve. It normally should be below watertable. The valve must in all cases be leak tight so that when water is added to the pump no water may leak through the flap of the valve. If it happens it might be difficult to prime the pumping set. All water added will leak through the valve and through the strainer into the formation. Although it is preferable to locate the reflex valve below the watertable but sometimes due to the fluctuation in the level of ground water, the reflex valve may remain above the water. If there is no leakage in the valve th·n repeated filling and working of the pump will remove the air.

The practice of installing a check valve in addition to a reflex valve is not common. It is usually done in a case of hand-pump. It will be preferable if such a valve is provided in the tubewell also. The type of a check valve suggested is shown in Fig. 10. It may

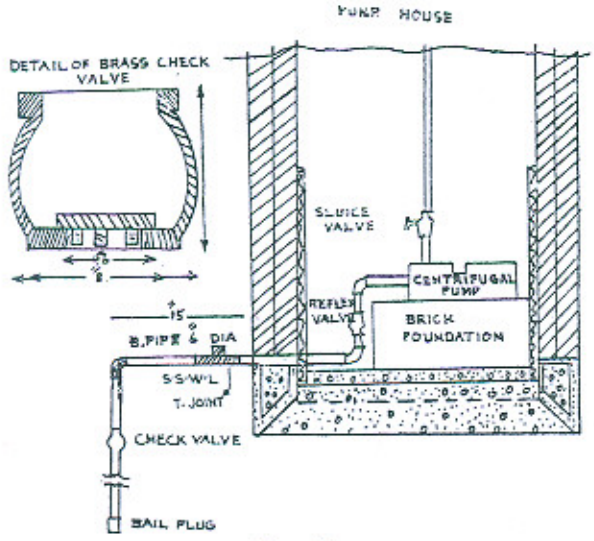


Fig. 10.

be located 8 to 10 ft. below the watertable and fixed in the blind pipe. This valve can help in priming even when the watertable falls below the reflex valve or it develops a leakage.

Vertical Centrifugal Pump

A horizontal centrifugal pump is very satisfactory but there are cases when the water level fluctuates and submerges the motor. This may happen while open pumping of sewage, the level of which changes with sudden inflow of large volume of water. In such cases it is preferable to install a centrifugal pump in a vertical direction. Its motor is fitted at top, a few feet above the pump to which power is transmitted through a shaft. A common type of such a pumping set is shown in Fig. 11.

Advantage of a Centrifugal Pump

A centrifugal pump is easy to manufacture. It has very few components and works under a wide range of conditions. All its components are more or less visible and approachable so that any defect can be easily detected. In West Pakistan, its manufacturing has

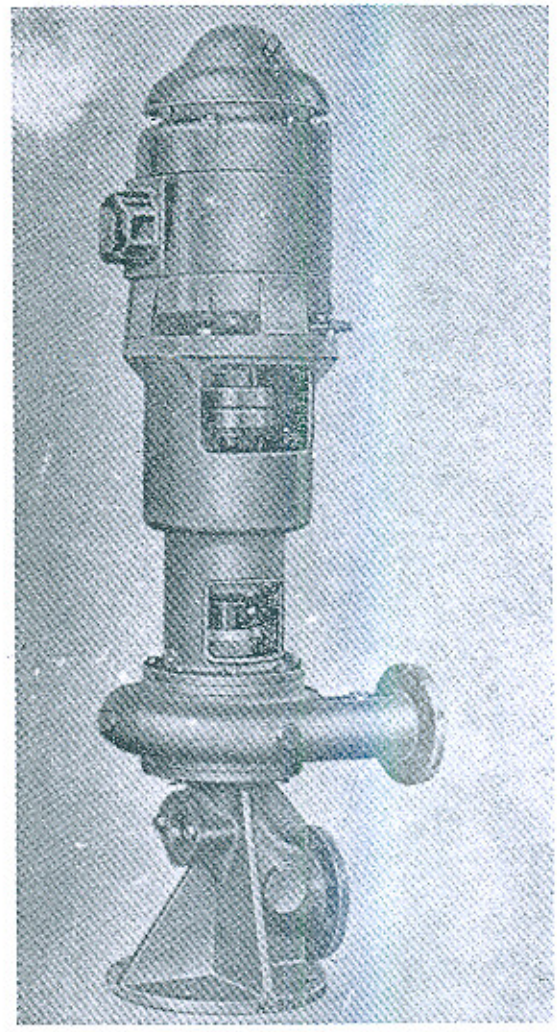


Fig. 11. Vertical Centrifugal Pump with Motor.

become very common, and these are being constructed in all big and small cities of the country. Pumps of BECO, K.S.B., Ittafaq etc., are now fairly standard and can compare with any foreign manufactured pump. These are relatively cheap, easy to construct and efficient to operate. Where electric power is not available these can be coupled to a diesel pumping set which are also being produced on a large scale in the country (Fig. 12) by BECO and other such manufacturers of the country. Very often a crude oil engine or a tractor is available as a source of power. The pumping set available for this purpose is shown in Fig. 13. The power is

transmitted through a belt. When water is not needed the engine can be put to various jobs. It can be used for grinding wheat and corn, for cutting fodder and such other purposes. Due to the multiple application of crude oil engine, farmers utilize this method for pump-

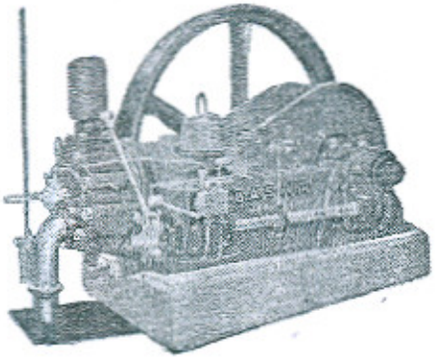
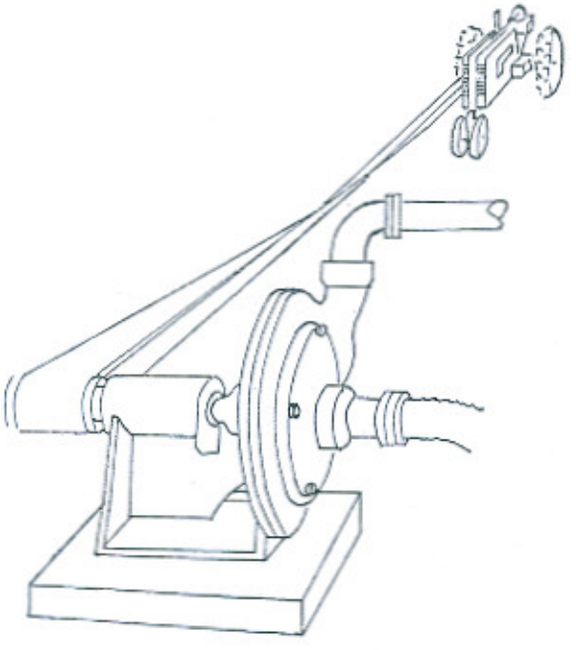


Fig. 12.



CENTRIFUGAL PUMP WITH FLAT BELT DRIVE

Fig. 13.

ing water to a great extent although electric-driven pump are cheaper and easy to operate. These are a few reasons for such a large-scale development of the horizontal centrifugal pump.

Using Centrifugal Pump with Multiple Strainers

There is another great advantage of using a centrifugal pumping set. It works on the principle of suction. It can be located in a pit close to the watertable. It can be connected to one or more strainers, located close by as shown in Fig. 14. In this way multiple strainers which have innumerable advantages can be connected together.

There are sites where the top ground water in shallow depth is of good quality and saline water exist in deeper regions. It is only possible to install shallow strainer. By using this type of pump more than one strainers, some distance apart can be connected together and any amount of discharge can be pumped. In this system even a four cusecs capacity pump can be installed as the depression will remain within the suction limit. This is another great advantage of using this type of pumping set. Sufficient volume of water can be made available to work the pump at its maximum efficiency.

So far a durable, long-life tubewell is yet to be designed. A tubewell strainer very often shows two to five percent fall in its yield per year. It is necessary to bring back the discharge to the original designed order. So far, chemical or mechanical renovation methods have not proved quite satisfactory but a centrifugal pump or a reciprocating pump is the only means by which more than one strainer can be connected together. If one strainer gets choked and gives low order of water, then another can be installed, a short distance away and connected to the same pumping set. This idea of multiple strainers is explained in Fig. 14 mentioned above. Utilizing of this principle will never fail to pump water, so long as the pumping set is in order. The yield can also be

INSTALLATION OF MULTIPLE SHALLOW T/W AT CHAK 95, KHANEWAL

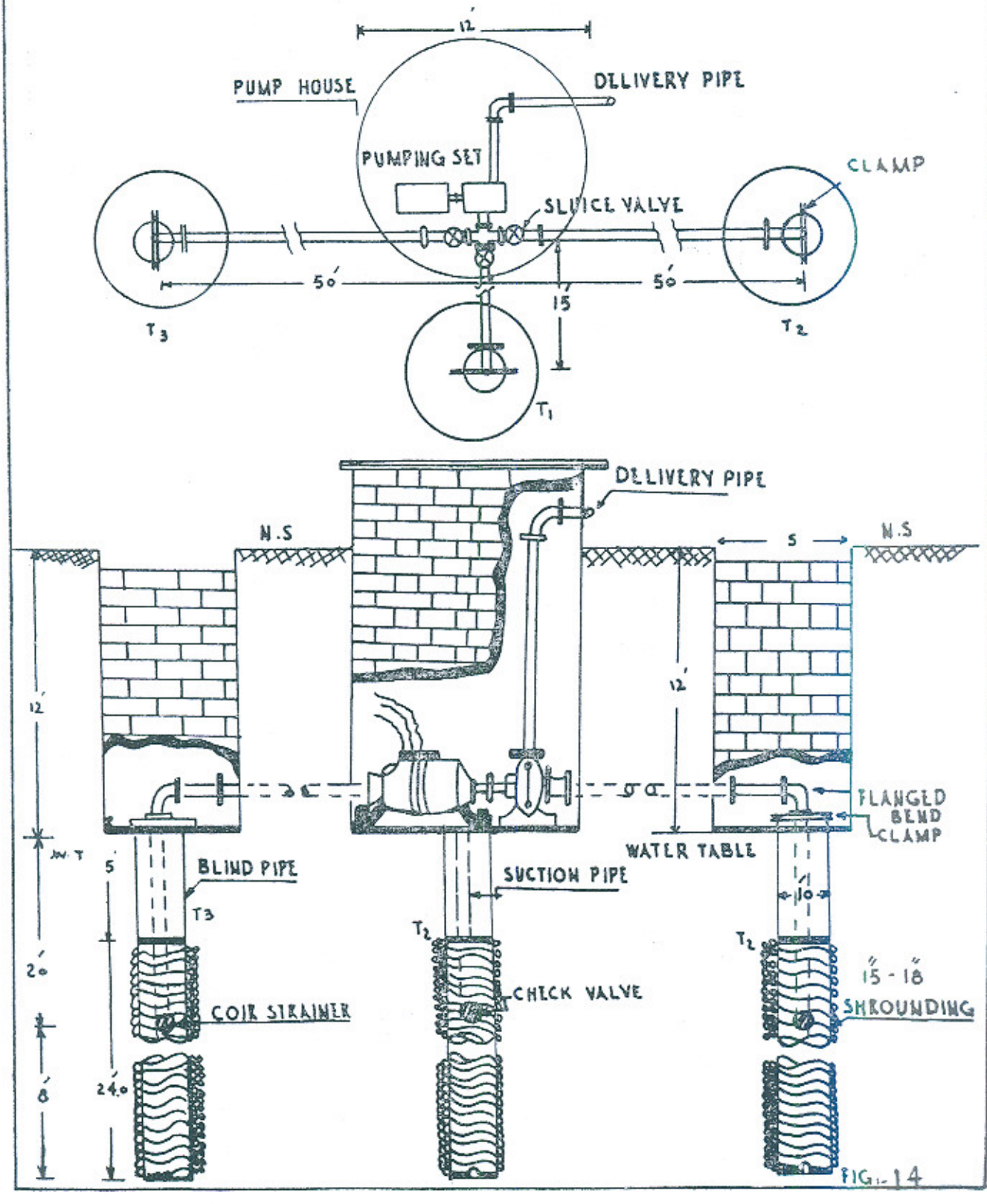


FIG.-14

kept to the maximum capacity of the pump. If one strainer fails it can be cut out of action and the other strainers will continue to yield water. This is possible only with a suction type pump. So long as the water table exist at a depth of 15-20 ft., pumping with a centrifugal pump is economical for this country.

Limitation of the Capacity of a Centrifugal Pump

It is a practice in this country to install a tubeweli by boring with a casing pipe or by excavating pit up to 18 to 22 inches in diameter. As the cost of boring increases with increase in diameter of the bore, the local farmers, in order to keep down the cost, use a boring pipe of 10 to 12 inches in diameter. The tubewell strainer is of a still smaller diameter, so that while pumping one cusec a depression of 8 to 12 ft. is created. This depression is a function of the water yielding capacity of the formation and for fine grade of medium sand it is about 8 to 12 ft. per cusec. In a centrifugal pump, the maximum suction practicable is of the order of 20 to 25 ft. Thus by a centrifugal pumping set a yield greater than 2 cusecs is not easily possible provided we increase the number of multiple strainers. In this system the interference of closely located strainers is a disadvantage. We thus limit the yield of a tubewell coupled with a centrifugal pump to two or two and half cusecs at the most.

Damage to the Impeller Working in Saline Water

It may be noted that cast iron is somewhat resistant to corrosion. It does not get corroded so quickly as wrought iron. Cast of pumping is justified only when the water is usable. Ground water with salts up to 1000 ppm. can normally be utilized for crops production without mixing with canal water. Ground water containing salts up to 3000 ppm. cannot be used economically for irrigation except by mixing with canal water to lower the salt percentage to less than 1000 ppm. If the salts present in water are more than 3000 ppm. then this water cannot be utilized for crops as the mixing ratio becomes uneconomical. For such water with salts less than 3000 ppm. impellers and casing of cast iron can be used. If the salts in ground water exceeds 3000 ppm., then the impellers start getting damaged. A specimen of the damaged impellers is shown in Fig. 15. For saline water having salts more than 5000 p.m., impellers of bronze only are satisfactory.

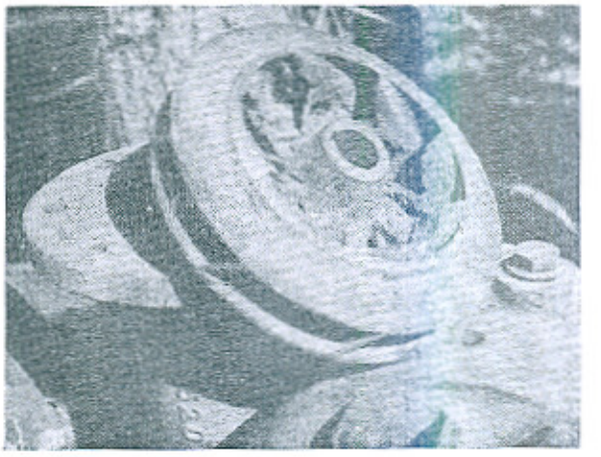
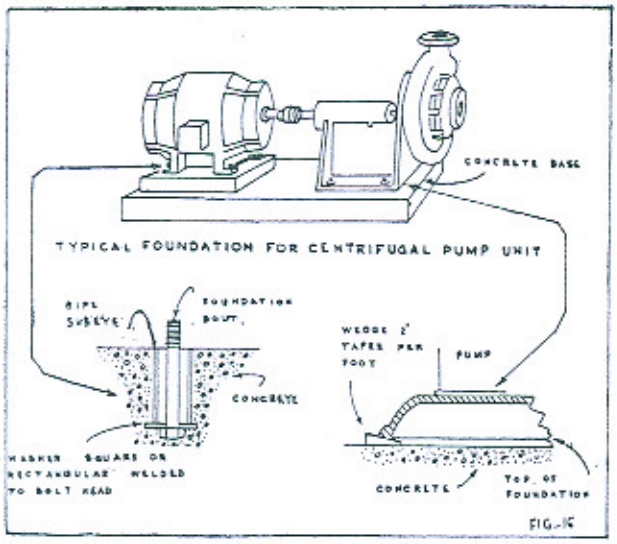


Fig. 15. Damaged Impeller.

Fixing the Centrifugal Pump

In order that a centrifugal pump may continue to operate for a long life, it should be correctly located having a good foundation and properly aligned. It is a practice to construct a pump-house and to install the pumping set as close to the water table as possible. The bed plate of the pumping set can be fixed to the concrete with bolts and nuts. The position of the bolts is



marked and then these are embedded in the concrete as shown in Fig. 16. After the cement has set, the pump-house is placed on the concrete foundation and it is bolted with it. Care is taken that the position of the pump remains in level at all times. If there is some defects, small wedge tapper pieces as shown in Fig. 16 can be used to raise the entire pumping unit. The wedges are left in place and covered by concrete.

Determining Head for Centrifugal or for Deep Turbine Pumps

The total lift in case of a centrifugal or a turbine pump is determined by measuring the following points:—

1. The depth of the water level inside the strainer when the pumping starts from the centre line of the pump.
2. The height of the discharging point of water above the centre line of the pumping set.
3. The total length of the pipe above the

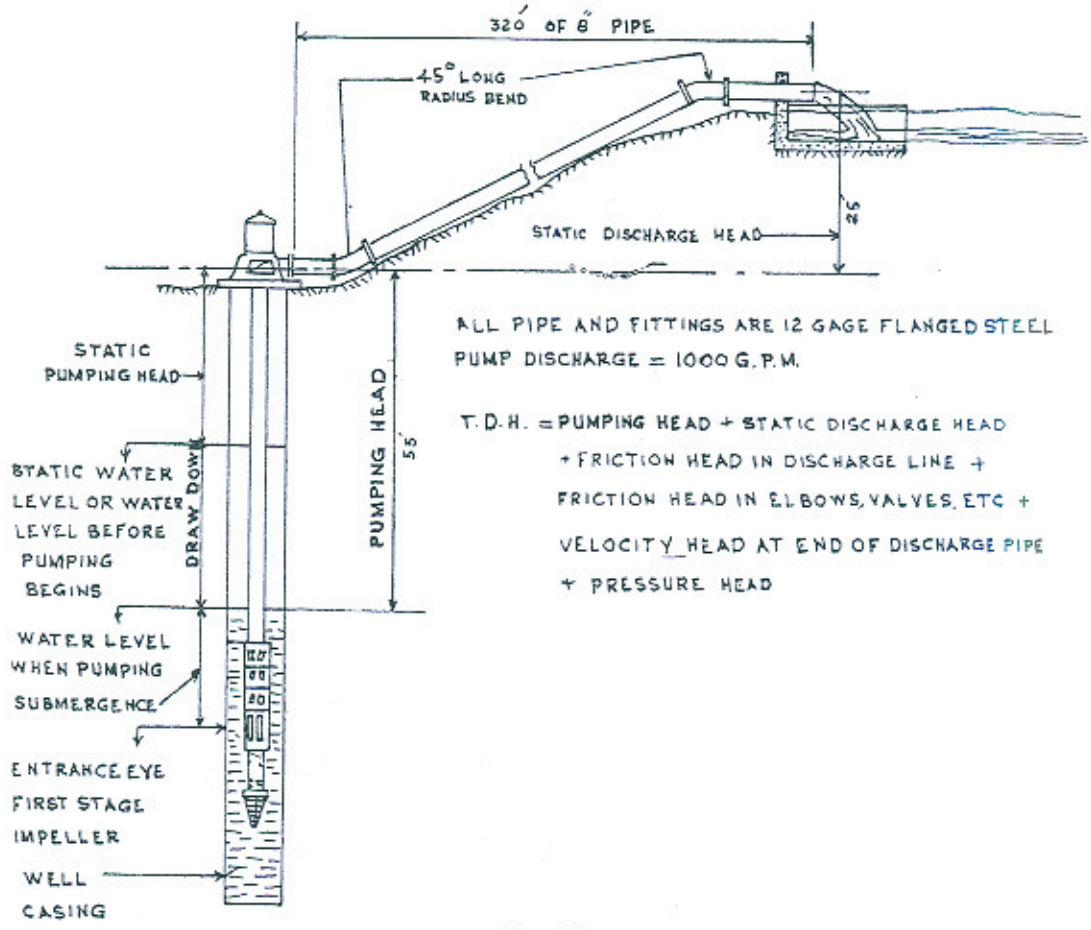


Fig. 17.

centre line of the pump and with reference to the diameter of the pipe and the discharge being pumped out using the frictional coefficient, the head loss is determined and this is added to the total head.

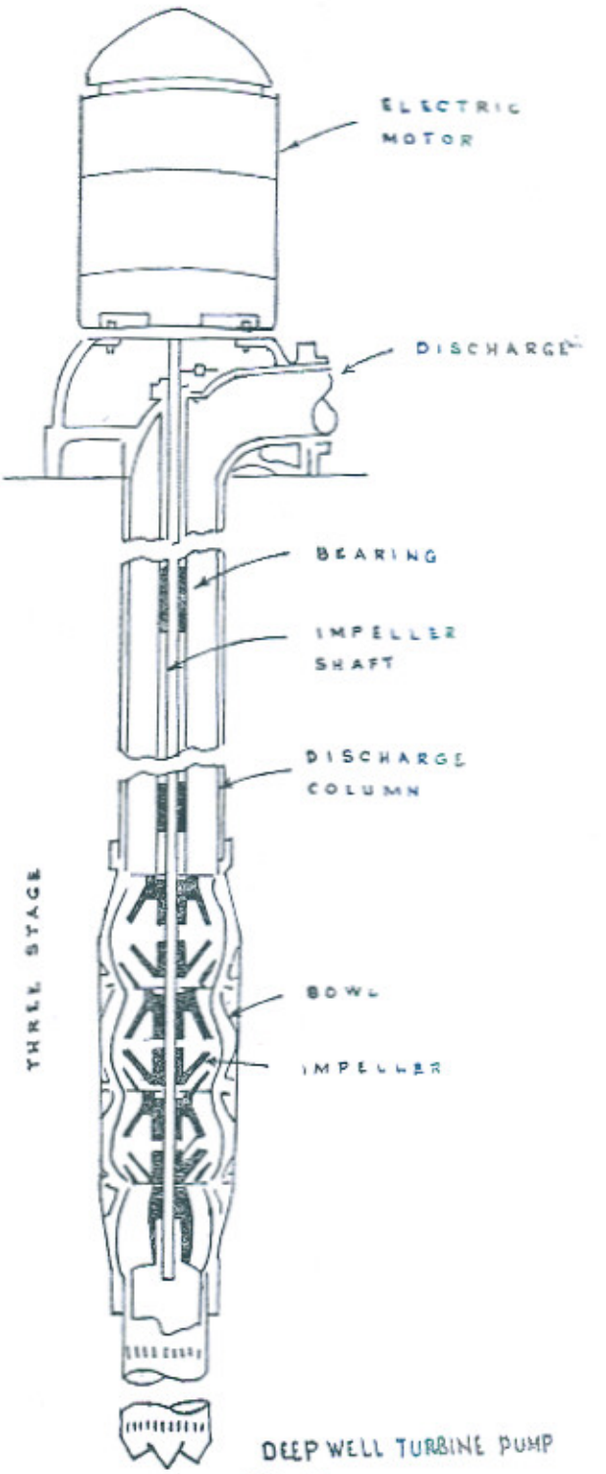
4. Determine the type of the bends, find out their number and determine the total head loss and add it to the total length of the pipe.

5. Determine the velocity head on the basis of the discharge taking place at the end of the pipe.

All these lifts are added together to determine the actual total head. An example of these calculations is shown in Fig. 17.

Turbine Pump

Where the ground water level is deeper than 20 ft., it is preferable to use a turbine pump. In fact a turbine pump is a submerged centrifugal pump. Its impellers are located deep in water, housed in a pipe, and are generally small in diameter. For this reason its pumping capacity is small. It thus becomes necessary to put in more than one impellers to increase its output. Very often two and three impellers can be installed to a turbine pump. These are called stages of a pump. The impellers are located several feet below water, although water can also be sucked if its level is below impellers. Under that condition, priming, so very necessary for a centrifugal pump, has to be done. However, it is very seldom that a turbine pump works on suction. The bowels are always kept so low as to work in a cushion of water. The head produced by a turbine pump is proportional to the number of stages of its impellers. A two stages pump will produce twice the head as a single stage pump. A turbine pump is equipped with either closed impellers or open impellers similar to those shown in Fig. 4 and 5 for a centrifugal pump. Such a pump develops a high pressure. There can be more than one such impellers fixed to a turbine (Fig. 18). A second type of



DEEP WELL TURBINE PUMP
Fig. 18.

impeller is like the propeller blades of a boat. These are set on the shaft at angles determined according to the required head and speed of the motor. The water is moved up as a result of lift given by the propeller blades. In this case the direction of flow does not change as it does in a centrifugal pump. The spiral motion of water resulting from the screw action of the blades is corrected by diffusion vanes.

In some pumps a combination of enclosed impellers and axial flow type impellers are used. These are called mixed flow impellers.

The impellers of a turbine pump are rotated by a shaft which is connected at top to a motor. The shaft has to rotate in bearings fixed to the column pipe. The bearings are kept closely apart, so that the shaft rotates smoothly without any vibrations.

The lubrication of the bearings is very important. Both oil and water lubricated bearings are used. A typical turbine pump with impellers and bearing is shown in Fig. 18. The performance of a turbine pump depends upon the design of the bowels and the speed of the impellers. Its capacity, efficiency, horse-power, are similar to those of a centrifugal pump with a slight change according to the design of the impellers. A turbine pump, however, cannot operate at a high efficiency over as wide a range of speed as a centrifugal pump. In fact in a turbine pump the flow is never in line with the direction of the vanes. The direction of flow leaving the impellers changes with the speed of the impellers. The turbulence so created results in reduced efficiency.

In case of deep turbine pump, correct alignment between the pump and the power unit is most important. No parts of the pump assembly should touch the wall of the casing. A slight misalignment will wear out the bushes and will cause vibrations and damage to the pumping set. A damaged impeller affects the performance of the pump. In Fig. 19 is shown a damaged casing and vanes of the impeller of a pump extracted from Jaranwala area, after it has operated for four years. This may be due to incorrect flow directions or turbulence.

It is important that the pump should be mounted on a strong cement concrete foundation, so as to maintain the correct alignment between the pump and the drive.

Damage to a Turbine Pumping Set

All components of a turbine pump are enclosed and not visible, it is thus very difficult to estimate the damage at an earlier stage. When the defects are fairly advanced, these result in vibrations and noise. From this consideration a turbine pump is not as satisfactory as a centrifugal pump.

Not Possible to Use with Multiple Strainers

It is not possible to connect multiple strainers to a turbine pump. If the strainer gets choked, as it should with time, then the whole installation has to be changed.

Submersible Pump

A submersible pump is also a turbine pump which is coupled to an electric motor which is lowered into a casing pipe and works close to the bowels. A typical submersible pump is shown in Fig. 20. A great advantage of a submersible pump is that the motor as well as the pump are located inside the pipe. An electric cable carries the power down to the motor. The motor and the pumps are held at top through a housing pipe.

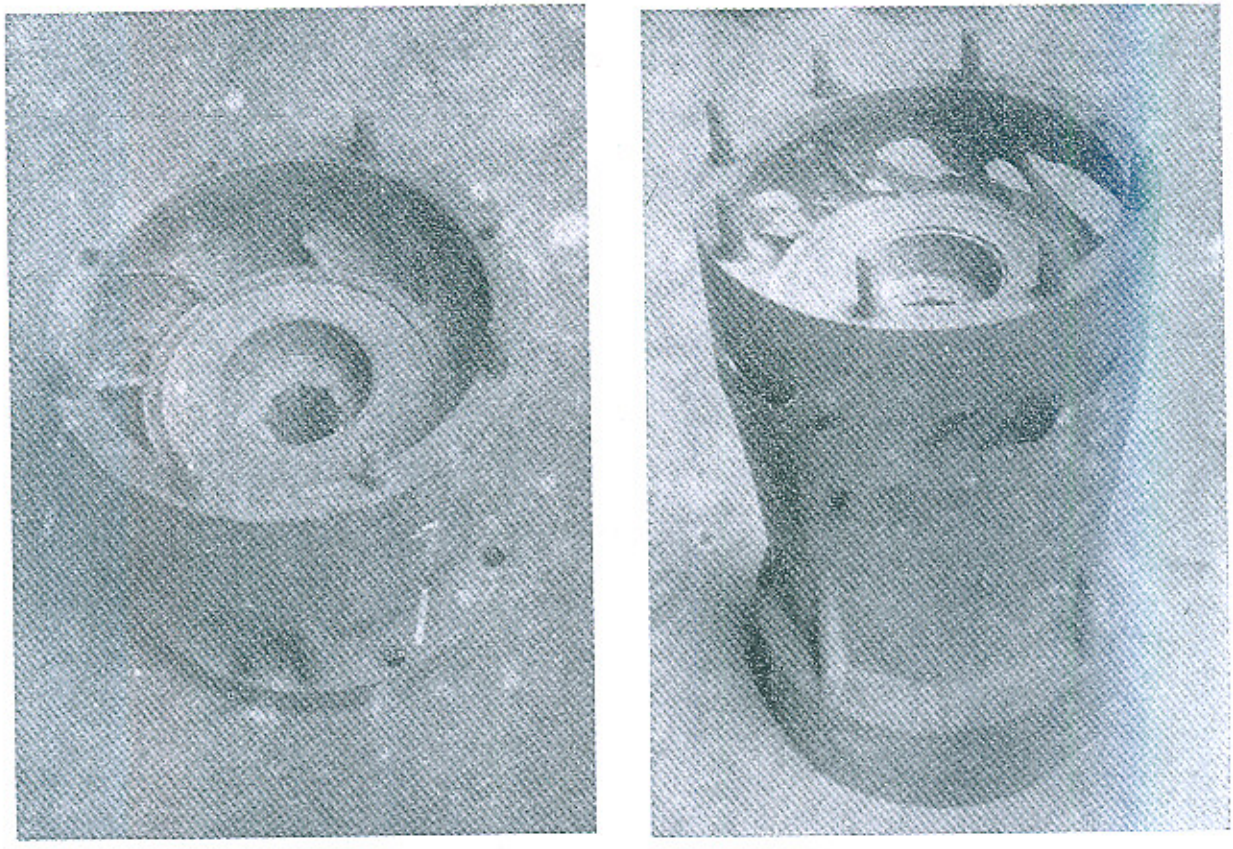


Fig. 19. Damaged Casing and Vanes of a Turbine Pump.

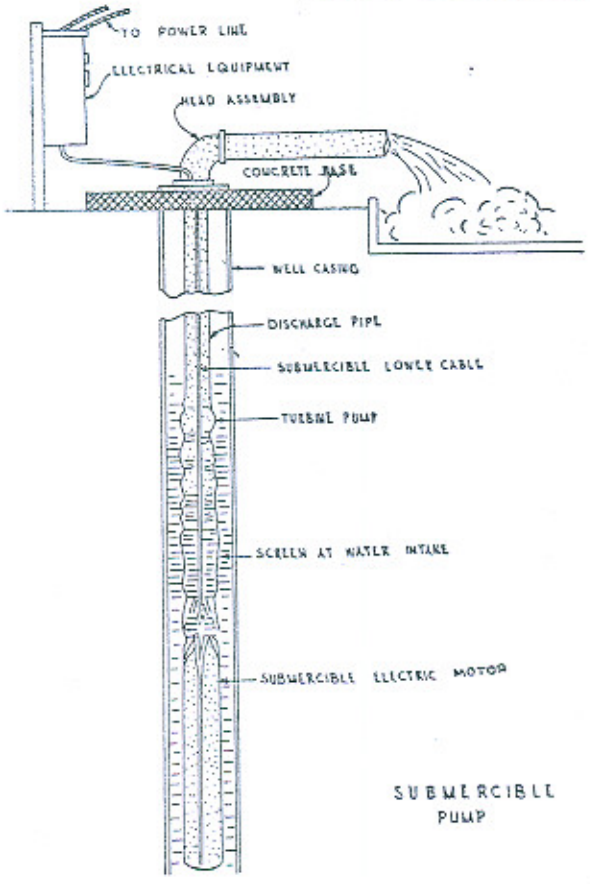


Fig. 20.

There are two types of motors, fitted to a submersible pump. One is hermitically sealed and is called a dry motor. It excludes the water of the well to enter into the motor. The motor runs in high die-electric oil kept under pressure. It submerges the windings, the bearings and the rotar. It prevents the water to enter into the motor.

The second type is the wet motor in which water enters in the motor. Its rotar and the bearings actually operate in water. The windings of the motor are usually sealed in an inner lining of thin, stainless steel. Although the initial cost of installing a submersible pump is low yet it has not found favour in our country.

Electric Motor

A pumping set works carefree if the power is transmitted through an electric

TABLE No. 3

Fuel Required in Gallons/hours

Discharge in gall/min.	Lift in feet.	WHP	Propane.	Diesel	Gasoline or Tractor fuel.	Natural Gas in cu. ft./hr.	Electri-city in k.w.h.
500 ..	100	13	2	1½	1½	210	14
500 ..	150	19	2¾	1¼	2¼	310	21
500 ..	200	25	3¾	2¼	3	410	29

TABLE No. 4

Pumping Plant Depreciation

Items	Estimated Useful Life
Well and Casing ..	20 years.
Plant housing.	20 years.
Pump, turbine :—	
Bowel (about 50% of cost of pump unit	8 years.
Column etc. ..	16 years.
Pump, centrifugal ..	16 years.
Power Transmission :—	
Gear head ..	15 years.
V-Belt ..	3 years.
Flat Belt, rubber and fabric ..	5 years.
Flat Belt, leather ..	10 years.
Electric motor ..	25 years.
Diesel engine ..	14 years.
Gasoline or distillate engine :—	
Air-cooled ..	4 years.
Water-cooled ..	9 years.
Propane engine ..	14 years.

motor which can be manufactured in various capacities. When the horse-power is low, say below 7.5, a single phase motor can be used. But for all high order of powers, three-phase motors working on 440 volts and on the frequency of the power produced in the country, are more suitable. Motor producing 1760 revolutions per minute are commonly used. Direct coupling of the motor with the pump is more appropriate. A motor must be fitted with a starter and a switch. In this country a common source of trouble is the burning of the windings due to the fall in voltage. This results in the flow of a large amperage which burns the motor. A motor must be fitted with a self-acting switch to disconnect the motor at the fall in the voltage. Damage to a motor is generally as a result of variation in voltage. So far no device has been perfected to safeguard it against the fall in voltage though the self-acting switch is helpful to a great extent.

Measuring Electric Consumption

Electricity Department supplies electric meters for recording the consumption of power. Electrical power is measured in units which is equal to one thousand watts consumed in one hour. It is called a kilo-watt hour. A watt is equal to volts multiplied by amperes consumed. Voltage of current is supposed to remain constant as supplied by the Electricity Department. The watts consumed can thus be determined provided the amperes consumed are known. If power consumed is to be estimated by measuring volt and ampere, a constant power factor has to be used. The power consumption of the motor can however be checked with reference to the electric meter which has a revolving disc with a red mark. The rotations of the disc corresponding to a k.w. are specified on the meter. Counting the revolutions of the disc in a given time, the power consumption can be estimated. A horse-power is equal to 746 watts. The motors used are of known horse-power. Observing the watts being consumed on the meter, the power being utilized by the motor can be worked out.

Measuring Efficiency of Pumping Set

Israelsen has referred to the work of Schleusener who has worked out maximum fuel requirement for a good pumping set. In Table 3 the relative production of energy produced by various materials to pump a given quantity of discharge is given.

Pumping Plant Depreciation

Just as a strainer has a limited life, similarly the components of the pumping plants last for some period after which these are required to be replaced. The United States Conservation Services has estimated the useful life period of various components of a tubewell. See Table No. 4.

Explanation of Paradoxes in Dupuit-Forchheimer Seepage Theory[1]

By **DON KIRKHAM**[2]

The Engineers of Lahore had the privilege of hearing Dr. Don Kirkham on 29th August, 1966 on a recent subject of his investigation. Those who have studied the ground water flow, its movement and measurement, need no introduction about the work of Dr. Don Kirkham. He is one of the world's topmost Scientists whose contributions to the sub-soil flow, drainage problems, technique of field permeability, measurements for soil are known and utilized throughout the world. His research publications number more than six scores. He is Curtiss Distinguished Professor of Agriculture and Director of Iowa State Water Resources Research Institute, and was on tour to West Pakistan with the Sponsorship of the office of Water Resources Research, United States Department of Interior, U.S.A.

The 'Engineering News' has the privilege to reproduce the lecture of Dr. Don Kirkham delivered at Lahore, for the benefit of its readers.

Cutting into a porous medium of a large number of vertical, parallel, infinitely permeable, equally spaced, infinitesimally thin slots produces a fictitious soil that follows exactly and without paradoxes Dupuit's assumptions and hence Dupuit-Forchheimer (D.F.) drainage theory in two dimensions. A soil having these infinitesimally thin slots is designated a D.F. soil. For this fictitious soil, a formula for the proper depth and spacing of ditches and drain tiles is derived. The formula is the same one found in conventional D.F. literature. The formula, valid for both tiles and ditches and known to hold approximately for actual soils, is exact for a D.F. soil. Dupuit's two-dimensional "parabolic seepage problem" and

1. Journal Paper No. J-5469 of the Iowa Agricultural and Home Economics Experiment Station, Ames, Iowa. Project No. 998. Contribution from the Department of Agronomy. Work supported in part with funds provided by the Department of the Interior, Office of Water Resources Research, under Public Law, 88-379. Part of this paper was presented at a meeting of Water Resources Engineers held in Lahore, West Pakistan, August 29, 1966.

Acknowledgement is made to the American Geophysical Union journal, Water Resources Research, where this article is scheduled for original publication, April 1967.

2. Professor of Agronomy (Soils) and Physics, and Director Iowa State Water Resources Research Institute, Iowa State University of Science and Technology, Ames.

others, may now be considered as exactly solvable for D.F. soils. For three-dimensional axially symmetric seepage flow, as into wells, the fictitious slots of a D.F. soil become concentric coaxial rings. D.F. streamlines are not horizontal ; they convergein a special way.

INTRODUCTION

Dupuit-Forchheimar (D.F.) theory is based on two assumptions of Dupuit (1863) which are :

1. for small inclinations of the free surface of a gravity-flow system, the streamlines can be taken as horizontal ;
2. the velocities associated with these streamlines are proportional to the slope of the free surface, but are independent of the depth.

These assumptions have been used extensively by Forchheimer [1930] and are being used by a number of workers, Kozeny [1953], Sutton [1960], Maasland [1961], van Schilfgaarde [1963], Dagan [1964], Schmid and Luthin [1964], Wooding and Chapman [1966] and others [see Luthin, 1957, 1966]. Busch [1963] and Kirkham [1966] list drainage formulas used over the world, a number are based on D.F. theory. The D.F. theory is of such wide use in the U.S.A. that a symposium on the subject was held by the American Society of Agricultural Engineers in New Orleans, December 8-11, 1964, principal participants being J. M. R. De Wiest, R. E. Glover, Herman Bouwer, Jan van Schilfgaarde and Morton Bittinger.

Despite the wide use of D.F. theory, Muskat [1946, pp. 360-362] showed that the two Dupuit assumptions on which it rests are "entirely contradictory to the implications of Darcy's law." Harr [1962] refers to the assumptions as "'paradoxical," but writes that many problem solutions based on the Dupuit assumptions compare favourably with those of more rigorous methods. Two triumphs of D.F. theory are the correct solutions for the discharge—but not the free surface—for semi-confined seepage from a canal to a ditch and from an aquifer to a well [see Hantush, 1962, and Kirkham, 1964]. Muskat [1946] has given little space in his treatise to D.F. theory because of its paradoxical nature.

Definition of a D.F. Soil

Because of the wide use and great interest in D.F. theory, our purpose here is to explain the paradoxes. We shall develop a theoretical model of a porous medium obeying D.F. theory that removes the paradoxes. For our model we shall, because of lack of space, fix our attention on the D.F. problem of tile and ditch drainage for soil over a barrier. We have applied the theoretical model to other problems. The model appears general wherever a "D.F. soil" is used. A "D.F. soil" is one, as we shall show, that contains numerous, vertical, infinitely permeable, infinitesimally thin slots. Soils with many cracks approximate D.F. soils ; soils that have been deep-chiseled [Swartzendruber, 1960] parallel to drains also approximate D.F. soils. In three-dimensional axially symmetric flow, as into wells, a D.F. soil is one where concentric infinitely permeable rings replace the infinitely permeable slots of two-dimensional flow.

TWO FALSE FLOW MODELS

Before we develop the details of a D.F. flow model, we consider two false models that illustrate paradoxes of the Dupuit

assumptions. Figure 1 is a model of vertical rainfall draining through soil to ditches. In the soil the "streamlines are taken as horizontal," according to the first Dupuit assumption. Two difficulties appear: (1) part of the discharge such as occurs at points P, Q, R and S, outlets into the soil rather than into the ditches ; (2) the source of draining water must be a vertical plane source located midway between ditches rather than the given horizontal plane uniform source of rain water.

In our second model (Figure 2) we have inserted a number of infinitesimally thin vertical strips such as EFG and IJK into the soil to assure that vertical flow enters the soil uniformly. We then continued the strips horizontally, as at GH and KL, to satisfy the first Dupuit assumption of horizontal flow. By the second D.F. assumption, head loss in the vertical direction as along EFG and IJK is neglected. The difficulty of the strips of Figure 2 and the neglected head loss in the vertical direction

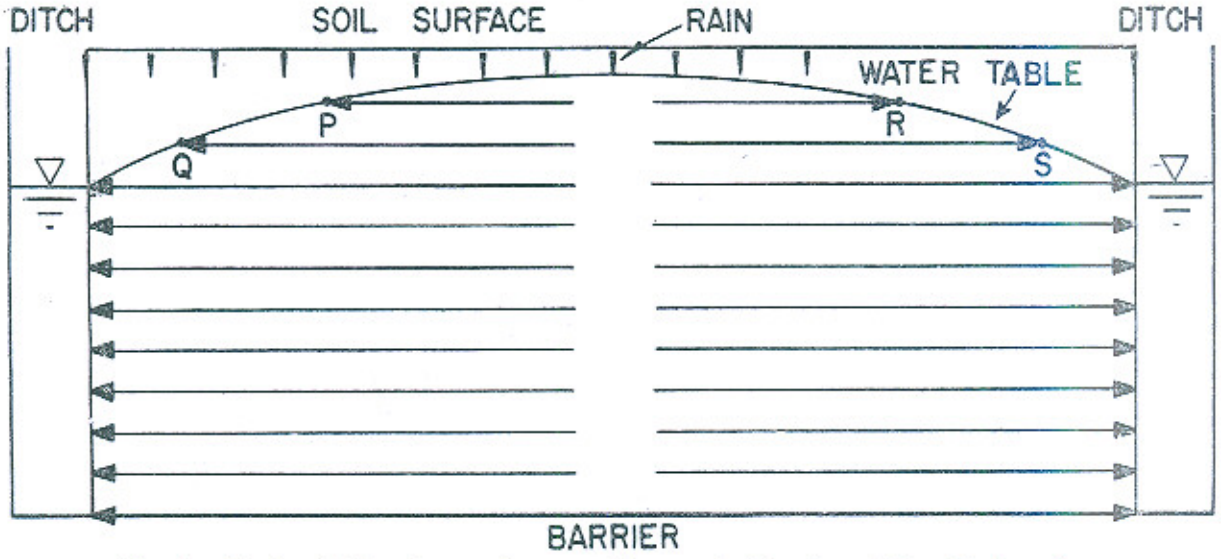


Fig. 1. Horizontal flow lines as (improperly) conceived for Dupuit-Forchheimer flow.

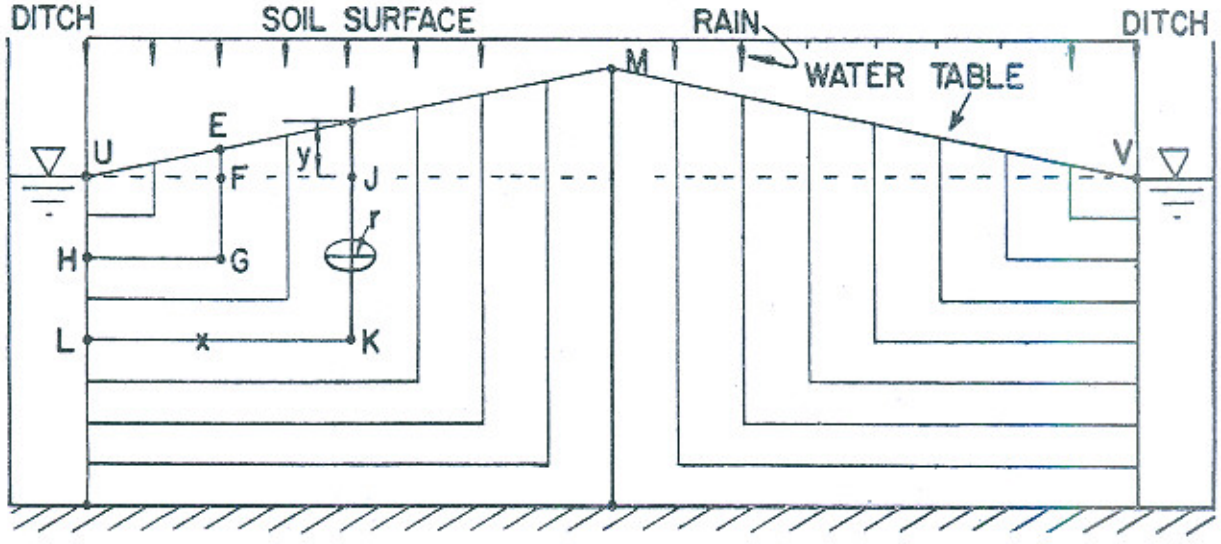


Fig. 2. Vertical and horizontal flow lines as (improperly) conceived for Dupuit-Forchheimer flow.

is, that instead of an ellipse-shape water table required by D.F. theory, we now obtain a tent-shape water table UEIMV. This is seen by applying Darcy's law to a typical stream tube as follows : We let the stream tube be of small cross section πr^2 with the streamline IJKL passing through its center. The head difference y across the tube is y=IJ. All this head loss is dissipated across the distance x=LK. If R is the rainfall rate, then the flux feeding the tube is $R\pi r^2$. So Darcy's law gives for the tube, if k is the hydraulic conductivity,

$$R\pi r^2 = k(y/x)\pi r^2$$

or

$$y = (k/R)x$$

which is the equation of the straight line UEIMV.

A CORRECT FLOW MODEL FOR A D. F. SOIL

Figure 3 shows a D.F. soil containing the numerous slots, called "D.F. slots" and numerous "slabs," columns of soil between adjacent slots. When these slots and slabs are sufficiently numerous and, hence, sufficiently thin, we shall see that the Dupuit assumptions apply properly to each slab and to the whole flow model shown and hence shall see that the model of Figure 3 gives D.F. theory without paradoxes. We first consider conventional D.F. theory which says nothing about slots and slabs; so we ignore these. We now choose an origin O of x, y coordinates at a point below the left drain on the barrier. We take R=rainfall rate, k=the conducitvity, S=the drain spacing and Q=RS, where Q is the total flow per unit length of drain. Other symbols are as shown. The flow moving to the left across line AB is $R[(S/2)-x]$, which is the same as $(Q/S)[(S/2)-x]$. An equal amount

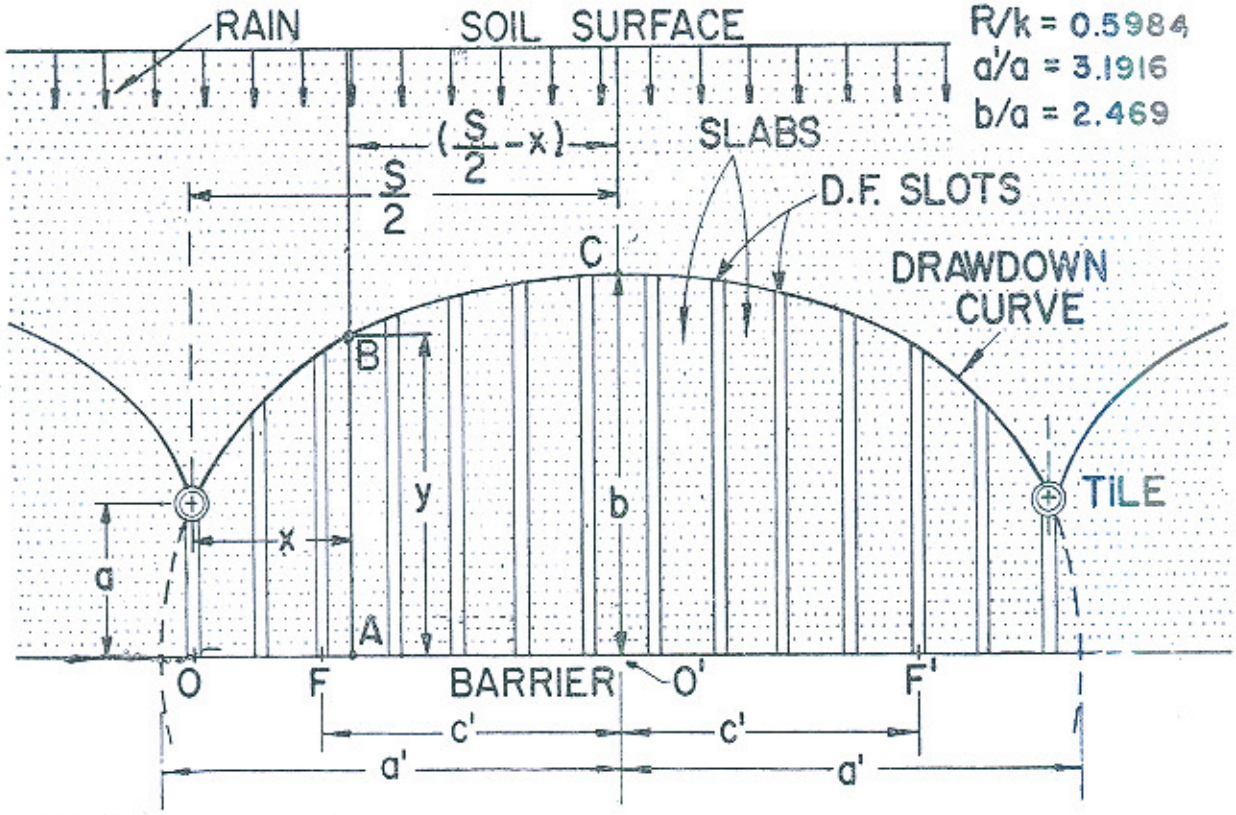


Fig. 3. D. F. slots and slabs of a rational physical model for Dupuit-Forchheimer drainage theory.

of flow moving from the line AB to the left is, by Darcy's law, $ky\ dy/dx$. Therefore, we have

$$(Q/S)[(S/2)-x]=ky\ dy/dx \qquad (1)$$

which, when integrated and the constant of integration evaluated, gives

$$y^2-a^2=[Q/(Sk)](Sx-x^2) \qquad (2)$$

in which $Q/(Sk)$ may be replaced by $RS/(Sk)=R/k$.

Equation (2) is the equation of the classic D.F. ellipse with foci at F and F′, at distances $\pm c'$ from the ellipse center O′, with c' given by

$$\pm c'=\pm\ (a'^2-b^2)^{1/2}$$

where a', the semi-major axis, is given by

$$a'^2=a^2\left(\frac{k}{R}+\frac{S^2}{4a^2}\right)$$

and b is the semi-minor axis O′C where C is the highest point on the ellipse. In (2), if we put $y=b$, $x=S/2$, $Q=Rs$ and solve for S^2, we get

$$S^2=(4k/R)(b^2-a^2) \qquad (3)$$

Equation (3) gives the drain spacing and has been derived independently by Donnan (Aronovici and Donnan, 1946) and by others reported by van Schilfgaarde, *et al.* [1956] and Luthin [1966, pp. 151-153]. Figure 4 shows the ellipse geometry for a more realistic situation than the situation in Fig. 3. In Figure 4 we have: $R/k=1/16$, $S/a=6$, $b/a=1.25$, $a'/a=5$, and $c'/a=4.84$. Since in Figures 3 and 4 we have

$$Q=RS \qquad (3a)$$

we may eliminate Q from (2) and rewrite the result as

$$y=[a^2+(R/k)(Sx-x^2)]^{1/2} \qquad (3b)$$

which is often more convenient than (2).

Derivation of the Ellipse Equation from a "Slot-Slab" Model

We now derive (1) and (2) from a slot-slab model (Figure 5). We let n be the number ($n=4$ in the figure) of "slot-slab" elements, each of width Δx. Then with c and d defined

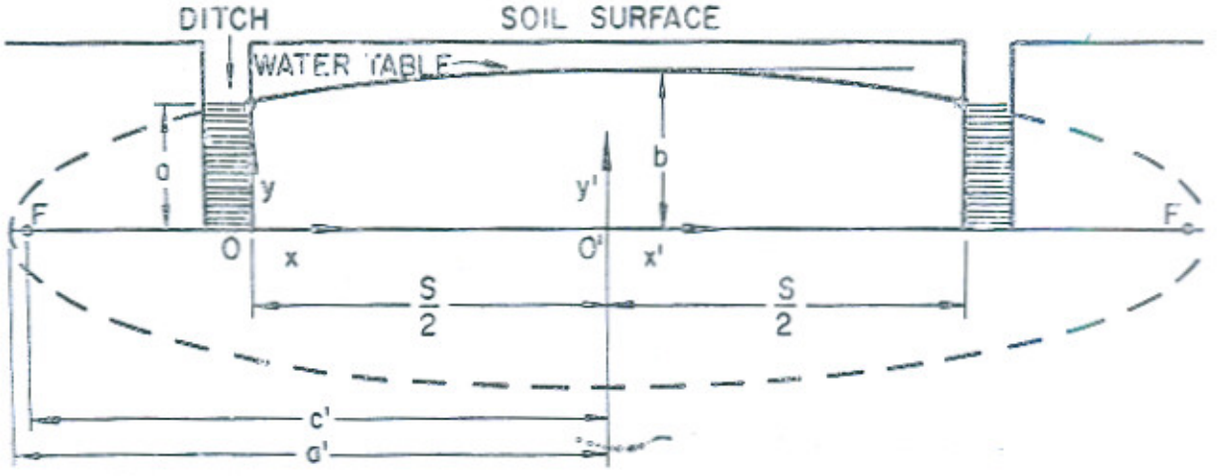


Fig. 4. Ditch drainage and the Dupuit-Forchheimer ellipse; a'=semi-major axis, b=semi-minor axis, and c'=distance from center of the ellipse to a focus F.

by c=slot width, and d=slab width, we get the relations

$$c+d=\Delta x \text{ and } n\Delta x=S/2 \qquad (4,5)$$

The slabs are of loam soil of which the conductivity is k. The slabs are of heights h_1 ($=a$), h_2, h_3 and h_4. A new feature is temporarily introduced into the model: on the slab tops are placed strips of "clay" of conductivity zero. These clay strips are of heights Δh_1, Δh_2, Δh_3, and Δh_4. The clay strips serve to prevent surfaces of seepage from forming. When the slots and slabs become infinitesimal, the clay strips may be

omitted because surfaces of seepage are then negligible; and the model becomes the slot-slab model previously discussed. We let Q/8 be the volume of water per unit time entering each of the slots in the figure so that Q/2 is the volume of water per unit time entering the right side of the ditch. The quantities Q/8 are to be "funnelled" uniformly, as indicated in the figure, into each slot. In the limit of small slots and slabs, the "funnels"

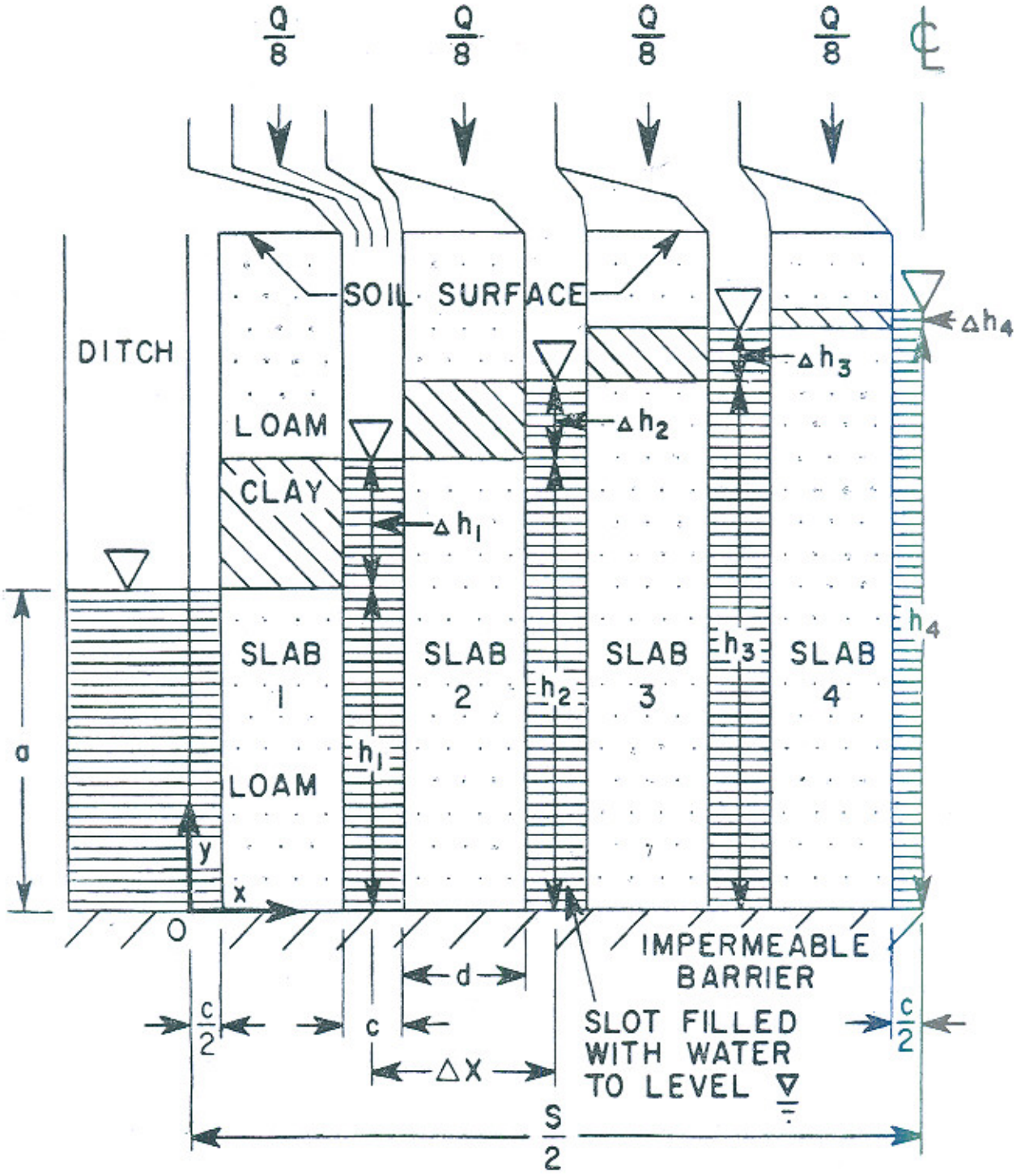


Fig. 5. Geometry of a slot-slab drainage model for a half-space between a pair of ditch drains when there are 8 slots and 8 slabs.

may, as is evident physically, be omitted. Darcy's law is now applied to the slabs. We find :

$$\text{Slab 1:} \qquad \frac{Q}{2}=k\,\frac{\Delta h_1}{\Delta x-c}\,h_1 \qquad (6)$$

$$\text{Slab 2:} \qquad \frac{S/2-1\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_2}{\Delta x-c}\,h_2 \qquad (7)$$

$$\text{Slab 3:} \qquad \frac{S/2-2\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_3}{\Delta x-c}\,h_3 \qquad (8)$$

$$\text{Slab 4:} \qquad \frac{S/2-3\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_4}{\Delta x-c}\,h_4 \qquad (9)$$

If, instead of 4 slabs, there are n slabs, one of which may be called the pth slab, the equations become (for the way the water is funneled in the figure).

$$\text{Slab 1:} \qquad \frac{Q}{2}=k\,\frac{\Delta h_1}{\Delta x-c}\,h_1 \qquad (10)$$

$$\text{Slab 2:} \qquad \frac{(S/2)-1\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_2}{\Delta x-c}\,h_2 \qquad (11)$$

$$\text{Slab } p\text{:} \qquad \frac{(S/2)-(p-1)\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_p}{\Delta x-c}\,h_p \qquad (12)$$

$$\text{Slab } n\text{:} \qquad \frac{(S/2)-(n-1)\Delta x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta h_n}{\Delta x-c}\,h_n \qquad (13)$$

where n is related to Δx and S by (5).

Now we choose an origin O of (x, y) coordinates at a distance $c/2$ to the left of slab 1 at a point on the barrier. Then the value of x at the middle of the slot just left of the $(p-1)st$ slab is $(p-1)\wedge x$. We let the height of the $(p-1)st$ slab be y and the height of the slab at the right of the $(p-1)st$ slab be $y+\Delta y$. Then (12) becomes

$$\frac{S/2-x}{S/2}\,\frac{Q}{2}=k\,\frac{\Delta y}{\Delta x-c}\,(y+\Delta y) \qquad (14)$$

Now we let c approach zero, and then let Δy and Δx approach zero. In this process, n approaches infinity; and $\Delta x/c$ and $\Delta y/c$ also each approach infinity; although Δx and Δy each approach zero. Equation (14) then becomes

$$\frac{(S/2)-x}{S/2}\,\frac{Q}{2}=k\,\frac{dy}{dx}\,y \qquad (15)$$

When integrated, this becomes

$$y^2-a^2=\frac{Q}{Sk}\,(Sx-x^2) \qquad (16)$$

Equations (15) and (16) are the same as (1) and (2) and show that the slot-slab model gives the same result as the D.F. assumptions. The slot width c and slab width d do not occur in (15) and (16). But the slots and slabs must still be imagined as existing in the soil for (15) and (16) to be physically valid. We see that the slots and slabs remove the paradoxes of D.F. theory.

A FINITE SLOT-SLAB MODEL TO APPROXIMATE D.F. FLOW

We shall now show, by a numerical example, that a real slot-slab model (a model that could be made with finite widths of slots and slabs) closely approximates the slot-slab model having an infinite number of fictitious slots and slabs. We need to modify (10)—(13). For specificity we may consider $n=4$ (eight slots), and hence see in Figure 5, the relations

$$h_1=a \qquad (17)$$

$$h_2=h_1+\Delta h_1 \qquad (18)$$

$$h_3=h_2+\Delta h_2 \qquad (19)$$

$$h_4=h_3+\Delta h_3 \qquad (20)$$

$$\Delta x=S/8, \quad \Delta x-c=d \qquad (21), (22)$$

Furthermore (3a) is

$$Q=RS \qquad (23)$$

We put the right sides of (17)-(23) in (6)-(9), to find

$$\frac{4}{4}\,\frac{RS}{2}=k\,\frac{\Delta h_1}{d}\,a \qquad (24)$$

$$\frac{3}{4}\,\frac{RS}{2}=k\,\frac{\Delta h_2}{d}\,(a+\Delta h_1) \qquad (25)$$

$$\frac{2}{4}\,\frac{RS}{2}=k\,\frac{\Delta h_2}{d}\,(a+\Delta h_1+\Delta h_2) \qquad (26)$$

$$\frac{1}{4}\,\frac{RS}{2}=k\,\frac{\Delta h_3}{d}\,(a+\Delta h_1+\Delta h_2+\Delta h_3) \qquad (27)$$

These may be rearranged to the dimensionless forms

$$\triangle h_1/a = \frac{4}{8}\,\frac{RSd}{kaa} \tag{28}$$

$$\triangle h_2/a = \frac{3}{8}\,\frac{RSd}{kaa}\,\frac{1}{1+(\triangle h_1/a)} \tag{29}$$

$$\triangle h_3/a = \frac{2}{8}\,\frac{RSd}{kaa}\,\frac{1}{1+(\triangle h_1/a)+(\triangle h_2/a)} \tag{30}$$

$$\triangle h_4/a = \frac{1}{8}\,\frac{RSd}{kaa}\,\frac{1}{1+(\triangle h_1/a)+(\triangle h_2/a)+(\triangle h_3/a)} \tag{31}$$

For the right side of (29) we use $(\triangle h_1/a)$ from (28) ; similarly for the right sides of (30) and (31). With (28)-(31) giving $\triangle h_1$, $\triangle h_2$, etc., we use these values in (18)—(20) to get h_2, h_3, and h_4.

(28)-(32) are : $h_1=1$, $\triangle h_1=0.7500$, $h_2=h_1+\triangle h_1=1.7500$; $\triangle h_2=0.3214$, $h_3=h_2+\triangle h_2=1.7500+0.3214=2.0714$; $\triangle h_3=0.1810$, $h_4=h_3+\triangle h_3=2.0714+0.1810=2.2524$; and $\triangle h_4=0.0832$. One may verify that the heights $h_1=1$, $h_2=1.7500$, $h_3=2.0714$ and $h_4=2.2524$, when multiplied by a, give heights of the four slabs of the left half of the figure. The figure also shows the D.F. ellipse as computed from (3*b*) where we have as in (32)

$$R/k=0.25\ ;\quad S=8a,\quad a=1 \tag{32a}$$

The fit of the ellipse to the tops of the slabs, especially where the edges of the slabs are closest to the ditches, shows that a finite system of eight or more slots and slabs gives a close approximation to an infinite number of slots and slabs (for which the ellipse

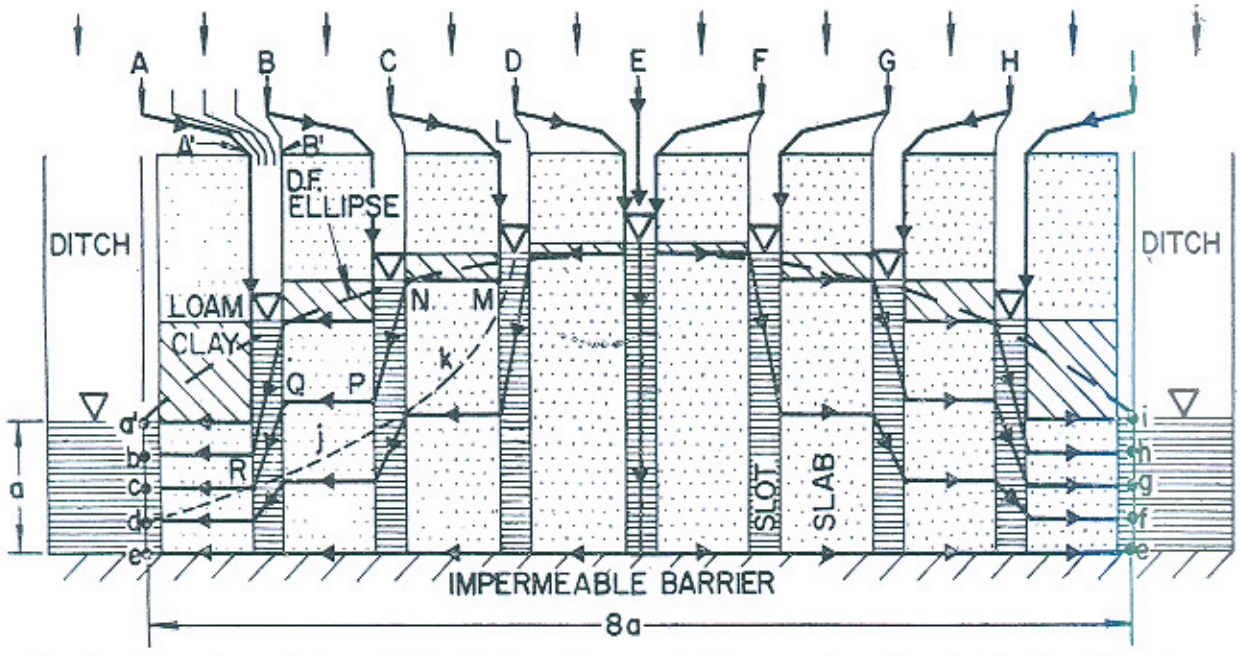


Fig. 6. Comparison of water heights in slots with water heights as given by the D. F. ellipse for a slot-slab "soil" having 8 slabs and 8 slots between a pair of ditch drains.

Figure 6 shows the slot-slab system calculated from (28)-(31) in conjunction with (17)-(21) when we used the values

$$R/k=0.25,\quad \triangle x=a,\quad S=8a,\quad d=0.75a,\quad \text{and}\quad a=1, \tag{32}$$

(which imply $h_1=a$, $n=4$, and $c=0.25a$). Specific values found from (17)-(21) and equation is exact.). At a point on the ellipse where $x=2\triangle x+c/2=2+0.25/2=2.125$, we have from (3*a*) and (32*a*), $y=2.030$, whereas the slab height (the height above the barrier of the point N in the figure) is $h_3=2.071$; the difference 2.071—2.030=0.041 is not discernible in the figure. The figure estab-

lishes the reality of a slot-slab model for rationalizing D.F. theory.

D.F. STREAMLINES

Graphically Obtained D.F. Streamlines for Finite Slot-Slabs

Figure 6 shows heavy zigzag streamlines, such as CLMNPQRc that are correct for the slot-slab system shown except that we do not know the shape of the streamline segments inside the slots. In the figure a broken-line streamline kjRd for a D.F. soil (infinitesimally thin slots and slabs) is also shown as a smooth curve. This curve is really a step function with steps too small to be evident but later we shall treat it as a smooth curve. To find graphically the shown zigzag streamlines Aa', Bb, C(LMNPQR)c, Dd and Ed, we notice that all the water falling above for four slabs and slots at the left half of the figure must pass through the extreme left slab, three-fourths through the next, two-fourths through the next, and one-fourth through the last. Furthermore, the flow in a slab must be divided equally per unit height of the slab. The streamlines are obtained by connecting the points of equal water division. We emphasize : the streamlines converge (they would also converge if the clay strips were omitted). The observed convergence is contrary to the representation of Figure 1. Users of Dupuit-Forchheimer theory have often said that D.F. theory does not account for convergence. A better statement would be : "D.F. theory does not *adequately* account for convergence." The broken-line streamline kjRd is for infinitesimal slot-slabs and may be obtained by the method of the next section.

Graphically Obtained D.F. Streamlines for Infinitesimal Slot-Slabs.

Figure 7 shows streamlines for a semi-space between ditch drains when $a=5$, $b=9$, $S/2=10$, and $R/k=0.56$. The equation of the water table ellipse is, therefore, by (3a),

$$y=[5^2+0.56\ (20x-x^2)]^{1/2} \qquad (33)$$

The points T, P, L, etc., of coordinates (0, 5), (2, 6.72), (4, 7.80), etc., were calculated from (33). Using these points we now can construct the streamlines of Figure 7. We proceed as for Figure 6 except that now we consider the flow through five infinitesimally thin slabs at OT, F′P, E′L, D′H, C″C, and BA, where $x=0$, $x=(1/5)S/2$, $x=(2/5)S/2$, ..., and $x=(5/5)S/2$. These slabs are divided into equal vertical sections at G, K, N, Q, F, J, M; etc., and these "points" at the divisions are connected as in Figure 6, except that now the streamlines are drawn continuously because the zigzags for the infinitesimally thin slabs (and slots) become undiscernible.

By inspection, the shown streamlines are seen to have the following values:

Line (point) T : $\psi=0$

Line PQ : $\psi=0.1RS$
Line LMN : $\psi=0.2RS$
Line HIJK : $\psi=0.3RS$
Line CDEFG: $\psi=0.4RS$
Line ABO : $\psi=0.5RS$

Analytically Derived D.F. Streamlines

To find the D.F. stream function for any point in the flow medium, a set of coordinates (x, w) are chosen (see Figure 7), with origin O the same as for (x, y) of (33). We notice, for points on the water table ellipse, the relation $x=y$. In Figure 7 we consider the point M, where $x=(1/5)(S/2)$ and $w=(3/4)$ (6.72).

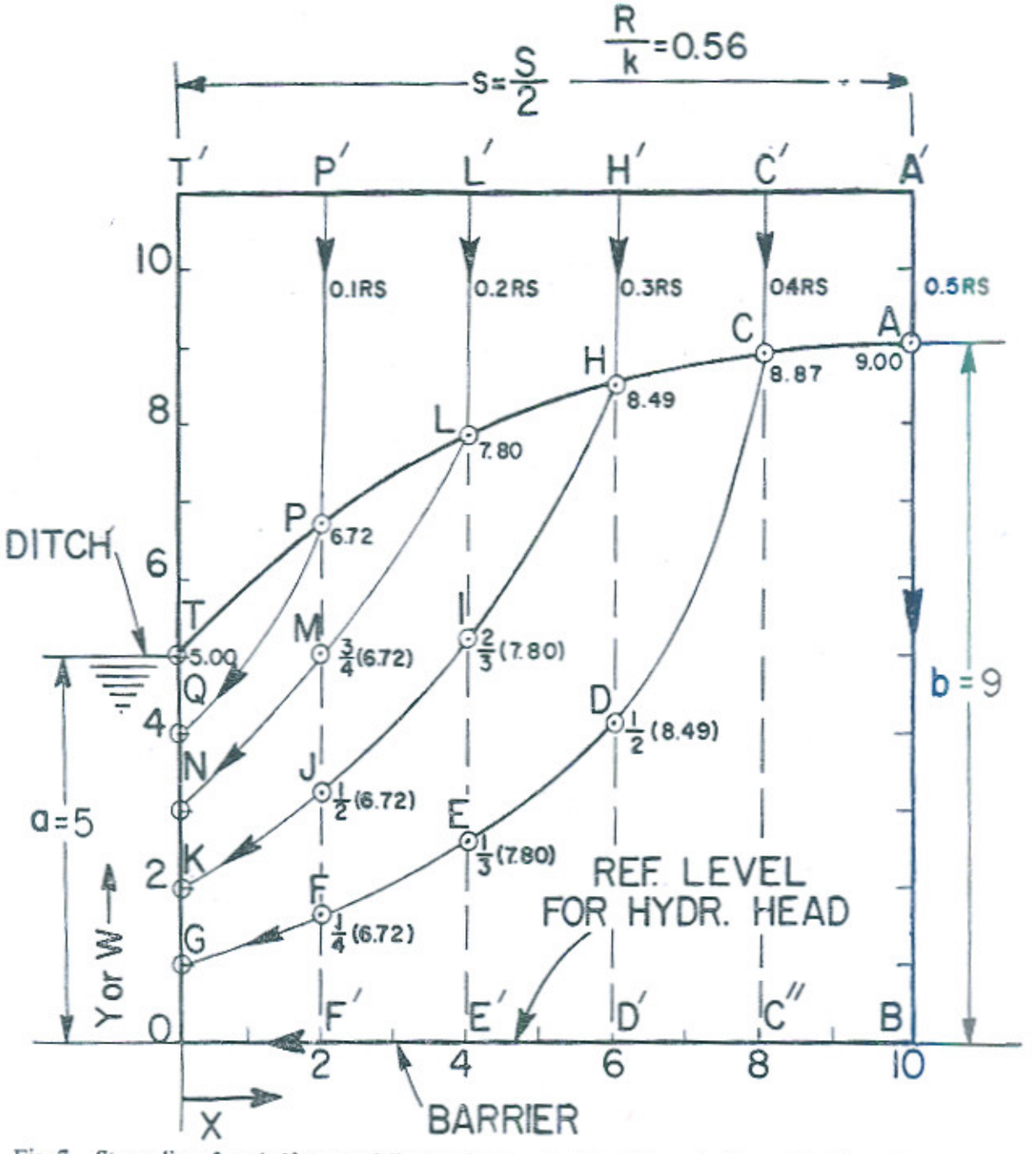


Fig. 7. Streamlines for a half-space of ditch drainage, when the slabs and slots as in Figure 6 approach zero width and when $R/k=0.56$, $S/2=10$, and $a=5$; the vertical lines are equipotentials and, with the streamlines, comprise a D.F. flow net.

We let ψ_M be the stream function for point M and let ψ_P be the stream function for a point P on the ellipse vertically above point M. We see, then, from the figure the relation

$$\psi_M=\psi_P+(\text{flow between P and M}) \quad (34)$$

But since, in the vertical slab PMJFF′, the horizontal flow velocity in D.F. theory is the same at all heights and since the total flow entering the slot at the right of the slab PMJFF′ is (0.5RS—0.1RS), we find

$$\psi_M=\psi_P+(PM/PF')(0.5RS-\psi_P) \quad (35)$$

From Figure 7 this is

$$\psi_M = 0.1RS + (1/4)(0.4RS) = 0.2RS$$

agreeing with the shown stream function value 0.2RS of the line L'LMN.

If, instead of point M, the point is any point (x, w) with the point above it on the ellipse being (x, y), we see that (34) is replaced by

$$\psi_{x,\,w} = \psi_{x,\,y} + \frac{y-w}{y}(0.5RS - \psi_{x,\,y}) \qquad (36)$$

or, since we have from the figure

$$\psi_{x,\,y} = Rx \qquad (37)$$

we get from (36) and (37), after some algebra,

$$\psi_{x,\,w} = 0.5RS\left[1 - \frac{w}{y}\left(1 - \frac{2x}{S}\right)\right] \qquad (38)$$

where if we use y as given by $(3a)$ we find, after dropping the subscripts on ψ,

$$\psi = 0.5RS\left\{1 - \frac{w(1-2x/S)}{[a^2 + (R/k)\,(Sx - x^2)]^{1/2}}\right\} \qquad (39)$$

This shows ψ to be a function of w and x, with the values R/k, S, a and RS (=Q) appearing as parameters. Equation (39) is the streamline equation for D.F. flow ψ has the same dimensions as RS.

If, in (39), ψ is given, (39) may be written as

$$w = \frac{(1-\psi/0.5RS)[a^2 + (R/k)(Sx - x^2)]^{1/2}}{1 - 2x/S} \qquad (40)$$

To get without interpolation, numerical values of the square roots in (39) and (40), we use Zimmerman [1946] for five significant figures ; or we use Barlow [1947] for four significant figures.

If we substitute into (39) the values for point M of Figure 7 [which are $x=2$, $w=(3/4)\,6.72$, $a=5$, $R/k=0.56$, $S=20$] we find $\psi=0.2RS$, which agrees with the value on the figure. If in (39) we put $w=y$ as in $(3b)$ the equation becomes $\psi = 0.5RS\{1-(1-2x/S)\} = Rx$, as it should be seen in Figure 7. Equation (39) is valid for $0 \leqslant x \leqslant S/2$; for other values of x one obtains ψ by symmetry. The streamline equation for the line hj Rd in Figure 6 is (because in (40) we have $R/k=1/4$, $S=8a$ and $\psi/0.5RS=0.75$) the expression

$$w = \frac{0.25[a^2 + 0.25(8ax - x^2)]^{1/2}}{1 - 2x/8a} \qquad (41)$$

Equation (41) gives, for $x=0$, the result $w=0.25a$; and gives, for $x=3a$, the value $w=0.18a$—values that may be verified by measuring the ordinates of the end points of the broken line streamline of Figure 6.

EQUIPOTENTIALS FOR D.F. THEORY

The equipotentials for D.F. theory are a set of vertical lines extending from the impermeable boundary to the water table. They are vertical lines because the slots, implied as we now know by D.F. theory, are equipotentials. In Figure 7, one may take a set of equipotentials to be the "lines" OT, F'P, E'L, D'H, C''C, and BA (even though we considered slabs to be at these "lines" earlier). If the level of the ditch water is taken as zero potential and the level at the highest point is taken as 100 per cent potential, then the equipotentials are seen to be : OT=zero, F'P=[(6.72—5.00)/(9—5)]×100=43 per cent; E'L=70 per cent; D'H=87.25 per cent ; C''C=97 per cent; and BA=100 per cent. These values show that the space rate of head loss near the ditch is greater than far from the ditch, agreeing with the convergence of the streamlines. The general equation for the equipotentials is obtained by writing down the piezometric height ϕ referred to the ditch water level; and this height is simply $\phi = y - a$, or in view of Equation $3b$ it is

$$\phi = [a^2 + (R/k)(Sx - x^2)]^{1/2} - a \qquad (42)$$

where the head units are the same length units used for x, a, and S. Equations (39) and (42) give the functions needed for a D.F.

flow net. D.F. flow nets differ from conventional flow nets in that D.F. flow nets are not orthogonal while conventional flow nets are.

Comparison of D.F. Streamlines and Correct Streamlines

Figures 8 and 9 show almost correct streamlines (and equipotentials) for ditch and tile drainage obtained by modifying (see the section before *Conclusions*) a pair of flow nets of Kirkham [1958], computed from potential theory. The "almost" is included in the last sentence because exact analytical theory for flow in a region containing a water table arch, except for the case of an infinitesimally thin water arch height, is not known. Although Kirkham's [1958] theory is exact only for an infinitesimally thin water table arch height, one sees that the orthogonality of the flow nets as examined in Figures 8 and 9 verifies their correctness for present purposes. For both nets the parameters are (as in Kirkham): $R/k = 0.01087$, $a = 10$ ft., $S = 100$ ft., drain radius (of Fig. 9 only) 0.5 ft. ; and the ψ values in per cents are 0, 20, 40, 60, 80 and 100 (instead of 0, 0.1RS, 0.2RS,...., 0.5RS). The parameters make the maximum water

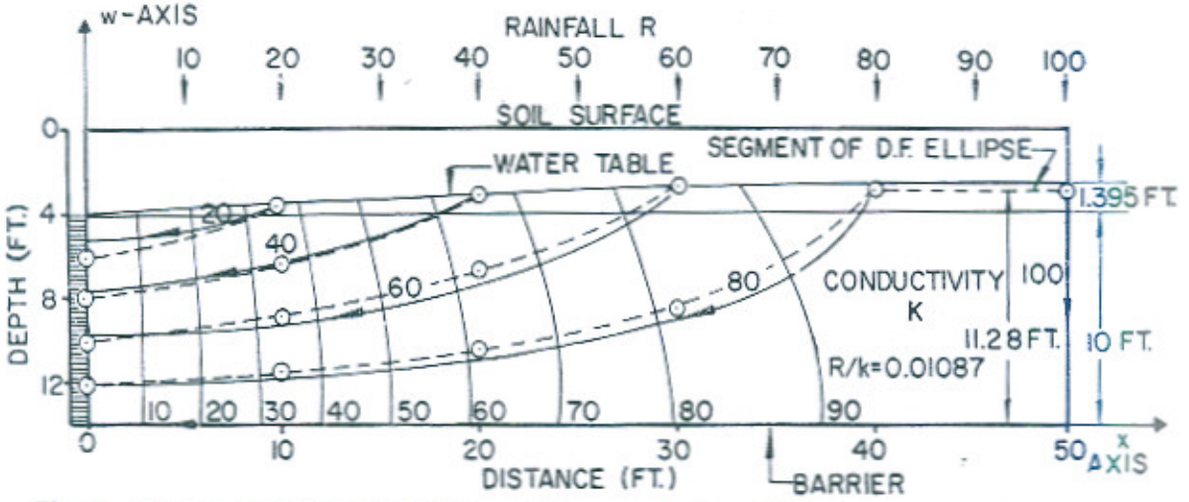


Fig. 8. Correct streamlines (solid lines with arrows) for ditch drainage compared with D.F. streamlines (broken lines) computed by equation (28)

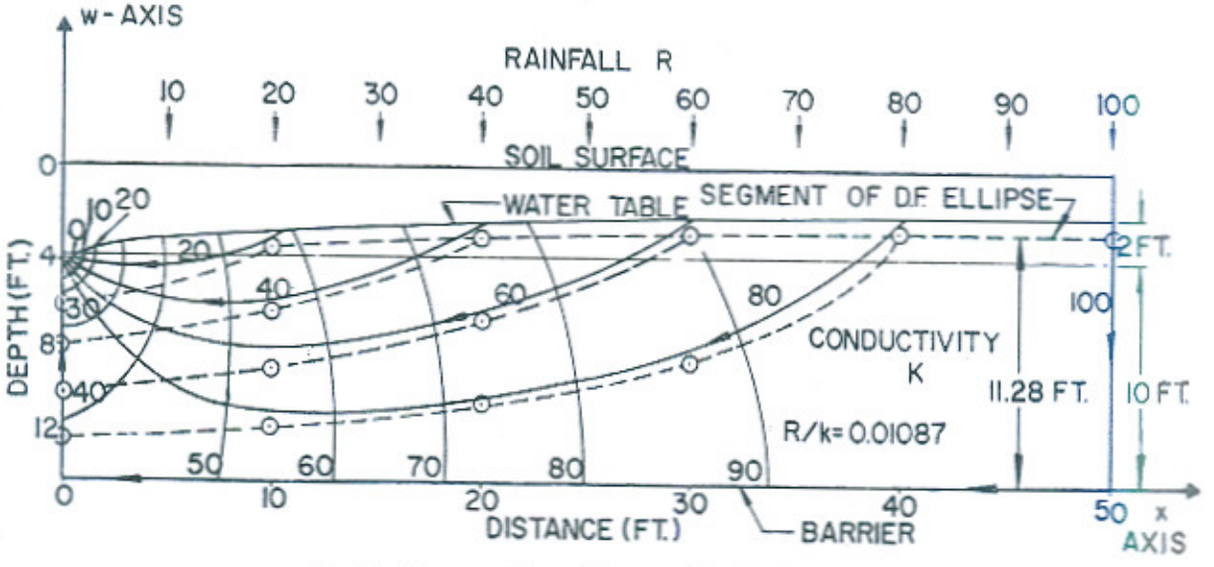


Fig. 9. Same as Figure 8, except for tile drainage

table height in Figure 8 have the value 1.395 ft. and in Figure 9, 2 ft.

The D.F. (dashed) streamlines for Figures 8 and 9 have the same geometrical locations because a slot implied by D.F. theory that must exist below the tile makes the tile situation a ditch situation. The streamline equations for the lines are easily obtained from (40) by use of the parameters noted. For the 0.8=80 per cent streamline, (40) becomes

$$w_{80}=\frac{(1-0.8)[10^2+0.01087(100x-x^2)]^{1/2}}{1-2x/100}\ \text{ft.} \tag{43}$$

for the 60 per cent streamline the equation is the same except that $(1-0.8)$ becomes $(1-0.6)$ and similarly for w_{40} and w_{20}. In Figure 8 the D.F. streamlines approximate closely the true streamlines. In Figure 9 the D.F. streamlines are in poor agreement with the true lines except near their beginnings. The D.F. Streamlines all start below the water table, seen especially in Figure 9. The reason is brought out in the next section.

D.F. THEORY GIVES TOO LOW WATER TABLE HEIGHTS

In Figure 8 the maximum water table height is 11.39 ft. above the impervious layer ; whereas the D.F. theory gives $b=[10^2+0.01087(50)^2]^{1/2}=11.28$ feet, a value 11.39—11.28=0.11 ft. too low [for $R/k=0.01087$, $a=10$ ft. and S=100 ft.]. In Figure 9, the maximum water table height is 12.00 ft. above the impervious layer ; whereas D.F. theory gives again 11.28 ft., a value 0.72 ft., too low. Thus, D.F. theory, although giving too low water table heights, gives a better approximation for water table heights for ditch drainage than for tile drainage. The physical reason why water table heights are too low for D.F. theory here is that the D.F. slots of zero impedance result in a flow medium of smaller impedance than one without the D.F. slots. This smaller impedance coupled with the fact that there is no increase in the amount of water per unit time to get to the drains means that the head (water table height) to drive the water to the drains must be less. Another reason that D.F. water tables are too low is that they don't provide for surfaces of seepage which imply higher water outflow levels at least in the neighbourhood of the outflow facility. Muskat [1946, pp. 314-316] has calculated the surface of seepage heights for seepage through a dam.

WHEN MAY D.F. THEORY PROPERLY BE USED?

If better theory is available one will not wish to use D.F. theory except as an approximation. Figure 10, from Toksöz-Kirkham [1961], takes drain sizes into account and gives an example of a drain spacing computation when $b=6.6$ *m*(eters), $a=6$ *m*, $k=1.20$ cm/day ($k/R=693$) and the drain diameter $2r$ is $2r=0.10$ *m*. The spacing S is then given in the inset of Figure 10 as

$$S=118.2\,m \tag{44}$$

D.F. theory, equation (3), gives

$$S=[4(694)(6.6^2-6^2)]^{1/2}=145.0\,m \tag{45}$$

For large values of S/a (shallow impermeable layers, wide spacings, and drain sizes not too small), D.F. theory may be used for good approximations of S. This is brought out by Figure 10 where for large S/a we see in the upper right of the figure that curves for different drain sizes all converge to a single curve. We consider in the upper right of this figure two points P and Q having a

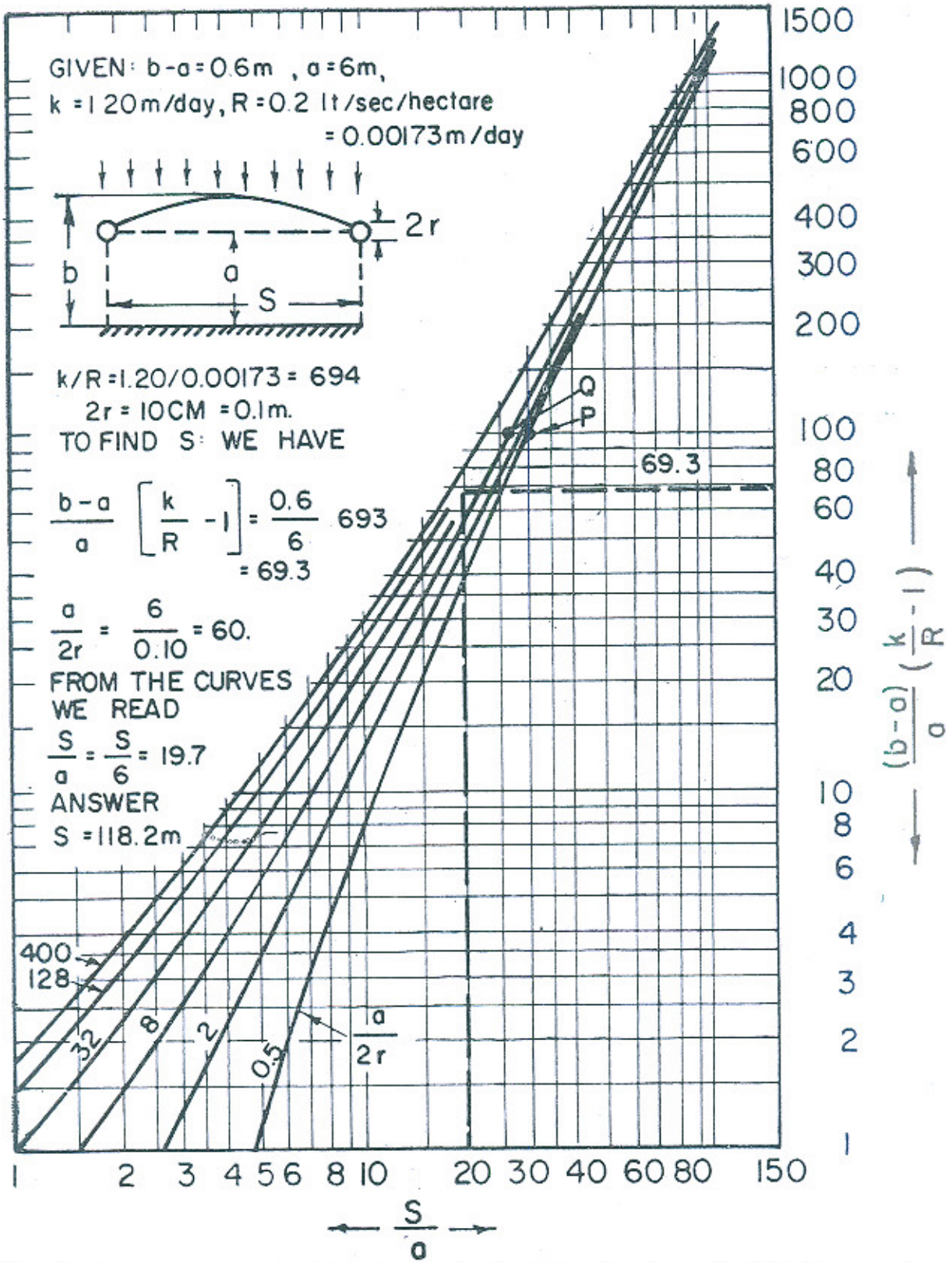


Fig. 10. Curves, and an example calculation, showing how drain radius, ignored in D.F. theory, enters into drainage design ; the curve at the far right, $a/2r=0.5$, more closely corresponds to D.F. theory than do the others ; after Toksöz and Kirkham [1961].